

Equations

2A Equations and Formulas

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2B Proportion and Percent

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Chapter Focus

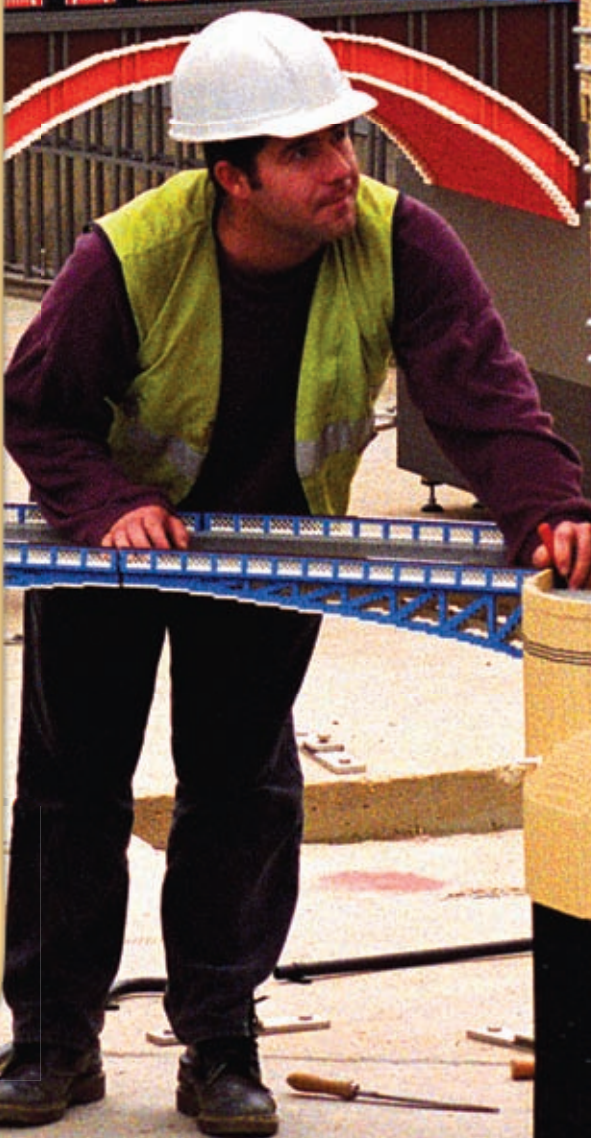
- Use properties of equality.
- Choose procedures to solve equations efficiently.
- Write and solve equations to solve problems.

All in Proportion

A common use of equations and proportional relationships is the construction of scale models.

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KEYWORD: MA7 ChProj



ARE YOU READY?

✓ Vocabulary

Match each term on the left with a definition on the right.

- | | |
|------------------------|--|
| 1. constant | A. a mathematical phrase that contains operations, numbers, and/or variables |
| 2. expression | B. a mathematical statement that two expressions are equivalent |
| 3. order of operations | C. a process for evaluating expressions |
| 4. variable | D. a symbol used to represent a quantity that can change |
| | E. a value that does not change |

✓ Order of Operations

Simplify each expression.

- | | |
|---------------------|-----------------------------|
| 5. $(7 - 3) \div 2$ | 6. $4 \cdot 6 \div 3$ |
| 7. $12 - 3 + 1$ | 8. $2 \cdot 10 \div 5$ |
| 9. $125 \div 5^2$ | 10. $7 \cdot 6 + 5 \cdot 4$ |

✓ Add and Subtract Integers

Add or subtract.

- | | | | |
|----------------|------------------|----------------|------------------|
| 11. $-15 + 19$ | 12. $-6 - (-18)$ | 13. $6 + (-8)$ | 14. $-12 + (-3)$ |
|----------------|------------------|----------------|------------------|

✓ Add and Subtract Fractions

Perform each indicated operation. Give your answer in the simplest form.

- | | | | |
|---------------------------------|----------------------------------|---------------------------------|---------------------------------|
| 15. $\frac{1}{4} + \frac{2}{3}$ | 16. $1\frac{1}{2} - \frac{3}{4}$ | 17. $\frac{3}{8} + \frac{2}{3}$ | 18. $\frac{3}{2} - \frac{2}{3}$ |
|---------------------------------|----------------------------------|---------------------------------|---------------------------------|

✓ Evaluate Expressions

Evaluate each expression for the given value of the variable.

- | | |
|---------------------------|--------------------------|
| 19. $2x + 3$ for $x = 7$ | 20. $3n - 5$ for $n = 7$ |
| 21. $13 - 4a$ for $a = 2$ | 22. $3y + 5$ for $y = 5$ |

✓ Connect Words and Algebra

- Janie bought 4 apples and 6 bananas. Each apple cost \$0.75, and each banana cost \$0.60. Write an expression representing the total cost.
- A rectangle has a width of 13 inches and a length of ℓ inches. Write an expression representing the area of the rectangle.
- Write a phrase that could be modeled by the expression $n + 2n$.

Where You've Been

Previously, you

- practiced using operations in algebra.
- used variables to represent quantities.
- wrote expressions to represent situations.
- simplified and evaluated expressions.

In This Chapter

You will study

- how to use inverse operations to solve equations containing variables.
- writing equations to represent situations.
- simplifying equations before solving.

Where You're Going

You can use the skills in this chapter

- to compare unit prices for consumer products.
- to calculate percentages in taxes, tips, interest, and commissions.
- to create or interpret scale models and drawings.
- to solve problems in science courses and all future math courses.

Key Vocabulary/Vocabulario

equation	ecuación
formula	fórmula
identity	identidad
indirect measurement	medición indirecta
literal equation	ecuación literal
percent	porcentaje
percent change	porcentaje de cambio
proportion	proporción
ratio	razón
unit rate	tasa unitaria

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word **equation** begins with the root *equa-*. List some other words that begin with *equa-*. What do all these words have in common?
2. The word *literal* means “of letters.” How might a **literal equation** be different from an equation like $3 + 5 = 8$?
3. One definition of **identity** is “exact sameness.” An equation consists of two expressions. If an equation is an *identity*, what do you think is true about the expressions?
4. The word *per* means “for each,” and the word *cent* means “hundred.” How can you use these meanings to understand the term **percent**?

Study Strategy: Use Your Own Words

Explaining a concept using your own words will help you better understand it. For example, learning to solve equations might seem difficult if the textbook doesn't use the same words that you would use.

As you work through each lesson:

- Identify the important ideas from the explanation in the book.
- Use your own words to explain the important ideas you identified.

What Arturo Reads

To evaluate an expression is to find its value.

To evaluate an algebraic expression, substitute numbers for the variables in the expression and then simplify the expression.

A replacement set is a set of numbers that can be substituted for a variable.

What Arturo Writes

*Evaluate an expression—
find the value.*

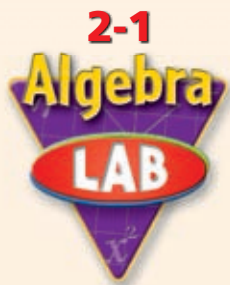
*Substitute a number for
each variable (letter), and
find the answer.*

*Replacement set—numbers
that can be substituted for
a letter.*

Try This

Rewrite each paragraph in your own words.

1. Two numbers are opposites if their sum is 0. A number and its opposite are on opposite sides of zero on a number line, but are the same distance from zero.
2. The Commutative and Associative Properties of Addition and Multiplication allow you to rearrange an expression to simplify it.
3. The terms of an expression are the parts to be added or subtracted. Like terms are terms that contain the same variables raised to the same powers. Constants are also like terms.

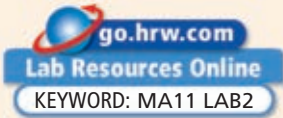


Model One-Step Equations

You can use algebra tiles and an equation mat to model and solve equations. To find the value of the variable, place or remove tiles to get the x -tile by itself on one side of the mat. You must place or remove the same number of yellow tiles or the same number of red tiles on both sides.

Use with Lesson 2-1

KEY	REMEMBER
= 1 = -1 = x	+ = 0



Activity

Use algebra tiles to model and solve $x + 6 = 2$.

MODEL	ALGEBRA
	$x + 6 = 2$
	$x + 6 + (-6) = 2 + (-6)$
	$x + 0 = 0 + (-4)$
	$x = -4$

Try This

Use algebra tiles to model and solve each equation.

1. $x + 2 = 5$

2. $x - 7 = 8$

3. $x - 5 = 9$

4. $x + 4 = 7$

2-1

Solving Equations by Adding or Subtracting



Objective

Solve one-step equations in one variable by using addition or subtraction.

Vocabulary

equation
solution of an equation

Who uses this?

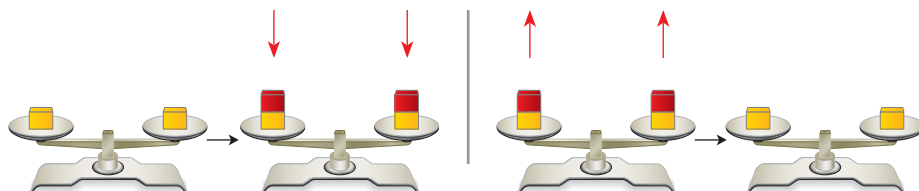
Athletes can use an equation to estimate their maximum heart rates. (See Example 4.)

An **equation** is a mathematical statement that two expressions are equal. A **solution of an equation** is a value of the variable that makes the equation true.

To find solutions, *isolate the variable*. A variable is isolated when it appears by itself on one side of an equation, and not at all on the other side. Isolate a variable by using inverse operations, which “undo” operations on the variable.

An equation is like a balanced scale. To keep the balance, perform the same operation on both sides.

Inverse Operations
Add x . \longleftrightarrow Subtract x .



EXAMPLE 1 Solving Equations by Using Addition

Solve each equation.

A $x - 10 = 4$
 $x - 10 = 4$
 $\quad \underline{+ 10} \quad \underline{+ 10}$
 $x = 14$

Since 10 is subtracted from x , add 10 to both sides to undo the subtraction.

Check $x - 10 = 4$
 $\quad \underline{14} - 10 \quad \underline{4}$
 $\quad \quad \quad 4 \quad \quad 4 \checkmark$

To check your solution, substitute 14 for x in the original equation.

B $\frac{2}{5} = m - \frac{1}{5}$
 $\frac{2}{5} = m - \frac{1}{5}$
 $\quad \underline{+ \frac{1}{5}} \quad \underline{+ \frac{1}{5}}$
 $\frac{3}{5} = m$

Since $\frac{1}{5}$ is subtracted from m , add $\frac{1}{5}$ to both sides to undo the subtraction.

Writing Math

Solutions are sometimes written in a *solution set*. For Example 1A, the solution set is $\{14\}$. For Example 1B, the solution set is $\{\frac{3}{5}\}$.



Solve each equation. Check your answer.

1a. $n - 3.2 = 5.6$

1b. $-6 = k - 6$

1c. $16 = m - 9$

EXAMPLE 2 Solving Equations by Using Subtraction

Solve each equation. Check your answer.

A $x + 7 = 9$

$$x + 7 = 9$$

$$\begin{array}{r} -7 \quad -7 \\ x + 7 = 9 \\ \hline x = 2 \end{array}$$

Since 7 is added to x , subtract 7 from both sides to undo the addition.

Check $x + 7 = 9$

$$\begin{array}{r|l} 2 + 7 & 9 \\ \hline 9 & 9 \checkmark \end{array}$$

To check your solution, substitute 2 for x in the original equation.

B $0.7 = r + 0.4$

$$0.7 = r + 0.4$$

$$\begin{array}{r} -0.4 \quad -0.4 \\ 0.7 = r + 0.4 \\ \hline 0.3 = r \end{array}$$

Since 0.4 is added to r , subtract 0.4 from both sides to undo the addition.

Check $0.7 = r + 0.4$

$$\begin{array}{r|l} 0.7 & 0.3 + 0.4 \\ \hline 0.7 & 0.7 \checkmark \end{array}$$

To check your solution, substitute 0.3 for r in the original equation.



Solve each equation. Check your answer.

2a. $d + \frac{1}{2} = 1$

2b. $-5 = k + 5$

2c. $6 + t = 14$

Remember that subtracting is the same as adding the opposite. When solving equations, you will sometimes find it easier to add an opposite to both sides instead of subtracting. For example, this method may be useful when the equation contains negative numbers.

EXAMPLE 3 Solving Equations by Adding the Opposite

Solve $-8 + b = 2$.

$$-8 + b = 2$$

$$\begin{array}{r} +8 \quad +8 \\ -8 + b = 2 \\ \hline b = 10 \end{array}$$

Since -8 is added to b , add 8 to both sides.



Solve each equation. Check your answer.

3a. $-2.3 + m = 7$

3b. $-\frac{3}{4} + z = \frac{5}{4}$

3c. $-11 + x = 33$

Student to Student

Zero As a Solution



Ama Walker
Carson High School

I used to get confused when I got a solution of 0. But my teacher reminded me that 0 is a number just like any other number, so it can be a solution of an equation. Just check your answer and see if it works.

$$\begin{array}{r} x + 6 = 6 \\ -6 \quad -6 \\ \hline x = 0 \end{array}$$

Check $x + 6 = 6$

$$\begin{array}{r|l} 0 + 6 & 6 \\ \hline 6 & 6 \checkmark \end{array}$$

EXAMPLE 4 Fitness Application



A person's maximum heart rate is the highest rate, in beats per minute, that the person's heart should reach. One method to estimate maximum heart rate states that your age added to your maximum heart rate is 220. Using this method, write and solve an equation to find the maximum heart rate of a 15-year-old.

Age added to maximum heart rate is 220.

$a + r = 220$

$$a + r = 220$$

$$15 + r = 220$$

$$\begin{array}{r} 15 + r = 220 \\ -15 \quad -15 \\ \hline r = 205 \end{array}$$

Write an equation to represent the relationship.

Substitute 15 for a . Since 15 is added to r , subtract 15 from both sides to undo the addition.

The maximum heart rate for a 15-year-old is 205 beats per minute. Since age added to maximum heart rate is 220, the answer should be less than 220. So 205 is a reasonable answer.



4. **What if...?** Use the method above to find a person's age if the person's maximum heart rate is 185 beats per minute.

The properties of equality allow you to perform inverse operations, as in the previous examples. These properties say that you can perform the same operation on both sides of an equation.



Properties of Equality

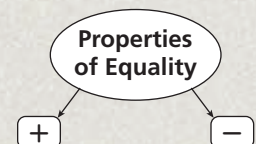
WORDS	NUMBERS	ALGEBRA
Addition Property of Equality You can add the same number to both sides of an equation, and the statement will still be true.	$3 = 3$ $3 + 2 = 3 + 2$ $5 = 5$	$a = b$ $a + c = b + c$
Subtraction Property of Equality You can subtract the same number from both sides of an equation, and the statement will still be true.	$7 = 7$ $7 - 5 = 7 - 5$ $2 = 2$	$a = b$ $a - c = b - c$

THINK AND DISCUSS

1. Describe how the Addition and Subtraction Properties of Equality are like a balanced scale.



2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of an equation that can be solved by using the given property, and solve it.



GUIDED PRACTICE

1. **Vocabulary** Will the *solution of an equation* such as $x - 3 = 9$ be a variable or a number? Explain.

Solve each equation. Check your answer.

SEE EXAMPLE 1

p. 77

2. $s - 5 = 3$

3. $17 = w - 4$

4. $k - 8 = -7$

5. $x - 3.9 = 12.4$

6. $8.4 = y - 4.6$

7. $\frac{3}{8} = t - \frac{1}{8}$

SEE EXAMPLE 2

p. 78

8. $t + 5 = -25$

9. $9 = s + 9$

10. $42 = m + 36$

11. $2.8 = z + 0.5$

12. $b + \frac{2}{3} = 2$

13. $n + 1.8 = 3$

SEE EXAMPLE 3

p. 78

14. $-10 + d = 7$

15. $20 = -12 + v$

16. $-46 + q = 5$

17. $2.8 = -0.9 + y$

18. $-\frac{2}{3} + c = \frac{2}{3}$

19. $-\frac{5}{6} + p = 2$

SEE EXAMPLE 4

p. 79

20. **Geology** In 1673, the Hope diamond was reduced from its original weight by about 45 carats, resulting in a diamond weighing about 67 carats. Write and solve an equation to find how many carats the original diamond weighed. Show that your answer is reasonable.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
21–30	1
31–40	2
41–48	3
49	4

Solve each equation. Check your answer.

21. $1 = k - 8$

22. $u - 15 = -8$

23. $x - 7 = 10$

24. $-9 = p - 2$

25. $\frac{3}{7} = p - \frac{1}{7}$

26. $q - 0.5 = 1.5$

27. $6 = t - 4.5$

28. $4\frac{2}{3} = r - \frac{1}{3}$

29. $6 = x - 3$

30. $1.75 = k - 0.75$

31. $19 + a = 19$

32. $4 = 3.1 + y$

33. $m + 20 = 3$

34. $-12 = c + 3$

35. $v + 2300 = -800$

36. $b + 42 = 300$

37. $3.5 = n + 4$

38. $b + \frac{1}{2} = \frac{1}{2}$

39. $x + 5.34 = 5.39$

40. $2 = d + \frac{1}{4}$

41. $-12 + f = 3$

42. $-9 = -4 + g$

43. $-1200 + j = 345$

44. $90 = -22 + a$

45. $26 = -4 + y$

46. $1\frac{3}{4} = -\frac{1}{4} + w$

47. $-\frac{1}{6} + h = \frac{1}{6}$

48. $-5.2 + a = -8$

49. **Finance** Luis deposited \$500 into his bank account. He now has \$4732. Write and solve an equation to find how much was in his account before the deposit. Show that your answer is reasonable.

50. **/// ERROR ANALYSIS ///** Below are two possible solutions to $x + 12.5 = 21.6$. Which is incorrect? Explain the error.

A

$$\begin{array}{r} x + 12.5 = 21.6 \\ - 12.5 - 12.5 \\ \hline x = 9.1 \end{array}$$

B

$$\begin{array}{r} x + 12.5 = 21.6 \\ + 12.5 + 12.5 \\ \hline x = 34.1 \end{array}$$

Write an equation to represent each relationship. Then solve the equation.

51. Ten less than a number is equal to 12.
52. A number decreased by 13 is equal to 7.
53. Eight more than a number is 16.
54. A number minus 3 is -8 .
55. The sum of 5 and a number is 6.
56. Two less than a number is -5 .
57. The difference of a number and 4 is 9.



Geology

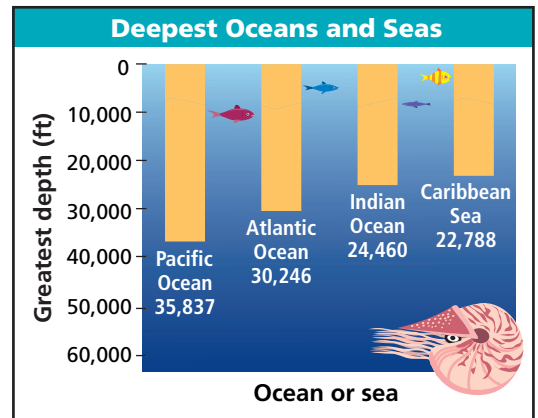


The ocean depths are home to many odd-looking creatures. The anglerfish pictured above, known as the common black devil, may appear menacing but reaches a maximum length of only about 5 inches.

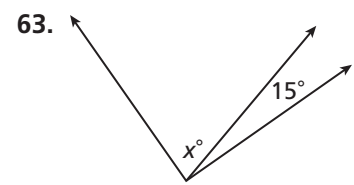
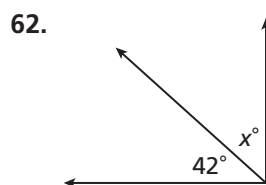
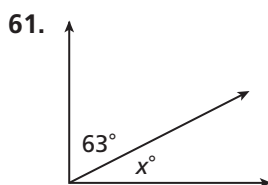
58. Geology The sum of the Atlantic Ocean's average depth (in feet) and its greatest depth is 43,126. Use the information in the graph to write and solve an equation to find the average depth of the Atlantic Ocean. Show that your answer is reasonable.

59. School Helene's marching band needs money to travel to a competition. Band members have raised \$560. They need to raise a total of \$1680. Write and solve an equation to find how much more they need. Show that your answer is reasonable.

60. Economics When you receive a loan to make a purchase, you often must make a down payment in cash. The amount of the loan is the purchase cost minus the down payment. Riva made a down payment of \$1500 on a used car. She received a loan of \$2600. Write and solve an equation to find the cost of the car. Show that your answer is reasonable.



Geometry The angles in each pair are complementary. Write and solve an equation to find each value of x . (*Hint: The measures of complementary angles add to 90° .*)



MULTI-STEP TEST PREP



64. This problem will prepare you for the Multi-Step Test Prep on page 118.

Rates are often used to describe how quickly something is moving or changing.

- a. A wildfire spreads at a rate of 1000 acres per day. How many acres will the fire cover in 2 days? Show that your answer is reasonable.
- b. How many acres will the fire cover in 5 days? Explain how you found your answer.
- c. Another wildfire spread for 7 days and covered a total of 780 square miles. How can you estimate the number of square miles the fire covered per day?

65. **Statistics** The range of a set of scores is 28, and the lowest score is 47. Write and solve an equation to find the highest score. (*Hint:* In a data set, the range is the difference between the highest and the lowest values.) Show that your answer is reasonable.
66. **Write About It** Describe a real-world situation that can be modeled by $x + 5 = 25$. Tell what the variable represents in your situation. Then solve the equation and tell what the solution means in the context of your problem.
67. **Critical Thinking** Without solving, tell whether the solution of $-3 + z = 10$ will be greater than 10 or less than 10. Explain.



68. Which situation is best represented by $x - 32 = 8$?
- (A) Logan withdrew \$32 from her bank account. After her withdrawal, her balance was \$8. How much was originally in her account?
- (B) Daniel has 32 baseball cards. Joseph has 8 fewer baseball cards than Daniel. How many baseball cards does Joseph have?
- (C) Room A contains 32 desks. Room B has 8 fewer desks. How many desks are in Room B?
- (D) Janelle bought a bag of 32 craft sticks for a project. She used 8 craft sticks. How many craft sticks does she have left?
69. For which equation is $a = 8$ a solution?
- (F) $15 - a = 10$ (G) $10 + a = 23$ (H) $a - 18 = 26$ (J) $a + 8 = 16$
70. **Short Response** Julianna used a gift card to pay for an \$18 haircut. The remaining balance on the card was \$22.
- a. Write an equation that can be used to determine the original value of the card.
- b. Solve your equation to find the original value of the card.

CHALLENGE AND EXTEND

Solve each equation. Check your answer.

71. $3\frac{1}{5} + b = \frac{4}{5}$ 72. $x - \frac{7}{4} = \frac{2}{3}$ 73. $x + \frac{7}{4} = \frac{2}{3}$ 74. $x - \frac{4}{9} = \frac{4}{9}$
75. If $p - 4 = 2$, find the value of $5p - 20$. 76. If $t + 6 = 21$, find the value of $-2t$.
77. If $x + 3 = 15$, find the value of $18 + 6x$. 78. If $2 + n = -11$, find the value of $6n$.

SPIRAL REVIEW

Multiply or divide. (*Lesson 1-3*)

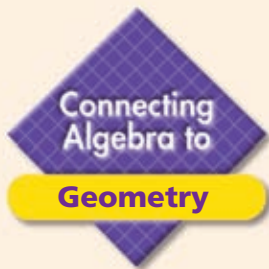
79. $-63 \div (-7)$ 80. $\frac{3}{7} \div \left(-\frac{4}{7}\right)$ 81. $(-12)(-6)$

Give the side length of a square with the given area. (*Lesson 1-5*)

82. 225 m^2 83. 36 ft^2 84. 100 cm^2



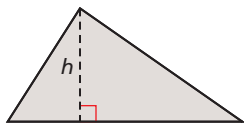
Simplify each expression. (*Lesson 1-6*)

85. $8[-5 - (3 + 2)]$ 86. $1 - [4^2 - (12 - 15)^2]$ 87. $\frac{-12 + (-6)}{6}$



Area of Composite Figures

Review the area formulas for squares, rectangles, and triangles in the table below.

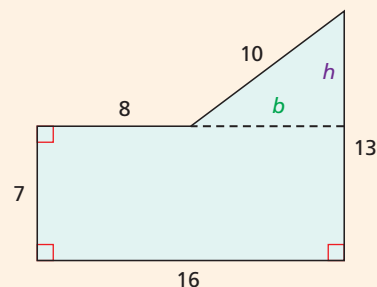
Squares	Rectangles	Triangles
 <p style="text-align: center;">s</p> <p style="text-align: center;">$A = s^2$</p>	 <p style="text-align: center;">l</p> <p style="text-align: center;">$A = \ell w$</p>	 <p style="text-align: center;">b</p> <p style="text-align: center;">$A = \frac{1}{2}bh$</p>

A *composite figure* is a figure that is composed of basic shapes. You can divide composite figures into combinations of squares, rectangles, and triangles to find their areas.

Example

Find the area of the figure shown.

Divide the figure into a rectangle and a right triangle. Notice that you do not know the base or the height of the triangle. Use b and h to represent these lengths.



The bottom of the rectangle is 16 units long; the top of the rectangle is 8 units long plus the base of the triangle. Use this information to write and solve an equation.

$$\begin{array}{r} b + 8 = 16 \\ \underline{-8 \quad -8} \\ b = 8 \end{array}$$

The right side of the figure is 13 units long; 7 units from the rectangle plus the height of the triangle. Use this information to write and solve an equation.

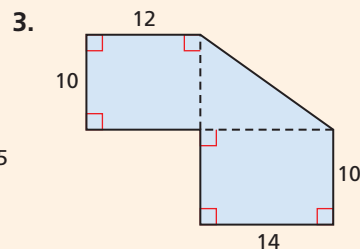
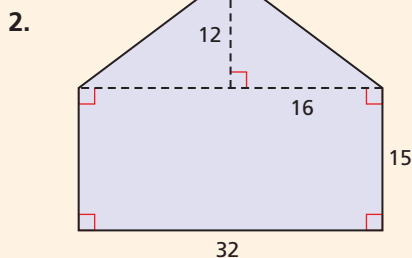
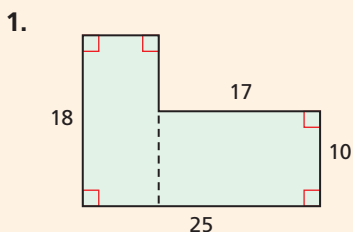
$$\begin{array}{r} h + 7 = 13 \\ \underline{-7 \quad -7} \\ h = 6 \end{array}$$

The area of the figure is the sum of the areas of the rectangle and the triangle.

$$\begin{array}{l} \text{Area of rectangle} \\ \downarrow \\ \text{Area of triangle} \\ A = \ell w + \frac{1}{2}bh \\ A = 16(7) + \frac{1}{2}(8)(6) \\ A = 112 + 24 \\ A = 136 \text{ square units} \end{array}$$

Try This

Find the area of each composite figure.



2-2

Solving Equations by Multiplying or Dividing

Objective

Solve one-step equations in one variable by using multiplication or division.

Who uses this?

Pilots can make quick calculations by solving one-step equations. (See Example 4.)



Solving an equation that contains multiplication or division is similar to solving an equation that contains addition or subtraction. Use inverse operations to undo the operations on the variable.

Remember that an equation is like a balanced scale. To keep the balance, whatever you do on one side of the equation, you must also do on the other side.

Inverse Operations

Multiply by x . \longleftrightarrow Divide by x .

EXAMPLE 1 Solving Equations by Using Multiplication

Solve each equation. Check your answer.

A $-4 = \frac{k}{-5}$
 $(-5)(-4) = (-5)\left(\frac{k}{-5}\right)$
 $20 = k$

Since k is divided by -5 , multiply both sides by -5 to undo the division.

Check $-4 = \frac{k}{-5}$
 $-4 \mid \frac{20}{-5}$
 $-4 \mid -4 \checkmark$

To check your solution, substitute 20 for k in the original equation.

B $\frac{m}{3} = 1.5$
 $(3)\left(\frac{m}{3}\right) = (3)(1.5)$
 $m = 4.5$

Since m is divided by 3, multiply both sides by 3 to undo the division.

Check $\frac{m}{3} = 1.5$
 $\frac{4.5}{3} \mid 1.5$
 $1.5 \mid 1.5 \checkmark$

To check your solution, substitute 1.5 for m in the original equation.



Solve each equation. Check your answer.

1a. $\frac{p}{5} = 10$

1b. $-13 = \frac{y}{3}$

1c. $\frac{c}{8} = 7$

EXAMPLE 2 Solving Equations by Using Division

Solve each equation. Check your answers.

A $7x = 56$

$$\frac{7x}{7} = \frac{56}{7}$$

$$x = 8$$

Since x is multiplied by 7, divide both sides by 7 to undo the multiplication.

Check $7x = 56$

$$\begin{array}{r|l} 7(8) & 56 \\ \hline 56 & 56 \checkmark \end{array}$$

To check your solution, substitute 8 for x in the original equation.

B $13 = -2w$

$$\frac{13}{-2} = \frac{-2w}{-2}$$

$$-6.5 = w$$

Since w is multiplied by -2 , divide both sides by -2 to undo the multiplication.

Check $13 = -2w$

$$\begin{array}{r|l} 13 & -2(-6.5) \\ \hline 13 & 13 \checkmark \end{array}$$

To check your solution, substitute -6.5 for w in the original equation.



Solve each equation. Check your answer.

2a. $16 = 4c$

2b. $0.5y = -10$

2c. $15k = 75$

Remember that dividing is the same as multiplying by the reciprocal. When solving equations, you will sometimes find it easier to multiply by a reciprocal instead of dividing. This is often true when an equation contains fractions.

EXAMPLE 3 Solving Equations That Contain Fractions

Solve each equation.

A $\frac{5}{9}v = 35$

$$\left(\frac{9}{5}\right)\frac{5}{9}v = \left(\frac{9}{5}\right)35$$

$$v = 63$$

The reciprocal of $\frac{5}{9}$ is $\frac{9}{5}$. Since v is multiplied by $\frac{5}{9}$, multiply both sides by $\frac{9}{5}$.

B $\frac{5}{2} = \frac{4y}{3}$

$$\frac{5}{2} = \frac{4y}{3}$$

$$\frac{5}{2} = \frac{4}{3}y$$

$\frac{4y}{3}$ is the same as $\frac{4}{3}y$.

$$\left(\frac{3}{4}\right)\frac{5}{2} = \left(\frac{3}{4}\right)\frac{4}{3}y$$

$$\frac{15}{8} = y$$

The reciprocal of $\frac{4}{3}$ is $\frac{3}{4}$. Since y is multiplied by $\frac{4}{3}$, multiply both sides by $\frac{3}{4}$.



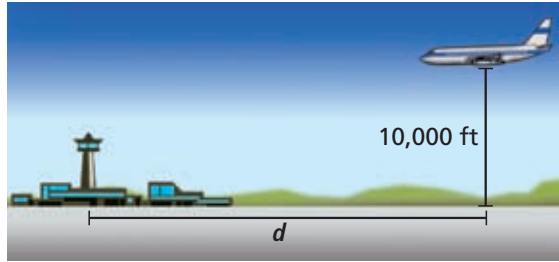
Solve each equation. Check your answer.

3a. $-\frac{1}{4} = \frac{1}{5}b$

3b. $\frac{4j}{6} = \frac{2}{3}$

3c. $\frac{1}{6}w = 102$

EXAMPLE 4 Aviation Application



The distance in miles from the airport that a plane should begin descending, divided by 3, equals the plane's height above the ground in thousands of feet. If a plane is 10,000 feet above the ground, write and solve an equation to find the distance at which the pilot should begin descending.

Caution!

The equation uses the plane's height above the ground in *thousands* of feet. So substitute 10 for h , not 10,000.

Distance divided by 3 equals height in thousands of feet.

$$\frac{d}{3} = h$$

Write an equation to represent the relationship.

$$\frac{d}{3} = 10$$

Substitute 10 for h . Since d is divided by 3, multiply both sides by 3 to undo the division.

$$(3)\frac{d}{3} = (3)10$$

$$d = 30$$

The pilot should begin descending 30 miles from the airport.



4. **What if...?** A plane began descending 45 miles from the airport. Use the equation above to find how high the plane was flying when the descent began.

You have now used four properties of equality to solve equations. These properties are summarized in the box below.

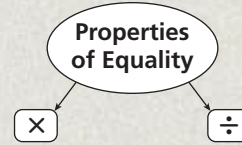


Properties of Equality

WORDS	NUMBERS	ALGEBRA
Addition Property of Equality You can add the same number to both sides of an equation, and the statement will still be true.	$3 = 3$ $3 + 2 = 3 + 2$ $5 = 5$	$a = b$ $a + c = b + c$
Subtraction Property of Equality You can subtract the same number from both sides of an equation, and the statement will still be true.	$7 = 7$ $7 - 5 = 7 - 5$ $2 = 2$	$a = b$ $a - c = b - c$
Multiplication Property of Equality You can multiply both sides of an equation by the same number, and the statement will still be true.	$6 = 6$ $6(3) = 6(3)$ $18 = 18$	$a = b$ $ac = bc$
Division Property of Equality You can divide both sides of an equation by the same nonzero number, and the statement will still be true.	$8 = 8$ $\frac{8}{4} = \frac{8}{4}$ $2 = 2$	$a = b$ $(c \neq 0)$ $\frac{a}{c} = \frac{b}{c}$

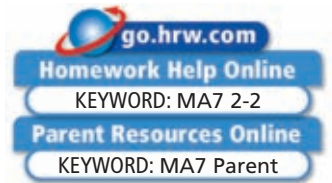
THINK AND DISCUSS

1. Tell how the Multiplication and Division Properties of Equality are similar to the Addition and Subtraction Properties of Equality.
2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of an equation that can be solved by using the given property, and solve it.



2-2

Exercises



GUIDED PRACTICE

Solve each equation. Check your answer.

SEE EXAMPLE 1
p. 84

1. $\frac{k}{4} = 8$

2. $\frac{z}{3} = -9$

3. $-2 = \frac{w}{-7}$

4. $6 = \frac{t}{-5}$

5. $\frac{g}{1.9} = 10$

6. $2.4 = \frac{b}{5}$

SEE EXAMPLE 2
p. 85

7. $4x = 28$

8. $-64 = 8c$

9. $-9j = -45$

10. $84 = -12a$

11. $4m = 10$

12. $2.8 = -2h$

SEE EXAMPLE 3
p. 85

13. $\frac{1}{2}d = 7$

14. $15 = \frac{5}{6}f$

15. $\frac{2}{3}s = -6$

16. $9 = -\frac{3}{8}r$

17. $\frac{1}{10} = \frac{4}{5}y$

18. $\frac{1}{4}v = -\frac{3}{4}$

SEE EXAMPLE 4
p. 86

19. **Recreation** The Baseball Birthday Batter Package at a minor league ballpark costs \$192. The package includes tickets, drinks, and cake for a group of 16 children. Write and solve an equation to find the cost per child.

20. **Nutrition** An orange contains about 80 milligrams of vitamin C, which is 10 times as much as an apple contains. Write and solve an equation to find the amount of vitamin C in an apple.

PRACTICE AND PROBLEM SOLVING

Solve each equation. Check your answer.

21. $\frac{x}{2} = 12$

22. $-40 = \frac{b}{5}$

23. $-\frac{j}{6} = 6$

24. $-\frac{n}{3} = -4$

25. $-\frac{q}{5} = 30$

26. $1.6 = \frac{d}{3}$

27. $\frac{v}{10} = 5.5$

28. $\frac{h}{8.1} = -4$

29. $5t = -15$

30. $49 = 7c$

31. $-12 = -12u$

32. $-7m = 63$

33. $-52 = -4c$

34. $11 = -2z$

35. $5f = 1.5$

36. $-8.4 = -4n$

Independent Practice

For Exercises	See Example
21–28	1
29–36	2
37–44	3
45	4

Extra Practice

Skills Practice p. S6

Application Practice p. S29

Solve each equation. Check your answer.

37. $\frac{5}{2}k = 5$ 38. $-9 = \frac{3}{4}d$ 39. $-\frac{5}{8}b = 10$ 40. $-\frac{4}{5}g = -12$
 41. $\frac{4}{7}t = -2$ 42. $-\frac{4}{5}p = \frac{2}{3}$ 43. $\frac{2}{3} = -\frac{1}{3}q$ 44. $-\frac{5}{8} = -\frac{3}{4}a$

45. **Finance** After taxes, Alexandra’s take-home pay is $\frac{7}{10}$ of her salary before taxes. Write and solve an equation to find Alexandra’s salary before taxes for the pay period that resulted in \$392 of take-home pay.
 46. **Earth Science** Your weight on the Moon is about $\frac{1}{6}$ of your weight on Earth. Write and solve an equation to show how much a person weighs on Earth if he weighs 16 pounds on the Moon. How could you check that your answer is reasonable?
 47. **/// ERROR ANALYSIS ///** For the equation $\frac{x}{3} = 15$, a student found the value of x to be 5. Explain the error. What is the correct answer?



Geometry The perimeter of a square is given. Write and solve an equation to find the length of each side of the square.

48. $P = 36$ in. 49. $P = 84$ in. 50. $P = 100$ yd 51. $P = 16.4$ cm



Statistics



American Robert P. Wadlow (1918–1940) holds the record for world’s tallest man—8 ft 11.1 in. He also holds world records for the largest feet and hands.

Source: Guinness World Records 2005

Write an equation to represent each relationship. Then solve the equation.

52. Five times a number is 45.
 53. A number multiplied by negative 3 is 12.
 54. A number divided by 4 is equal to 10.
 55. The quotient of a number and 3 is negative 8.
 56. **Statistics** The mean height of the students in Marta’s class is 60 in. There are 18 students in her class. Write and solve an equation to find the total measure of all students’ heights. (*Hint:* The mean is found by dividing the sum of all data values by the number of data values.)
 57. **Finance** Lisa earned \$6.25 per hour at her after-school job. Each week she earned \$50. Write and solve an equation to show how many hours she worked each week.
 58. **Critical Thinking** Will the solution of $\frac{x}{2.1} = 4$ be greater than 4 or less than 4? Explain.
 59. **Consumer Economics** Dion’s long-distance phone bill was \$13.80. His long-distance calls cost \$0.05 per minute. Write and solve an equation to find the number of minutes he was charged for. Show that your answer is reasonable.
 60. **Nutrition** An 8 oz cup of coffee has about 184 mg of caffeine. This is 5 times as much caffeine as in a 12 oz soft drink. Write and solve an equation to find about how much caffeine is in a 12 oz caffeinated soft drink. Round your answer to the nearest whole number. Show that your answer is reasonable.

Use the equation $8y = 4x$ to find y for each value of x .

	x	$4x$	$8y = 4x$	y
61.	-4	$4(-4) = -16$	$8y = -16$	■
62.	-2	■	■	■
63.	0	■	■	■
64.	2	■	■	■



65. This problem will prepare you for the Multi-Step Test Prep on page 118.
- The formula for the mean of a data set is $\text{mean} = \frac{\text{sum of data values}}{\text{number of data values}}$. One summer, there were 1926 wildfires in Arizona. Which value does this number represent in the formula?
 - The mean number of acres burned by each wildfire was 96.21. Which value does this number represent in the formula?
 - Use the formula and information given to find how many acres were burned by wildfires in Arizona that summer. Round your answer to the nearest acre. Show that your answer is reasonable.


Solve each equation. Check your answer.

66. $\frac{m}{6} = 1$ 67. $4x = 28$ 68. $1.2h = 14.4$ 69. $\frac{1}{5}x = 121$
70. $2w = 26$ 71. $4b = \frac{3}{4}$ 72. $5y = 11$ 73. $\frac{n}{1.9} = 3$

Biology Use the table for Exercises 74 and 75.

Average Weight			
Animal	At Birth (g)	Adult Female (g)	Adult Male (g)
Hamster	2	130	110
Guinea pig	85	800	1050
Rat	5	275	480

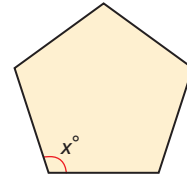


74. The mean weight of an adult male rat is 16 times the mean weight of an adult male mouse. Write and solve an equation to find the mean weight of an adult male mouse. Show that your answer is reasonable.
75. On average, a hamster at birth weighs $\frac{2}{3}$ the weight of a gerbil at birth. Write and solve an equation to find the average weight of a gerbil at birth. Show that your answer is reasonable.
-  76. **Write About It** Describe a real-world situation that can be modeled by $3x = 42$. Solve the equation and tell what the solution means in the context of your problem.



77. Which situation does NOT represent the equation $\frac{d}{2} = 10$?
- Leo bought a box of pencils. He gave half of them to his brother. They each got 10 pencils. How many pencils were in the box Leo bought?
 - Kasey evenly divided her money from baby-sitting into two bank accounts. She put \$10 in each account. How much did Kasey earn?
 - Gilbert cut a piece of ribbon into 2-inch strips. When he was done, he had ten 2-inch strips. How long was the ribbon to start?
 - Mattie had 2 more CDs than her sister Leona. If Leona had 10 CDs, how many CDs did Mattie have?
78. Which equation below shows a correct first step for solving $3x = -12$?
- $3x + 3 = -12 + 3$
 - $3(3x) = 3(-12)$
 - $3x - 3 = -12 - 3$
 - $\frac{3x}{3} = \frac{-12}{3}$

79. In a regular pentagon, all of the angles are equal in measure. The sum of the angle measures is 540° . Which of the following equations could be used to find the measure of each angle?



- (A) $\frac{x}{540} = 5$ (C) $540x = 5$
 (B) $5x = 540$ (D) $\frac{x}{5} = 540$
80. For which equation is $m = 10$ a solution?
 (F) $5 = 2m$ (G) $5m = 2$ (H) $\frac{m}{2} = 5$ (J) $\frac{m}{10} = 2$
81. **Short Response** Luisa bought 6 cans of cat food that each cost the same amount. She spent a total of \$4.80.
- Write an equation to determine the cost of one can of cat food. Tell what each part of your equation represents.
 - Solve your equation to find the cost of one can of cat food. Show each step.

CHALLENGE AND EXTEND

Solve each equation. Check your answer.

82. $\left(3\frac{1}{5}\right)b = \frac{4}{5}$ 83. $\left(1\frac{1}{3}\right)x = 2\frac{2}{3}$ 84. $\left(5\frac{4}{5}\right)x = -52\frac{1}{5}$
 85. $\left(-2\frac{9}{10}\right)k = -26\frac{1}{10}$ 86. $\left(1\frac{2}{3}\right)w = 15\frac{1}{3}$ 87. $\left(2\frac{1}{4}\right)d = 4\frac{1}{2}$

Find each indicated value.

88. If $2p = 4$, find the value of $6p + 10$. 89. If $6t = 24$, find the value of $-5t$.
 90. If $3x = 15$, find the value of $12 - 4x$. 91. If $\frac{n}{2} = -11$, find the value of $6n$.
 92. To isolate x in $ax = b$, what should you divide both sides by?
 93. To isolate x in $\frac{x}{a} = b$, what operation should you perform on both sides of the equation?
 94. **Travel** The formula $d = rt$ gives the distance d that is traveled at a rate r in time t .
 a. If $d = 400$ and $r = 25$, what is the value of t ?
 b. If $d = 400$ and $r = 50$, what is the value of t ?
 c. **What if...?** How did t change when r increased from 25 to 50?
 d. **What if...?** If r is doubled while d remains the same, what is the effect on t ?

SPIRAL REVIEW

Find each square root. (Lesson 1-5)

95. $\sqrt{144}$ 96. $\sqrt{196}$ 97. $\sqrt{625}$ 98. $-\sqrt{9}$

Write and solve an equation that could be used to answer each question. (Lesson 2-1)

99. Lisa's age plus Sean's age is 17. Sean is 11 years old. How old is Lisa?
 100. The length of a rectangle is 6 feet more than the width of the rectangle. The length is 32 feet. What is the width of the rectangle?

Solve each equation. Check your answer. (Lesson 2-1)

101. $2 = a - 4$ 102. $x - 12 = -3$ 103. $z - 5 = 11$ 104. $-4 = x + 5$



Solve Equations by Graphing

You can use graphs to solve equations. As you complete this activity, you will learn some of the connections between graphs and equations.

Use with Lesson 2-3

Activity

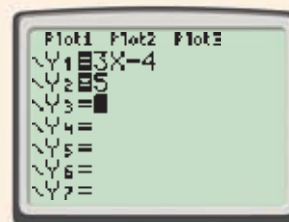
Solve $3x - 4 = 5$.

- 1 Press **Y=**. In Y_1 , enter the left side of the equation, $3x - 4$.

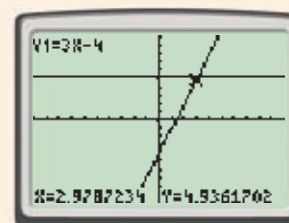
Y= 3 **X,T,θ,n** **-** 4 **ENTER**

In Y_2 , enter the right side of the equation, 5.

Y= 5 **ENTER**

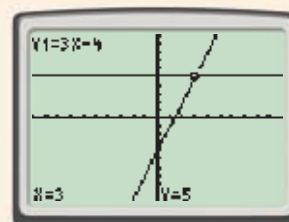


- 2 Press **GRAPH**. Press **TRACE**. The display will show the x - and y -values of a point on the first line. Press the right arrow key several times. Notice that the x - and y -values change.



- 3 Continue to trace as close as possible to the intersection of the two lines. The x -value of this point 2.9787..., is an approximation of the solution. The solution is about 3.

- 4 While still in trace mode, to check, press **3** **ENTER**. The display will show the y -value when $x = 3$. When $x = 3$, $y = 5$. So 3 is the solution. You can also check this solution by substituting 3 for x in the equation:



Check

$3x - 4 = 5$	
$3(3) - 4$	5
$9 - 4$	5
5	5 ✓

Try This

- Solve $3x - 4 = 2$, $3x - 4 = 17$, and $3x - 4 = -7$ by graphing.
- What does each line represent?
- Describe a procedure for finding the solution of $3x - 4 = y$ for any value of y .
- Solve $\frac{1}{2}x - 7 = -4$, $\frac{1}{2}x - 7 = 0$, and $\frac{1}{2}x - 7 = 2$ by graphing.



2-3

Solving Two-Step and Multi-Step Equations



Objective

Solve equations in one variable that contain more than one operation.

Why learn this?

Equations containing more than one operation can model real-world situations, such as the cost of a music club membership.

Alex belongs to a music club. In this club, students can buy a student discount card for \$19.95. This card allows them to buy CDs for \$3.95 each. After one year, Alex has spent \$63.40.

To find the number of CDs c that Alex bought, you can solve an equation.

$$\text{Cost per CD} \rightarrow 3.95c + \overset{\substack{\text{Cost of discount card} \\ \downarrow}}{19.95} = 63.40 \leftarrow \text{Total cost}$$

Notice that this equation contains multiplication and addition. Equations that contain more than one operation require more than one step to solve. Identify the operations in the equation and the order in which they are applied to the variable. Then use inverse operations and work backward to undo them one at a time.

$$3.95c + 19.95 = 63.40$$

Operations in the Equation

- 1 First c is multiplied by 3.95.
- 2 Then 19.95 is added.

To Solve

- 1 Subtract 19.95 from both sides of the equation.
- 2 Then divide both sides by 3.95.

Work Backward

EXAMPLE 1 Solving Two-Step Equations

Solve $10 = 6 - 2x$. Check your answer.

$$10 = 6 - 2x$$

$$\underline{-6} \quad \underline{-6}$$

$$4 = -2x$$

$$\underline{\frac{4}{-2}} = \underline{\frac{-2x}{-2}}$$

$$-2 = 1x$$

$$-2 = x$$

First x is multiplied by -2 . Then 6 is added.

Work backward: Subtract 6 from both sides.

Since x is multiplied by -2 , divide both sides by -2 to undo the multiplication.

Check

$10 = 6 - 2x$	
10	$6 - 2(-2)$
10	$6 - (-4)$
10	$10 \checkmark$



Solve each equation. Check your answer.

- 1a. $-4 + 7x = 3$ 1b. $1.5 = 1.2y - 5.7$ 1c. $\frac{n}{7} + 2 = 2$

EXAMPLE 2 Solving Two-Step Equations That Contain Fractions

Solve $\frac{q}{15} - \frac{1}{5} = \frac{3}{5}$.

Method 1 Use fraction operations.

$$\frac{q}{15} - \frac{1}{5} = \frac{3}{5}$$

Since $\frac{1}{5}$ is subtracted from $\frac{q}{15}$, add $\frac{1}{5}$ to both sides to undo the subtraction.

$$\frac{q}{15} + \frac{1}{5} - \frac{1}{5} = \frac{3}{5} + \frac{1}{5}$$

$$\frac{q}{15} = \frac{4}{5}$$

Since q is divided by 15, multiply both sides by 15 to undo the division.

$$15\left(\frac{q}{15}\right) = 15\left(\frac{4}{5}\right)$$

Simplify.

$$q = \frac{15 \cdot 4}{5}$$

$$q = \frac{60}{5}$$

$$q = 12$$

Method 2 Multiply by the least common denominator (LCD) to clear the fractions.

$$\frac{q}{15} - \frac{1}{5} = \frac{3}{5}$$

$$15\left(\frac{q}{15} - \frac{1}{5}\right) = 15\left(\frac{3}{5}\right)$$

Multiply both sides by 15, the LCD of the fractions.

$$15\left(\frac{q}{15}\right) - 15\left(\frac{1}{5}\right) = 15\left(\frac{3}{5}\right)$$

Distribute 15 on the left side.

$$q - 3 = 9$$

Simplify.

$$\frac{q}{15} + \frac{1}{5} = \frac{3}{5} + \frac{1}{5}$$

$$q = 12$$

Since 3 is subtracted from q , add 3 to both sides to undo the subtraction.



Solve each equation. Check your answer.

2a. $\frac{2x}{5} - \frac{1}{2} = 5$

2b. $\frac{3}{4}u + \frac{1}{2} = \frac{7}{8}$

2c. $\frac{1}{5}n - \frac{1}{3} = \frac{8}{3}$

Equations that are more complicated may have to be simplified before they can be solved. You may have to use the Distributive Property or combine like terms before you begin using inverse operations.

EXAMPLE 3 Simplifying Before Solving Equations

Solve each equation.

A $6x + 3 - 8x = 13$

$$6x + 3 - 8x = 13$$

$$6x - 8x + 3 = 13$$

Use the Commutative Property of Addition.

$$-2x + 3 = 13$$

Combine like terms.

$$\frac{-2x}{15} + \frac{1}{5} = \frac{3}{5} + \frac{1}{5}$$

Since 3 is added to $-2x$, subtract 3 from both sides to undo the addition.

$$-2x = 10$$

$$\frac{-2x}{-2} = \frac{10}{-2}$$

Since x is multiplied by -2 , divide both sides by -2 to undo the multiplication.

$$x = -5$$

Helpful Hint

You can think of an opposite sign as a coefficient of -1 .
 $-(x + 2) = -1(x + 2)$
and $-x = -1x$.

Solve each equation.

$$\mathbf{B} \quad 9 = 6 - (x + 2)$$

$$9 = 6 + (-1)(x + 2)$$

$$9 = 6 + (-1)(x) + (-1)(2)$$

$$9 = 6 - x - 2$$

$$9 = 6 - 2 - x$$

$$9 = 4 - x$$

$$\begin{array}{r} -4 \quad -4 \\ 5 = \quad -x \end{array}$$

$$\begin{array}{r} 5 \\ -1 \end{array} = \begin{array}{r} -x \\ -1 \end{array}$$

$$-5 = x$$

Write subtraction as addition of the opposite.

Distribute -1 on the right side.

Simplify.

Use the Commutative Property of Addition.

Combine like terms.

Since 4 is added to $-x$, subtract 4 from both sides to undo the addition.

Since x is multiplied by -1 , divide both sides by -1 to undo the multiplication.



Solve each equation. Check your answer.

3a. $2a + 3 - 8a = 8$

3b. $-2(3 - d) = 4$

3c. $4(x - 2) + 2x = 40$

EXAMPLE 4 Problem-Solving Application



Alex belongs to a music club. In this club, students can buy a student discount card for \$19.95. This card allows them to buy CDs for \$3.95 each. After one year, Alex has spent \$63.40. Write and solve an equation to find how many CDs Alex bought during the year.



1 Understand the Problem

The answer will be the number of CDs that Alex bought during the year.

List the important information:

- Alex paid \$19.95 for a student discount card.
- Alex pays \$3.95 for each CD purchased.
- After one year, Alex has spent \$63.40.

2 Make a Plan

Let c represent the number of CDs that Alex purchased. That means Alex has spent $3.95c$. However, Alex must also add the amount spent on the card.

Write an equation to represent this situation.

$$\text{total cost} = \text{cost of compact discs} + \text{cost of discount card}$$

$$63.40 = 3.95c + 19.95$$

3 Solve

$$\begin{array}{r} 63.40 = 3.95c + 19.95 \\ - 19.95 \quad \quad - 19.95 \\ \hline 43.45 = 3.95c \\ \frac{43.45}{3.95} = \frac{3.95c}{3.95} \\ 11 = c \end{array}$$

Since 19.95 is added to 3.95c, subtract 19.95 from both sides to undo the addition.

Since c is multiplied by 3.95, divide both sides by 3.95 to undo the multiplication.

Alex bought 11 CDs during the year.

4 Look Back

Check that the answer is reasonable. The cost per CD is about \$4, so if Alex bought 11 CDs, this amount is about $11(4) = \$44$.

Add the cost of the discount card, which is about \$20: $44 + 20 = 64$. So the total cost was about \$64, which is close to the amount given in the problem, \$63.40.



4. Sara paid \$15.95 to become a member at a gym. She then paid a monthly membership fee. Her total cost for 12 months was \$735.95. How much was the monthly fee?

EXAMPLE 5 Solving Equations to Find an Indicated Value

If $3a + 12 = 30$, find the value of $a + 4$.

Step 1 Find the value of a .

$$\begin{array}{r} 3a + 12 = 30 \\ - 12 \quad - 12 \\ \hline 3a = 18 \\ \frac{3a}{3} = \frac{18}{3} \\ a = 6 \end{array}$$

Since 12 is added to $3a$, subtract 12 from both sides to undo the addition.

Since a is multiplied by 3, divide both sides by 3 to undo the multiplication.

Step 2 Find the value of $a + 4$.

$$\begin{array}{r} a + 4 \\ 6 + 4 \\ 10 \end{array}$$

To find the value of $a + 4$, substitute 6 for a . Simplify.



5. If $2x + 4 = -24$, find the value of $3x$.

THINK AND DISCUSS

1. Explain the steps you would follow to solve $2x + 1 = 7$. How is this procedure different from the one you would follow to solve $2x - 1 = 7$?
2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write and solve a multi-step equation. Use addition, subtraction, multiplication, and division at least one time each.

Solving Multi-Step Equations

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GUIDED PRACTICE

Solve each equation. Check your answer.

SEE EXAMPLE 1

p. 92

1. $4a + 3 = 11$

2. $8 = 3r - 1$

3. $42 = -2d + 6$

4. $x + 0.3 = 3.3$

5. $15y + 31 = 61$

6. $9 - c = -13$

SEE EXAMPLE 2

p. 93

7. $\frac{x}{6} + 4 = 15$

8. $\frac{1}{3}y + \frac{1}{4} = \frac{5}{12}$

9. $\frac{2}{7}j - \frac{1}{7} = \frac{3}{14}$

10. $15 = \frac{a}{3} - 2$

11. $4 - \frac{m}{2} = 10$

12. $\frac{x}{8} - \frac{1}{2} = 6$

SEE EXAMPLE 3

p. 93

13. $28 = 8x + 12 - 7x$

14. $2y - 7 + 5y = 0$

15. $2.4 = 3(m + 4)$

16. $3(x - 4) = 48$

17. $4t + 7 - t = 19$

18. $5(1 - 2w) + 8w = 15$

SEE EXAMPLE 4

p. 94

19. **Transportation** Paul bought a student discount card for the bus. The card cost \$7 and allows him to buy daily bus passes for \$1.50. After one month, Paul spent \$29.50. How many daily bus passes did Paul buy?

SEE EXAMPLE 5

p. 95

20. If $3x - 13 = 8$, find the value of $x - 4$.

21. If $3(x + 1) = 7$, find the value of $3x$.

22. If $-3(y - 1) = 9$, find the value of $\frac{1}{2}y$.

23. If $4 - 7x = 39$, find the value of $x + 1$.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises See Example

24–29 1

30–35 2

36–41 3

42 4

43–46 5

Extra Practice

Skills Practice p. S6

Application Practice p. S29

Solve each equation. Check your answer.

24. $5 = 2g + 1$

25. $6h - 7 = 17$

26. $0.6v + 2.1 = 4.5$

27. $3x + 3 = 18$

28. $0.6g + 11 = 5$

29. $32 = 5 - 3t$

30. $2d + \frac{1}{5} = \frac{3}{5}$

31. $1 = 2x + \frac{1}{2}$

32. $\frac{z}{2} + 1 = \frac{3}{2}$

33. $\frac{2}{3} = \frac{4j}{6}$

34. $\frac{3}{4} = \frac{3}{8}x - \frac{3}{2}$

35. $\frac{1}{5} - \frac{x}{5} = -\frac{2}{5}$

36. $6 = -2(7 - c)$

37. $5(h - 4) = 8$

38. $-3x - 8 + 4x = 17$

39. $4x + 6x = 30$

40. $2(x + 3) = 10$

41. $17 = 3(p - 5) + 8$

42. **Consumer Economics** Jennifer is saving money to buy a bike. The bike costs \$245. She has \$125 saved, and each week she adds \$15 to her savings. How long will it take her to save enough money to buy the bike?

43. If $2x + 13 = 17$, find the value of $3x + 1$.

44. If $-(x - 1) = 5$, find the value of $-4x$.

45. If $5(y + 10) = 40$, find the value of $\frac{1}{4}y$.

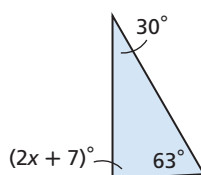
46. If $9 - 6x = 45$, find the value of $x - 4$.



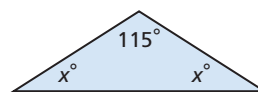
Geometry Write and solve an equation to find the value of x for each triangle.

(Hint: The sum of the angle measures in any triangle is 180° .)

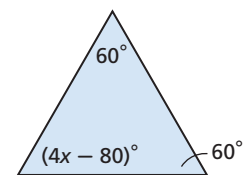
47.



48.



49.





History



Martin Luther King Jr. entered college at age 15. During his life he earned 3 degrees and was awarded 20 honorary degrees.

Source: lib.lsu.edu

Write an equation to represent each relationship. Solve each equation.

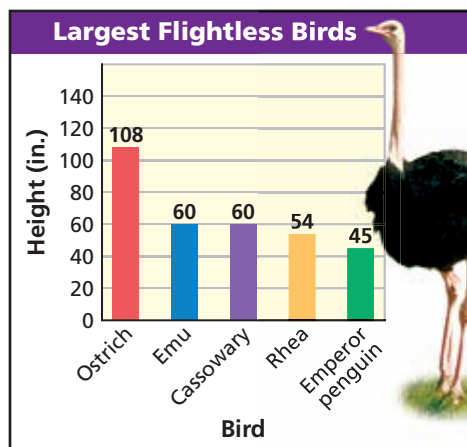
50. Seven less than twice a number equals 19.
51. Eight decreased by 3 times a number equals 2.
52. The sum of two times a number and 5 is 11.
53. **History** In 1963, Dr. Martin Luther King Jr. began his famous “I have a dream” speech with the words “Five score years ago, a great American, in whose symbolic shadow we stand, signed the Emancipation Proclamation.” The proclamation was signed by President Abraham Lincoln in 1863.
- Using the dates given, write and solve an equation that can be used to find the number of years in a score.
 - How many score would represent 60?

Solve each equation. Check your answer.

54. $3t + 44 = 50$ 55. $3(x - 2) = 18$ 56. $15 = \frac{c}{3} - 2$ 57. $2x + 6.5 = 15.5$
58. $3.9w - 17.9 = -2.3$ 59. $17 = x - 3(x + 1)$ 60. $5x + 9 = 39$ 61. $15 + 5.5m = 70$

Biology Use the graph for Exercises 62 and 63.

62. The height of an ostrich is 20 inches more than 4 times the height of a kiwi. Write and solve an equation to find the height of a kiwi. Show that your answer is reasonable.
63. Five times the height of a kakapo minus 70 equals the height of an emu. Write and solve an equation to find the height of a kakapo. Show that your answer is reasonable.
64. The sum of two consecutive whole numbers is 57. What are the two numbers? (*Hint: Let n represent the first number. Then $n + 1$ is the next consecutive whole number.*)
65. Stan’s, Mark’s, and Wayne’s ages are consecutive whole numbers. Stan is the youngest, and Wayne is the oldest. The sum of their ages is 111. Find their ages.
66. The sum of two consecutive even whole numbers is 206. What are the two numbers? (*Hint: Let n represent the first number. What expression can you use to represent the second number?*)



Source: The Top Ten of Everything

MULTI-STEP TEST PREP



67. This problem will prepare you for the Multi-Step Test Prep on page 118.
- The cost of fighting a certain forest fire is \$225 per acre. Complete the table.
 - Write an equation for the relationship between the cost c of fighting the fire and the number of acres n .

Cost of Fighting Fire	
Acres	Cost (\$)
100	22,500
200	■
500	■
1000	■
1500	■
n	■

68. **Critical Thinking** The equation $2(m - 8) + 3 = 17$ has more than one solution method. Give at least two different “first steps” to solve this equation.



69. **Write About It** Write a series of steps that you can use to solve any multi-step equation.



70. Lin sold 4 more shirts than Greg. Fran sold 3 times as many shirts as Lin. In total, the three sold 51 shirts. Which represents the number of shirts Greg sold?
(A) $3g = 51$ (B) $3 + g = 51$ (C) $8 + 5g = 51$ (D) $16 + 5g = 51$

71. If $\frac{4m - 3}{7} = 3$, what is the value of $7m - 5$?
(F) 6 (G) 10.5 (H) 37 (J) 68.5

72. The equation $c = 48 + 0.06m$ represents the cost c of renting a car and driving m miles. Which statement best describes this cost?
(A) The cost is a flat rate of \$0.06 per mile.
(B) The cost is \$0.48 for the first mile and \$0.06 for each additional mile.
(C) The cost is a \$48 fee plus \$0.06 per mile.
(D) The cost is a \$6 fee plus \$0.48 per mile.

73. **Gridded Response** A telemarketer earns \$150 a week plus \$2 for each call that results in a sale. Last week she earned a total of \$204. How many of her calls resulted in sales?

CHALLENGE AND EXTEND

Solve each equation. Check your answer.

74. $\frac{9}{2}x + 18 + 3x = \frac{11}{2}$

75. $\frac{15}{4}x - 15 = \frac{33}{4}$

76. $(x + 6) - (2x + 7) - 3x = -9$

77. $(4x + 2) - (12x + 8) + 2(5x - 3) = 6 + 11$

78. Find a value for b so that the solution of $4x + 3b = -1$ is $x = 2$.

79. Find a value for b so that the solution of $2x - 3b = 0$ is $x = -9$.

80. **Business** The formula $p = nc - e$ gives the profit p when a number of items n are each sold at a cost c and expenses e are subtracted.

a. If $p = 2500$, $n = 2000$, and $e = 800$, what is the value of c ?

b. If $p = 2500$, $n = 1000$, and $e = 800$, what is the value of c ?

c. **What if...?** If n is divided in half while p and e remain the same, what is the effect on c ?

SPIRAL REVIEW

Write all classifications that apply to each real number. (Lesson 1-5)

81. $\sqrt{3}$

82. -58

83. $2\frac{1}{3}$

84. 0.17

Write each product using the Distributive Property. Then simplify. (Lesson 1-7)

85. $8(61)$

86. $9(28)$

87. $11(28)$

88. $13(21)$

Solve each equation. Check your answer. (Lesson 2-1)

89. $17 = k + 4$

90. $x - 18 = 3$

91. $a + 6 = -12$

92. $-7 = q - 7$

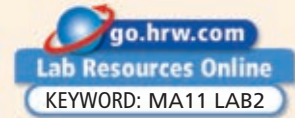


Model Equations with Variables on Both Sides

Algebra tile models can help you understand how to solve equations with variables on both sides.

Use with Lesson 2-4

KEY		$+ = 1$ $- = -1$ $+ = x$ $- = -x$	REMEMBER		$+ + - = 0$
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Activity

Use algebra tiles to model and solve $5x - 2 = 2x + 10$.

MODEL	ALGEBRA
<p>Model $5x - 2$ on the left side of the mat and $2x + 10$ on the right side. Remember that $5x - 2$ is the same as $5x + (-2)$.</p>	$5x - 2 = 2x + 10$
<p>Remove 2 x-tiles from both sides. This represents subtracting $2x$ from both sides of the equation.</p>	$5x - 2 - 2x = 2x - 2x + 10$ $3x - 2 = 10$
<p>Place 2 yellow tiles on both sides. This represents adding 2 to both sides of the equation. Remove zero pairs.</p>	$3x - 2 + 2 = 10 + 2$ $3x = 12$
<p>Separate each side into 3 equal groups. Each group is $\frac{1}{3}$ of the side. One x-tile is equivalent to 4 yellow tiles.</p>	$\frac{1}{3}(3x) = \frac{1}{3}(12)$ $x = 4$

Try This

Use algebra tiles to model and solve each equation.

- $3x + 2 = 2x + 5$
- $5x + 12 = 2x + 3$
- $9x - 5 = 6x + 13$
- $x = -2x + 9$

2-4

Solving Equations with Variables on Both Sides

Objective

Solve equations in one variable that contain variable terms on both sides.

Vocabulary

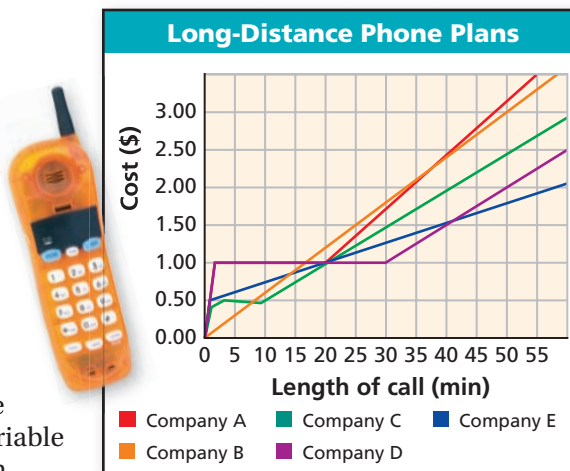
identity

Why learn this?

You can compare prices and find the best value.

Many phone companies offer low rates for long-distance calls without requiring customers to sign up for their services. To compare rates, solve an equation with variables on both sides.

To solve an equation like this, use inverse operations to “collect” variable terms on one side of the equation.



EXAMPLE 1 Solving Equations with Variables on Both Sides

Solve each equation.

A $7k = 4k + 15$
 $7k = 4k + 15$
 $\underline{-4k \quad -4k}$
 $3k = 15$
 $\frac{3k}{3} = \frac{15}{3}$
 $k = 5$

To collect the variable terms on one side, subtract $4k$ from both sides.

Since k is multiplied by 3, divide both sides by 3 to undo the multiplication.

B $5x - 2 = 3x + 4$
 $5x - 2 = 3x + 4$
 $\underline{-3x \quad -3x}$
 $2x - 2 = 4$
 $\underline{+2 \quad +2}$
 $2x = 6$
 $\frac{2x}{2} = \frac{6}{2}$
 $x = 3$

To collect the variable terms on one side, subtract $3x$ from both sides.

Since 2 is subtracted from $2x$, add 2 to both sides to undo the subtraction.

Since x is multiplied by 2, divide both sides by 2 to undo the multiplication.

Check $5x - 2 = 3x + 4$
 $\frac{5(3) - 2}{15 - 2} \quad \frac{3(3) + 4}{9 + 4}$
 $13 \quad 13 \checkmark$

To check your solution, substitute 3 for x in the original equation.

Helpful Hint

Equations are often easier to solve when the variable has a positive coefficient. Keep this in mind when deciding on which side to “collect” variable terms.



Solve each equation. Check your answer.

1a. $4b + 2 = 3b$

1b. $0.5 + 0.3y = 0.7y - 0.3$

To solve more complicated equations, you may need to first simplify by using the Distributive Property or combining like terms.

EXAMPLE 2 Simplifying Each Side Before Solving Equations

Solve each equation.

A $2(y + 6) = 3y$

$$\begin{aligned} 2(y + 6) &= 3y \\ 2(y) + 2(6) &= 3y \\ 2y + 12 &= 3y \\ \underline{-2y} \quad \underline{-2y} & \\ 12 &= y \end{aligned}$$

*Distribute 2 to the expression in parentheses.**To collect the variable terms on one side, subtract 2y from both sides.**To check your solution, substitute 12 for y in the original equation.*

Check $2(y + 6) = 3y$

$2(12 + 6)$	$3(12)$
$2(18)$	36
36	36 ✓

B $2k - 5 = 3(1 - 2k)$

$$\begin{aligned} 2k - 5 &= 3(1 - 2k) \\ 2k - 5 &= 3(1) - 3(2k) \\ 2k - 5 &= 3 - 6k \\ \underline{+6k} \quad \underline{+6k} & \\ 8k - 5 &= 3 \\ \underline{+5} \quad \underline{+5} & \\ 8k &= 8 \\ \frac{8k}{8} &= \frac{8}{8} \\ k &= 1 \end{aligned}$$

*Distribute 3 to the expression in parentheses.**To collect the variable terms on one side, add 6k to both sides.**Since 5 is subtracted from 8k, add 5 to both sides.**Since k is multiplied by 8, divide both sides by 8.*

C $3 - 5b + 2b = -2 - 2(1 - b)$

$$\begin{aligned} 3 - 5b + 2b &= -2 - 2(1 - b) \\ 3 - 5b + 2b &= -2 - 2(1) - 2(-b) \\ 3 - 5b + 2b &= -2 - 2 + 2b \\ 3 - 3b &= -4 + 2b \\ \underline{+3b} \quad \underline{+3b} & \\ 3 &= -4 + 5b \\ \underline{+4} \quad \underline{+4} & \\ 7 &= 5b \\ \frac{7}{5} &= \frac{5b}{5} \\ 1.4 &= b \end{aligned}$$

*Distribute -2 to the expression in parentheses.**Combine like terms. Add 3b to both sides.**Since -4 is added to 5b, add 4 to both sides.**Since b is multiplied by 5, divide both sides by 5.*

Solve each equation. Check your answer.

2a. $\frac{1}{2}(b + 6) = \frac{3}{2}b - 1$

2b. $3x + 15 - 9 = 2(x + 2)$

An **identity** is an equation that is always true, no matter what value is substituted for the variable. The solutions of an identity are all real numbers. Some equations are always false. These equations have no solutions.

EXAMPLE 3 Infinitely Many Solutions or No Solutions

Solve each equation.

A $x + 4 - 6x = 6 - 5x - 2$
 $x + 4 - 6x = 6 - 5x - 2$ *Identify like terms.*
 $4 - 5x = 4 - 5x$ *Combine like terms on the left and the right.*
 $\quad + 5x \quad + 5x$ *Add 5x to both sides.*
 $4 = 4 \checkmark$ *True statement*

The equation $x + 4 - 6x = 6 - 5x - 2$ is an identity. All values of x will make the equation true. All real numbers are solutions.

B $-8x + 6 + 9x = -17 + x$
 $-8x + 6 + 9x = -17 + x$ *Identify like terms.*
 $x + 6 = -17 + x$ *Combine like terms.*
 $\quad - x \quad - x$ *Subtract x from both sides.*
 $6 = -17 \times$ *False statement*

The equation $-8x + 6 + 9x = -17 + x$ is always false. There is no value of x that will make the equation true. There are no solutions.

Writing Math

The solution set for Example 3B is an empty set—it contains no elements. The empty set can be written as \emptyset or $\{\}$.



Solve each equation.

3a. $4y + 7 - y = 10 + 3y$

3b. $2c + 7 + c = -14 + 3c + 21$

EXAMPLE 4 Consumer Application

The long-distance rates of two phone companies are shown in the table. How long is a call that costs the same amount no matter which company is used? What is the cost of that call?

Phone Company	Charges
Company A	36¢ plus 3¢ per minute
Company B	6¢ per minute

Let m represent minutes, and write expressions for each company's cost.

When is 36¢ plus 3¢ per minute times number of minutes the same as 6¢ per minute times number of minutes?

$$36 + 3m = 6m$$

$$36 + 3m = 6m$$

$$\frac{36}{3} = \frac{3m}{3}$$

$$12 = m$$

To collect the variable terms on one side, subtract 3m from both sides.

Since m is multiplied by 3, divide both sides by 3 to undo the multiplication.

The charges will be the same for a 12-minute call using either phone service. To find the cost of this call, evaluate either expression for $m = 12$:

$$36 + 3m = 36 + 3(12) = 36 + 36 = 72 \quad 6m = 6(12) = 72$$

The cost of a 12-minute call through either company is 72¢.



4. Four times Greg's age, decreased by 3 is equal to 3 times Greg's age, increased by 7. How old is Greg?

THINK AND DISCUSS

1. Tell which of the following is an identity. Explain your answer.
a. $4(a + 3) - 6 = 3(a + 3) - 6$ **b.** $8.3x - 9 + 0.7x = 2 + 9x - 11$
2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of an equation that has the indicated number of solutions.



An equation with variables on both sides can have...

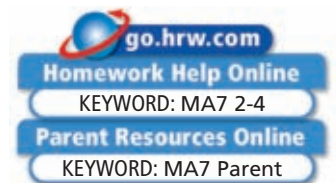
one solution:

many solutions:

no solution:

2-4

Exercises



GUIDED PRACTICE

1. **Vocabulary** How can you recognize an identity?

Solve each equation. Check your answer.

SEE EXAMPLE 1
p. 100

2. $2c - 5 = c + 4$

3. $8r + 4 = 10 + 2r$

p. 100

4. $2x - 1 = x + 11$

5. $28 - 0.3y = 0.7y - 12$

SEE EXAMPLE 2
p. 101

6. $-2(x + 3) = 4x - 3$

7. $3c - 4c + 1 = 5c + 2 + 3$

p. 101

8. $5 + 3(q - 4) = 2(q + 1)$

9. $5 - (t + 3) = -1 + 2(t - 3)$

SEE EXAMPLE 3
p. 102

10. $7x - 4 = -2x + 1 + 9x - 5$

11. $8x + 6 - 9x = 2 - x - 15$

p. 102

12. $6y = 8 - 9 + 6y$

13. $6 - 2x - 1 = 4x + 8 - 6x - 3$

SEE EXAMPLE 4
p. 102

14. **Consumer Economics** A house-painting company charges \$376 plus \$12 per hour. Another painting company charges \$280 plus \$15 per hour.

- a.** How long is a job for which both companies will charge the same amount?
b. What will that cost be?

PRACTICE AND PROBLEM SOLVING

Solve each equation. Check your answer.

15. $7a - 17 = 4a + 1$

16. $2b - 5 = 8b + 1$

17. $4x - 2 = 3x + 4$

18. $2x - 5 = 4x - 1$

19. $8x - 2 = 3x + 12.25$

20. $5x + 2 = 3x$

21. $3c - 5 = 2c + 5$

22. $-17 - 2x = 6 - x$

23. $3(t - 1) = 9 + t$

24. $5 - x - 2 = 3 + 4x + 5$

25. $2(x + 4) = 3(x - 2)$

26. $3m - 10 = 2(4m - 5)$

27. $5 - (n - 4) = 3(n + 2)$

28. $6(x + 7) - 20 = 6x$

29. $8(x + 1) = 4x - 8$

30. $x - 4 - 3x = -2x - 3 - 1$

31. $-2(x + 2) = -2x + 1$

32. $2(x + 4) - 5 = 2x + 3$

Independent Practice

For Exercises	See Example
15–22	1
23–29	2
30–32	3
33	4

Extra Practice

Skills Practice p. S6

Application Practice p. S29

33. **Sports** Justin and Tyson are beginning an exercise program to train for football season. Justin weighs 150 lb and hopes to gain 2 lb per week. Tyson weighs 195 lb and hopes to lose 1 lb per week.

- If the plan works, in how many weeks will the boys weigh the same amount?
- What will that weight be?

Write an equation to represent each relationship. Then solve the equation.

34. Three times the sum of a number and 4 is the same as 18 more than the number.
35. A number decreased by 30 is the same as 14 minus 3 times the number.
36. Two less than 2 times a number is the same as the number plus 64.

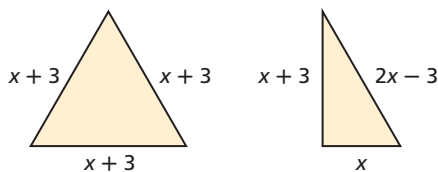
Solve each equation. Check your answer.

37. $2x - 2 = 4x + 6$ 38. $3x + 5 = 2x + 2$ 39. $4x + 3 = 5x - 4$
40. $-\frac{2}{5}p + 2 = \frac{1}{5}p + 11$ 41. $5x + 24 = 2x + 15$ 42. $5x - 10 = 14 - 3x$
43. $12 - 6x = 10 - 5x$ 44. $5x - 7 = -6x - 29$ 45. $1.8x + 2.8 = 2.5x + 2.1$
46. $2.6x + 18 = 2.4x + 22$ 47. $1 - 3x = 2x + 8$ 48. $\frac{1}{2}(8 - 6h) = h$
49. $3(x + 1) = 2x + 7$ 50. $9x - 8 + 4x = 7x + 16$ 51. $3(2x - 1) + 5 = 6(x + 1)$
52. **Travel** Rapid Rental Car company charges a \$40 rental fee, \$15 for gas, and \$0.25 per mile driven. For the same car, Capital Cars charges \$45 for rental and gas and \$0.35 per mile.

- Find the number of miles for which the companies' charges will be the same. Then find that charge. Show that your answers are reasonable.
- The Barre family estimates that they will drive about 95 miles during their vacation to Hershey, Pennsylvania. Which company should they rent their car from? Explain.
- What if...?** The Barres have extended their vacation and now estimate that they will drive about 120 miles. Should they still rent from the same company as in part b? Why or why not?
- Give a general rule for deciding which company to rent from.



53. **Geometry** The triangles shown have the same perimeter. What is the value of x ?



**MULTI-STEP
TEST PREP**



54. This problem will prepare you for the Multi-Step Test Prep on page 118.
- A fire currently covers 420 acres and continues to spread at a rate of 60 acres per day. How many total acres will be covered in the next 2 days? Show that your answer is reasonable.
 - Write an expression for the total area covered by the fire in d days.
 - The firefighters estimate that they can put out the fire at a rate of 80 acres per day. Write an expression for the total area that the firefighters can put out in d days.
 - Set the expressions in parts b and c equal. Solve for d . What does d represent?



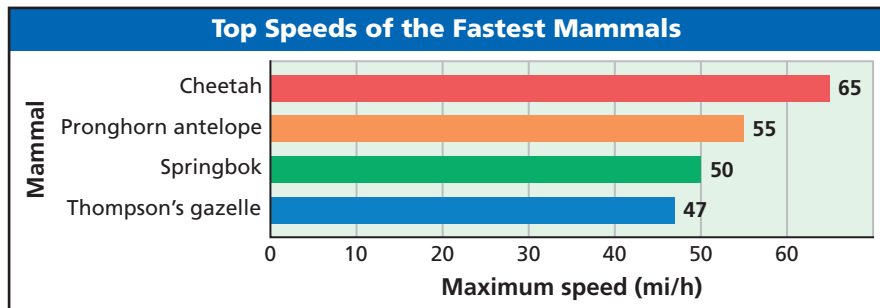
Biology



A cheetah's body is well designed for fast running. Its tail acts like a boat's rudder to help it make sharp turns. Its spine acts like a spring to propel it forward.

Source:
www.cheetahspot.com

55. **Critical Thinking** Write an equation with variables on both sides that has no solution.
56. **Biology** The graph shows the maximum recorded speeds of the four fastest mammals.



Source: *The Top 10 of Everything*

- Write an expression for the distance in miles that a Thompson's gazelle can run at top speed in x hours.
 - Write an expression for the distance in miles that a cheetah can run at top speed in x hours.
 - A cheetah and a Thompson's gazelle are running at their top speeds. The cheetah is one mile behind the gazelle. Write an expression for the distance the cheetah must run to catch up with the gazelle.
 - Write and solve an equation that represents how long the cheetah will have to run at top speed to catch up with the gazelle.
 - A cheetah can maintain its top speed for only 300 yards. Will the cheetah be able to catch the gazelle? Explain.
57. **Write About It** Write a series of steps that you can use to solve any equation with variables on both sides.



58. Lindsey's monthly magazine subscription costs \$1.25 per issue. Kenzie's monthly subscription costs \$1.50 per issue, but she received her first 2 issues free. Which equation can be used to find the number of months after which the girls will have paid the same amount?
- (A) $1.25m = 1.50m - 2$ (C) $1.25m = 1.50(m - 2)$
 (B) $1.25m = 1.50m - 2m$ (D) $1.25m = 3m - 1.50$
59. What is the numerical solution of the equation 7 times a number equals 3 less than 5 times that number?
- (F) -1.5 (G) 0.25 (H) $\frac{2}{3}$ (J) 4
60. Three packs of markers cost \$9.00 less than 5 packs of markers. Which equation best represents this situation?
- (A) $5x + 9 = 3x$ (B) $3x + 9 = 5x$ (C) $3x - 9 = 5x$ (D) $9 - 3x = 5x$
61. Nicole has \$120. If she saves \$20 per week, in how many days will she have \$500?
- (F) 19 (G) 25 (H) 133 (J) 175
62. **Gridded Response** Solve $-2(x - 1) + 5x = 2(2x - 1)$.

CHALLENGE AND EXTEND

Solve each equation.

63. $4x + 2[4 - 2(x + 2)] = 2x - 4$

64. $\frac{x+5}{2} + \frac{x-1}{2} = \frac{x-1}{3}$

65. $\frac{2}{3}w - \frac{1}{4} = \frac{2}{3}\left(w - \frac{1}{4}\right)$

66. $-5 - 7 - 3f = -f - 2(f + 6)$

67. $\frac{2}{3}x + \frac{1}{2} = \frac{3}{5}x - \frac{5}{6}$

68. $x - \frac{1}{4} = \frac{x}{3} + 7\frac{3}{4}$

69. Find three consecutive integers such that twice the greatest integer is 2 less than 3 times the least integer.

70. Find three consecutive integers such that twice the least integer is 12 more than the greatest integer.

71. Rob had twice as much money as Sam. Then Sam gave Rob 1 quarter, 2 nickels, and 3 pennies. Rob then gave Sam 8 dimes. If they now have the same amount of money, how much money did Rob originally have? Check your answer.

SPIRAL REVIEW

Write an expression for the perimeter of each figure. (Lesson 1-1)

72. square with side x cm

73. equilateral triangle with side y cm

Multiply or divide. (Lesson 1-3)

74. $6.1 \div 0$

75. $3(-21)$

76. $0 \div \frac{7}{8}$

77. $\frac{2}{5} \div \frac{1}{10}$

78. $5 \div (-5)$

79. $\frac{-16}{-8}$

80. $-1000 \div (-0.001)$

81. $500(-0.25)$

Solve each equation. Check your answer. (Lesson 2-3)

82. $4x - 44 = 8$

83. $2(x - 3) = 24$

84. $-1 = \frac{x}{4} - 3$

85. $2x + 6 = 12$

Career Path



Beth Simmons
Biology major

Q: What math classes did you take in high school?

A: Algebra 1 and 2, Geometry, and Precalculus

Q: What math classes have you taken in college?

A: Two calculus classes and a calculus-based physics class

Q: How do you use math?

A: I use math a lot in physics. Sometimes I would think a calculus topic was totally useless, and then we would use it in physics class! In biology, I use math to understand populations.

Q: What career options are you considering?

A: When I graduate, I could teach, or I could go to graduate school and do more research. I have a lot of options.

go.hrw.com
Career Resources Online
KEYWORD: MA7 Career

2-5

Solving for a Variable



Objectives

Solve a formula for a given variable.

Solve an equation in two or more variables for one of the variables.

Vocabulary

formula

literal equation

Who uses this?

Athletes can “rearrange” the distance formula to calculate their average speed.

Many wheelchair athletes compete in marathons, which cover about 26.2 miles. Using the time t it took to complete the race, the distance d , and the formula $d = rt$, racers can find their average speed r .

A **formula** is an equation that states a rule for a relationship among quantities.

In the formula $d = rt$, d is isolated. You can “rearrange” a formula to isolate any variable by using inverse operations. This is called *solving for a variable*.



Solving for a Variable

Step 1 Locate the variable you are asked to solve for in the equation.

Step 2 Identify the operations on this variable and the order in which they are applied.

Step 3 Use inverse operations to undo operations and isolate the variable.

EXAMPLE 1 Sports Application

In 2004, Ernst Van Dyk won the wheelchair race of the Boston Marathon with a time of about 1.3 hours. The race was about 26.2 miles. What was his average speed? Use the formula $d = rt$ and round your answer to the nearest tenth.

The question asks for speed, so first solve the formula $d = rt$ for r .

$$d = rt$$

Locate r in the equation.

$$\frac{d}{t} = \frac{rt}{t}$$

Since r is multiplied by t , divide both sides by t to undo the multiplication.

$$\frac{d}{t} = r, \text{ or } r = \frac{d}{t}$$

Now use this formula and the information given in the problem.

$$r = \frac{d}{t} \approx \frac{26.2}{1.3} \\ \approx 20.2$$

Van Dyk's average speed was about 20.2 miles per hour.

Helpful Hint

A number divided by itself equals 1. For $t \neq 0$, $\frac{t}{t} = 1$.



1. Solve the formula $d = rt$ for t . Find the time in hours that it would take Van Dyk to travel 26.2 miles if his average speed was 18 miles per hour. Round to the nearest hundredth.

EXAMPLE 2 Solving Formulas for a Variable

- A** The formula for a Fahrenheit temperature in terms of degrees Celsius is $F = \frac{9}{5}C + 32$. Solve for C .

$$F = \frac{9}{5}C + 32 \quad \text{Locate } C \text{ in the equation.}$$

$$\begin{array}{r} -32 \\ \hline F - 32 = \frac{9}{5}C \end{array} \quad \begin{array}{l} \text{Since } 32 \text{ is added to } \frac{9}{5}C, \text{ subtract } 32 \text{ from both} \\ \text{sides to undo the addition.} \end{array}$$

$$\left(\frac{5}{9}\right)(F - 32) = \left(\frac{5}{9}\right)\frac{9}{5}C \quad \begin{array}{l} \text{Since } C \text{ is multiplied by } \frac{9}{5}, \text{ divide both} \\ \text{sides by } \frac{9}{5} \text{ (multiply by } \frac{5}{9}\text{) to undo the} \\ \text{multiplication.} \end{array}$$

$$\frac{5}{9}(F - 32) = C$$

- B** The formula for a person's typing speed is $s = \frac{w - 10e}{m}$, where s is speed in words per minute, w is number of words typed, e is number of errors, and m is number of minutes typing. Solve for w .

$$s = \frac{w - 10e}{m} \quad \text{Locate } w \text{ in the equation.}$$

$$m(s) = m\left(\frac{w - 10e}{m}\right) \quad \begin{array}{l} \text{Since } w - 10e \text{ is divided by } m, \text{ multiply both} \\ \text{sides by } m \text{ to undo the division.} \end{array}$$

$$ms = w - 10e$$

$$\begin{array}{r} +10e \\ \hline ms + 10e = w \end{array} \quad \begin{array}{l} \text{Since } 10e \text{ is subtracted from } w, \text{ add } 10e \text{ to} \\ \text{both sides to undo the subtraction.} \end{array}$$



2. The formula for an object's final velocity f is $f = i - gt$, where i is the object's initial velocity, g is acceleration due to gravity, and t is time. Solve for i .

A formula is a type of *literal equation*. A **literal equation** is an equation with two or more variables. To solve for one of the variables, use inverse operations.

EXAMPLE 3 Solving Literal Equations for a Variable

- A** Solve $m - n = 5$ for m .

$$m - n = 5 \quad \text{Locate } m \text{ in the equation.}$$

$$\begin{array}{r} +n \\ \hline m = 5 + n \end{array} \quad \begin{array}{l} \text{Since } n \text{ is subtracted from } m, \text{ add } n \text{ to both sides to} \\ \text{undo the subtraction.} \end{array}$$

- B** Solve $\frac{m}{k} = x$ for k .

$$\frac{m}{k} = x \quad \text{Locate } k \text{ in the equation.}$$

$$k\left(\frac{m}{k}\right) = kx \quad \begin{array}{l} \text{Since } k \text{ appears in the denominator, multiply both} \\ \text{sides by } k. \end{array}$$

$$m = kx$$

$$\begin{array}{r} \frac{m}{x} = \frac{kx}{x} \\ \hline \frac{m}{x} = k \end{array} \quad \begin{array}{l} \text{Since } k \text{ is multiplied by } x, \text{ divide both sides by } x \text{ to} \\ \text{undo the multiplication.} \end{array}$$



- 3a. Solve $5 - b = 2t$ for t .

3b. Solve $D = \frac{m}{V}$ for V .

Remember!

Dividing by a fraction is the same as multiplying by the reciprocal.

THINK AND DISCUSS

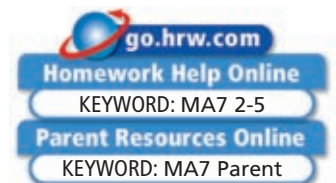
- Describe a situation in which a formula could be used more easily if it were “rearranged.” Include the formula in your description.
- Explain how to solve $P = 2\ell + 2w$ for w .
- GET ORGANIZED** Copy and complete the graphic organizer. Write a formula that is used in each subject. Then solve the formula for each of its variables.

Common Formulas	
Subject	Formula
Geometry	
Physical science	
Earth science	



2-5

Exercises



GUIDED PRACTICE

SEE EXAMPLE 1
p. 107

- Vocabulary** Explain why a *formula* is a type of *literal equation*.
- Construction** The formula $a = 46c$ gives the floor area a in square meters that can be wired using c circuits.
 - Solve $a = 46c$ for c .
 - If a room is 322 square meters, how many circuits are required to wire this room?

SEE EXAMPLE 2
p. 108

- The formula for the volume of a rectangular prism with length ℓ , width w , and height h is $V = \ell wh$. Solve this formula for w .

SEE EXAMPLE 3
p. 108

- Solve $st + 3t = 6$ for s .
- Solve $m - 4n = 8$ for m .
- Solve $\frac{f+4}{g} = 6$ for f .
- Solve $b + c = \frac{10}{a}$ for a .

PRACTICE AND PROBLEM SOLVING

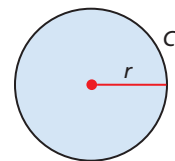
Independent Practice

For Exercises	See Example
8	1
9	2
10–13	3

Extra Practice

Skills Practice p. S6
Application Practice p. S29

- Geometry** The formula $C = 2\pi r$ relates the circumference C of a circle to its radius r . (Recall that π is the constant ratio of circumference to diameter.)
 - Solve $C = 2\pi r$ for r .
 - If a circle's circumference is 15 inches, what is its radius? Leave the symbol π in your answer.
- Finance** The formula $A = P + I$ shows that the total amount of money A received from an investment equals the principal P (the original amount of money invested) plus the interest I . Solve this formula for I .



C is the distance around the circle.

r is the distance from the center of the circle to any point on the circle.

- Solve $-2 = 4r + s$ for s .
- Solve $xy - 5 = k$ for x .
- Solve $\frac{m}{n} = p - 6$ for n .
- Solve $\frac{x-2}{y} = z$ for y .

Solve for the indicated variable.

14. $S = 180n - 360$ for n 15. $\frac{x}{5} - g = a$ for x 16. $A = \frac{1}{2}bh$ for b
 17. $y = mx + b$ for x 18. $a = 3n + 1$ for n 19. $PV = nRT$ for T
 20. $T + M = R$ for T 21. $M = T - R$ for T 22. $PV = nRT$ for R
 23. $2a + 2b = c$ for b 24. $5p + 9c = p$ for c 25. $ax + r = 7$ for r
 26. $3x + 7y = 2$ for y 27. $4y + 3x = 5$ for x 28. $y = 3x + 3b$ for b

29. **Estimation** The table shows the flying time and distance traveled for five flights on a certain airplane.
- Use the data in the table to write a rule that *estimates* the relationship between flying time t and distance traveled d .
 - Use your rule from part **a** to estimate the time that it takes the airplane to fly 1300 miles.
 - Solve your rule for d .
 - Use your rule from part **c** to estimate the distance the airplane can fly in 8 hours.

Flying Times		
Flight	Time (h)	Distance (mi)
A	2	1018
B	3	1485
C	4	2103
D	5	2516
E	6	2886

30. **Sports** To find a baseball pitcher's earned run average (ERA), you can use the formula $Ei = 9r$, where E represents ERA, i represents number of innings pitched, and r represents number of earned runs allowed. Solve the equation for E . What is a pitcher's ERA if he allows 5 earned runs in 18 innings pitched?



31. **Meteorology** For altitudes up to 36,000 feet, the relationship between temperature and altitude can be described by the formula $t = -0.0035a + g$, where t is the temperature in degrees Fahrenheit, a is the altitude in feet, and g is the ground temperature in degrees Fahrenheit. Solve this formula for a .



32. **Write About It** In your own words, explain how to solve a literal equation for one of the variables.
33. **Critical Thinking** How is solving $a - ab = c$ for a different from the problems in this lesson? How might you solve this equation for a ?

**MULTI-STEP
TEST PREP**



34. This problem will prepare you for the Multi-Step Test Prep on page 118.
- Suppose firefighters can extinguish a wildfire at a rate of 60 acres per day. Use this information to complete the table.
 - Use the last row in the table to write an equation for acres A extinguished in terms of the number of days d .
 - Graph the points in the table with *Days* on the horizontal axis and *Acres* on the vertical axis. Describe the graph.

Days	Acres
1	60
2	■
3	180
4	■
5	■
d	■

35. Which equation is the result of solving $9 + 3x = 2y$ for x ?
 Ⓐ $\frac{9 + 3y}{2} = x$ Ⓑ $\frac{2}{3}y - 9 = x$ Ⓒ $x = \frac{2}{3}y - 3$ Ⓓ $x = 2y - 3$
36. Which of the following is a correct method for solving $2a - 5b = 10$ for b ?
 Ⓕ Add $5b$ to both sides, then divide both sides by 2.
 Ⓖ Subtract $5b$ from both sides, then divide both sides by 2.
 Ⓗ Divide both sides by 5, then add $2a$ to both sides.
 Ⓙ Subtract $2a$ from both sides, then divide both sides by -5 .
37. The formula for the volume of a rectangular prism is $V = \ell wh$. Anna wants to make a cardboard box with a length of 7 inches, a width of 5 inches, and a volume of 210 cubic inches. Which variable does Anna need to solve for in order to build her box?
 Ⓐ V Ⓑ ℓ Ⓒ w Ⓓ h

CHALLENGE AND EXTEND

Solve for the indicated variable.

38. $3.3x + r = 23.1$ for x 39. $\frac{2}{5}a - \frac{3}{4}b = c$ for a 40. $\frac{3}{5}x + 1.4y = \frac{2}{5}$ for y
41. $t = \frac{d}{500} + \frac{1}{2}$ for d 42. $s = \frac{1}{2}gt^2$ for g 43. $v^2 = u^2 + 2as$ for s
44. Solve $y = mx + 6$ for m . What can you say about y if $m = 0$?

45. **Entertainment** The formula $S = \frac{h \cdot w \cdot f \cdot t}{35,000}$ gives the approximate size in kilobytes (Kb) of a compressed video. The variables h and w represent the height and width of the frame measured in pixels, f is the number of frames per second (fps) the video plays, and t is the time the video plays in seconds. Estimate the time a movie trailer will play if it has a frame height of 320 pixels, has a frame width of 144 pixels, plays at 15 fps, and has a size of 2370 Kb.



SPIRAL REVIEW

46. Jill spent $\frac{1}{4}$ of the money she made baby-sitting. She made \$40 baby-sitting. How much did she spend? (*Previous course*)
47. In one class, $\frac{3}{5}$ of the students are boys. There are 30 students in the class. How many are girls? (*Previous course*)

Evaluate each expression for the given value of x . (*Lesson 1-6*)

48. $3 + 2 \cdot x + 4$ for $x = 3$ 49. $24 \div 4 - x$ for $x = 12$ 50. $43 - 62 + x$ for $x = 15$

Solve each equation. Check your answer. (*Lesson 2-1*)

51. $18 = -2 + w$ 52. $2 = -3 + c$ 53. $-8 + k = 4$ 54. $-15 + a = -27$

2-6

Solving Absolute-Value Equations

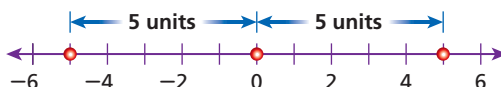
Objective

Solve equations in one variable that contain absolute-value expressions.

Why learn this?

Engineers can solve absolute-value equations to calculate the length of the deck of a bridge. (See Example 3.)

Recall that the absolute value of a number is that number's distance from zero on a number line. For example, $|-5| = 5$ and $|5| = 5$.



For any nonzero absolute value, there are exactly two numbers with that absolute value. For example, both 5 and -5 have an absolute value of 5.

To write this statement using algebra, you would write $|x| = 5$. This equation asks, "What values of x have an absolute value of 5?" The solutions are 5 and -5 . Notice that this equation has two solutions.



Absolute-Value Equations

WORDS	NUMBERS
The equation $ x = a$ asks, "What values of x have an absolute value of a ?" The solutions are a and the opposite of a .	$ x = 5$ $x = 5$ or $x = -5$
GRAPH	ALGEBRA
	$ x = a$ $x = a$ or $x = -a$ ($a \geq 0$)

To solve absolute-value equations, perform inverse operations to isolate the absolute-value expression on one side of the equation. Then you must consider two cases.

EXAMPLE 1 Solving Absolute-Value Equations

Helpful Hint

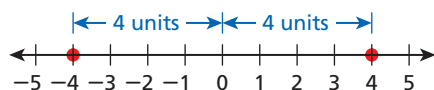
Be sure to check both solutions when you solve an absolute-value equation.

$$\begin{array}{r|l} |x| = 4 & |x| = 4 \\ \hline |-4| & 4 \\ 4 & 4 \checkmark \end{array} \quad \begin{array}{r|l} |x| = 4 & |x| = 4 \\ \hline |4| & 4 \\ 4 & 4 \checkmark \end{array}$$

Solve each equation.

A $|x| = 4$
 $|x| = 4$

Think: What numbers are 4 units from 0?



Case 1 | **Case 2** Rewrite the equation as two cases.

$x = -4$ | $x = 4$

The solutions are -4 and 4 .

Solve each equation.

B $4|x + 2| = 24$

$$\frac{4|x + 2|}{4} = \frac{24}{4}$$

$$|x + 2| = 6$$

Since $|x + 2|$ is multiplied by 4, divide both sides by 4 to undo the multiplication.

Think: What numbers are 6 units from 0?

Case 1

$$x + 2 = -6$$

$$\frac{-2}{-2} \quad \frac{-2}{-2}$$

$$x = -8$$

Case 2

$$x + 2 = 6$$

$$\frac{-2}{-2} \quad \frac{-2}{-2}$$

$$x = 4$$

Rewrite the equation as two cases. Since 2 is added to x , subtract 2 from both sides of the equation.

The solutions are -8 and 4 .



Solve each equation. Check your answer.

1a. $|x| - 3 = 4$

1b. $8 = |x - 2.5|$

The table summarizes the steps for solving absolute-value equations.



Solving an Absolute-Value Equation	
1.	Use inverse operations to isolate the absolute-value expression.
2.	Rewrite the resulting equation as two cases that do not involve absolute values.
3.	Solve the equation in each of the two cases.

Not all absolute-value equations have two solutions. If the absolute-value expression equals 0, there is one solution. If an equation states that an absolute value is negative, there are no solutions.

EXAMPLE 2 Special Cases of Absolute-Value Equations

Solve each equation.

A $|x + 3| + 4 = 4$

$$|x + 3| + 4 = 4$$

$$\frac{-4}{-4} \quad \frac{-4}{-4}$$

$$|x + 3| = 0$$

$$x + 3 = 0$$

$$\frac{-3}{-3} \quad \frac{-3}{-3}$$

$$x = -3$$

Since 4 is added to $|x + 3|$, subtract 4 from both sides to undo the addition.

There is only one case. Since 3 is added to x , subtract 3 from both sides to undo the addition.

B $5 = |x + 2| + 8$

$$5 = |x + 2| + 8$$

$$\frac{-8}{-8} \quad \frac{-8}{-8}$$

$$-3 = |x + 2| \quad \times$$

Since 8 is added to $|x + 2|$, subtract 8 from both sides to undo the addition.

Absolute value cannot be negative.

This equation has no solution.

Remember!

Absolute value must be nonnegative because it represents a distance.



Solve each equation.

2a. $2 - |2x - 5| = 7$

2b. $-6 + |x - 4| = -6$

EXAMPLE 3 Engineering Application

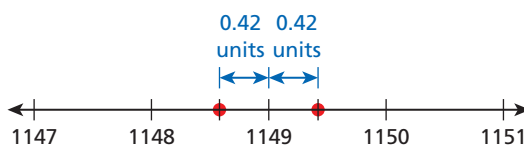
Sydney Harbour Bridge in Australia is 1149 meters long. Because of changes in temperature, the bridge can expand or contract by as much as 420 millimeters. Write and solve an absolute-value equation to find the minimum and maximum lengths of the bridge.



First convert millimeters to meters.

$$420 \text{ mm} = 0.420 \text{ m} \quad \text{Move the decimal point three places to the left.}$$

The length of the bridge can vary by 0.42 m, so find two numbers that are 0.42 units away from 1149 on a number line.



You can find these numbers by using the absolute-value equation $|x - 1149| = 0.42$. Solve the equation by rewriting it as two cases.

Case 1

$$\begin{array}{r} x - 1149 = -0.42 \\ \underline{+ 1149 \quad + 1149} \\ x = 1148.58 \end{array}$$

Case 2

$$\begin{array}{r} x - 1149 = 0.42 \\ \underline{+ 1149 \quad + 1149} \\ x = 1149.42 \end{array}$$

Since 1149 is subtracted from x , add 1149 to both sides of each equation.

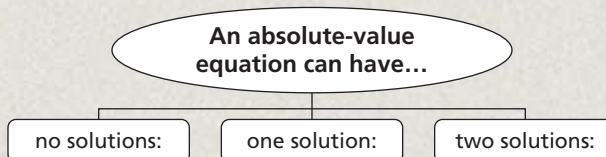
The minimum length of the bridge is 1148.58 m, and the maximum length is 1149.42 m.



3. Sydney Harbour Bridge is 134 meters tall. The height of the bridge can rise or fall by 180 millimeters because of changes in temperature. Write and solve an absolute-value equation to find the minimum and maximum heights of the bridge.

THINK AND DISCUSS

1. Explain the steps you would use to solve the equation $\frac{1}{5}|x - 3| = 2$.
2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of an absolute-value equation that has the indicated number of solutions, and then solve.



GUIDED PRACTICE

Solve each equation.

SEE EXAMPLE 1

p. 112

1. $|x| = 6$

2. $9 = |x + 5|$

3. $|3x| + 2 = 8$

4. $2|x| = 18$

5. $\left|x + \frac{1}{2}\right| = 1$

6. $|x - 3| - 6 = 2$

SEE EXAMPLE 2

p. 113

7. $-8 = |x|$

8. $|x| = 0$

9. $|x + 4| = -7$

10. $7 = |3x + 9| + 7$

11. $|2.8 - x| + 1.5 = 1.5$

12. $5|x + 7| + 14 = 8$

SEE EXAMPLE 3

p. 114

13. **Communication** Barry's walkie-talkie has a range of 2 mi. Barry is traveling on a straight highway and is at mile marker 207. Write and solve an absolute-value equation to find the minimum and maximum mile marker from 207 that Barry's walkie-talkie will reach.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14–22	1
23–28	2
29	3

Extra Practice

Skills Practice p. S6

Application Practice p. S29

Solve each equation.

14. $|x| = \frac{1}{5}$

15. $|2x - 4| = 22$

16. $18 = 3|x - 1|$

17. $-2|x| = -4$

18. $3|x| - 12 = 18$

19. $|x - 42.04| = 23.24$

20. $\left|\frac{2}{3}x - \frac{2}{3}\right| = \frac{2}{3}$

21. $|3x + 1| = 13$

22. $|-2x + 3| = 5.8$

23. $|4x| + 9 = 9$

24. $8 = 7 - |x|$

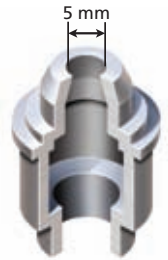
25. $|x| + 6 = 12 - 6$

26. $|x - 3| + 14 = 5$

27. $0 = \left|\frac{2}{3} - x\right|$

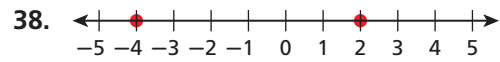
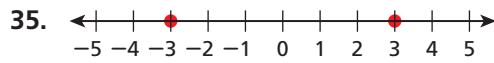
28. $3 + |x - 1| = 3$

29. **Space Shuttle** The diameter of a valve for the space shuttle must be within 0.001 mm of 5 mm. Write and solve an absolute-value equation to find the boundary values for the acceptable diameters of the valve.



30. The two numbers that are 5 units from 3 on the number line are represented by the equation $|n - 3| = 5$. What are these two numbers? Graph the solutions.
31. Write and solve an absolute-value equation that represents two numbers x that are 2 units from 7 on a number line. Graph the solutions.
32. **Manufacturing** A quality control inspector at a bolt factory examines random bolts that come off the assembly line. Any bolt whose diameter differs by more than 0.04 mm from 6.5 mm is sent back. Write and solve an absolute-value equation to find the maximum and minimum diameters of an acceptable bolt.
33. **Construction** A brick company guarantees to fill a contractor's order to within 5% accuracy. A contractor orders 1500 bricks. Write and solve an absolute-value equation to find the maximum and minimum number of bricks guaranteed.
34. **Multi-Step** A machine prints posters and then trims them to the correct size. The equation $|\ell - 65.1| = 0.2$ gives the maximum and minimum acceptable lengths for the posters in inches. Does a poster with a length of 64.8 inches fall within the acceptable range? Why or why not?

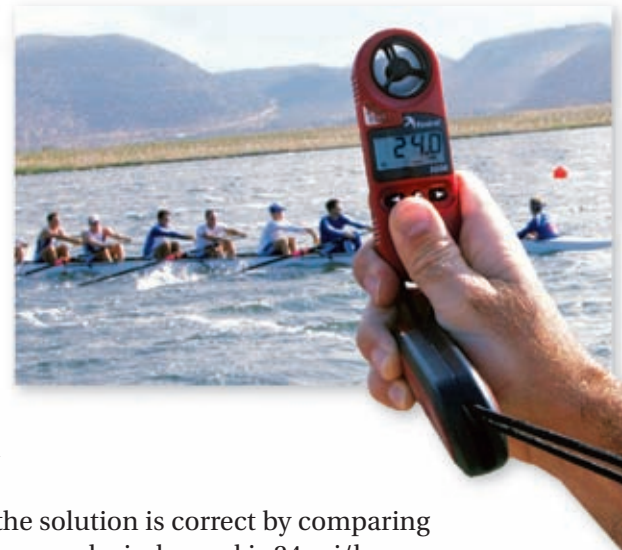
Write an absolute-value equation whose solutions are graphed on the number line.



Tell whether each statement is sometimes, always, or never true. Explain.

39. An absolute-value equation has two solutions.
40. The value of $|x + 4|$ is equal to the value of $|x| + 4$.
41. The absolute value of a number is nonnegative.
42. **Temperature** A thermostat is set so that the temperature in a laboratory freezer stays within 2.5°F of 2°F . Write and solve an absolute-value equation to find the maximum and minimum temperatures in the freezer.
43. **Recreation** To ensure safety, boaters must be aware of wind conditions while they are on the water. A particular anemometer gives a measurement of wind speed within a certain amount of the true wind speed, as shown in the table.

Measured Wind Speed (mi/h)	True Wind Speed (mi/h)
20	15–25
22	17–27
24	19–29
26	21–31
28	23–33
30	25–35



- a. Use the table to write an absolute-value equation for the minimum and maximum possible true wind speeds t for the measured wind speed shown on the anemometer.
- b. Solve your equation from part a. Check that the solution is correct by comparing it to the values given in the table when the measured wind speed is 24 mi/h.
- c. Will your equation work for all of the values in the table? Explain.
- d. Explain what your equation says about the instrument's measurements.

**MULTI-STEP
TEST PREP**



44. This problem will help prepare you for the Multi-Step Test Prep on page 118. The water pumps on a wildland fire apparatus can pump at various rates. The center of the acceptable range of pumping rates is 55 gallons per minute.
- a. Write an absolute-value expression that gives the distance on the number line of a pump's rate r from 55.
- b. The smallest and largest pumps have rates that differ by 45 gallons per minute from the rate at the center of the range. Write an absolute-value equation for the rates of these pumps.
- c. Find the least and greatest rates that are acceptable for a wildland fire apparatus.



45. **Write About It** Do you agree with the following statement: “To solve an absolute-value equation, you need to solve two equations.” Why or why not?
46. **Critical Thinking** Is there a value of a for which the equation $|x - a| = 1$ has exactly one solution? Explain.



47. Which situation could be modeled by the equation $|x - 65| = 3$?
- (A) Two numbers on the number line are 65 units away from 3.
 (B) The length of a carpet is 3 inches less than 65 inches.
 (C) The maximum and minimum weights of wrestlers on the team are within 3 kg of 65 kg.
 (D) The members of an exercise club for seniors are all between 63 and 67 years old.
48. For which of the following is $n = -3$ a solution?
- (F) $|n - 1| = 2$ (G) $|n + 2| = -1$ (H) $|n - 2| = 1$ (J) $|n + 1| = 2$
49. The minimum and maximum sound levels at a rock concert are 90 decibels and 95 decibels. Which equation models this situation?
- (A) $|x - 90| = 5$ (B) $|x - 92.5| = 2.5$ (C) $|x - 92.5| = 5$ (D) $|x - 95| = 2.5$

CHALLENGE AND EXTEND

50. The perimeter of a rectangle is 100 inches. The length of the rectangle is $|2x - 4|$ inches, and the width is x inches. What are the possible values of x ? Explain.
51. Fill in the missing reasons to justify each step in solving the equation $3|2x + 1| = 21$.

Statements	Reasons
1. $3 2x + 1 = 21$	1. Given
2. $ 2x + 1 = 7$	2. _____ ?
3. $2x + 1 = -7$ or $2x + 1 = 7$	3. Definition of absolute value
4. $2x = -8$ or $2x = 6$	4. _____ ?
5. $x = -4$ or $x = 3$	5. _____ ?

52. Solve $|x| = |x + 1|$. (*Hint*: Consider two cases: $x \geq 0$ and $x < 0$.)

SPIRAL REVIEW

53. Sabrina is building a bookcase for her room. She has a piece of wood that is $4\frac{3}{8}$ ft long and estimates that she wants to use about $\frac{2}{5}$ of it. How much wood does she want to use? (*Lesson 1-3*)

Solve each equation. Check your answer. (*Lesson 2-1*)

54. $5 = p - 4.5$ 55. $-2 = y + 6\frac{1}{2}$ 56. $-12 + q = 3$ 57. $y - 4.3 = -5.7$

Solve for the indicated variable. (*Lesson 2-5*)

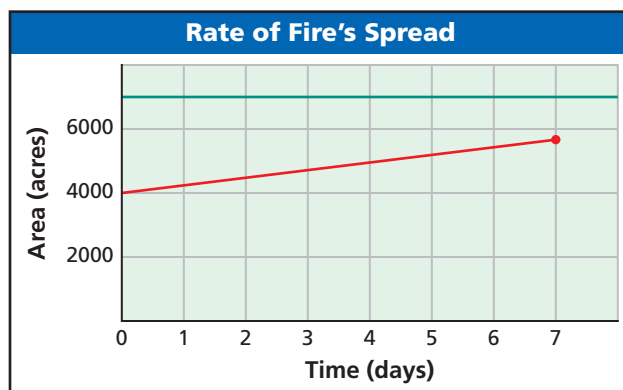
58. $m + 5n = 7$ for m 59. $S = T + R$ for T 60. $2y + 3x = 1$ for y
 61. $\frac{3 + w}{z} = x$ for w 62. $c + d = \frac{5}{e}$ for e 63. $6M - N = S$ for N



Equations and Formulas

All Fired Up A large forest fire in the western United States burns for 14 days, spreading to cover approximately 3850 acres. Firefighters do their best to contain the fire, but hot temperatures and high winds may prompt them to request additional support.

1. The fire spreads at an average rate of how many acres per day?
2. Officials estimate that the fire will spread to cover 9075 acres before it is contained. At this rate, how many more days will it take for the fire to cover an area of 9075 acres? Answer this question using at least two different methods.
3. Additional help arrives, and the firefighters contain the fire in 7 more days. In total, how many acres does the fire cover before it is contained?
4. If the fire had spread to cover an area of 7000 acres, it would have reached Bowman Valley. Explain how the graph shows that firefighters stopped the spread of the fire before it reached Bowman Valley.



5. The total cost of fighting the fire for 21 days was approximately \$1,440,000. What was the approximate cost per acre of fighting the fire?

Quiz for Lessons 2-1 Through 2-6

2-1 Solving Equations by Adding or Subtracting

Solve each equation.

1. $x - 32 = -18$ 2. $1.1 = m - 0.9$ 3. $j + 4 = -17$ 4. $\frac{9}{8} = g + \frac{1}{2}$

5. When she first purchased it, Soledad's computer had 400 GB of hard drive space. After six months, there were only 313 GB available. Write and solve an equation to find the amount of hard drive space that Soledad used in the first six months.

2-2 Solving Equations by Multiplying or Dividing

Solve each equation.

6. $\frac{h}{3} = -12$ 7. $-2.8 = \frac{w}{-3}$ 8. $42 = 3c$ 9. $-0.1b = 3.7$

10. A fund-raiser raised \$2400, which was $\frac{3}{5}$ of the goal. Write and solve an equation to find the amount of the goal.

2-3 Solving Two-Step and Multi-Step Equations

Solve each equation.

11. $2r + 20 = 200$ 12. $\frac{3}{5}k + 5 = 7$ 13. $5n + 6 - 3n = -12$ 14. $4(x - 7) = 2$

15. A taxicab company charges \$2.10 plus \$0.80 per mile. Carmen paid a fare of \$11.70. Write and solve an equation to find the number of miles she traveled.

2-4 Solving Equations with Variables on Both Sides

Solve each equation.

16. $4x - 3 = 2x + 5$ 17. $3(2x - 5) = 2(3x - 2)$
 18. $2(2t - 3) = 6(t + 2)$ 19. $7(x + 5) = -7(x + 5)$

20. On the first day of the year, Diego had \$700 in his savings account and started spending \$35 a week. His brother Juan had \$450 and started saving \$15 a week. After how many weeks will the brothers have the same amount? What will that amount be?

2-5 Solving for a Variable

21. Solve $2x + 3y = 12$ for x . 22. Solve $\frac{x}{7} = v$ for x .
 23. Solve $5j + s = t - 2$ for t . 24. Solve $h + p = 3(k - 8)$ for k .

2-6 Solving Absolute-Value Equations

Solve each equation.

25. $|r| = 7$ 26. $|h + 4| = 11$
 27. $|2x + 4| = 0$ 28. $16 = 7|p + 3| + 30$

2-7

Rates, Ratios, and Proportions

Objectives

Write and use ratios, rates, and unit rates.

Write and solve proportions.

Vocabulary

ratio	proportion
rate	cross products
scale	scale drawing
unit rate	scale model
conversion factor	dimensional analysis

Why learn this?

Ratios and proportions are used to draw accurate maps. (See Example 5.)

A **ratio** is a comparison of two quantities by division. The ratio of a to b can be written $a:b$ or $\frac{a}{b}$, where $b \neq 0$. Ratios that name the same comparison are said to be *equivalent*.

A statement that two ratios are equivalent, such as $\frac{1}{12} = \frac{2}{24}$, is called a **proportion**.



EXAMPLE 1 Using Ratios

Reading Math

Read the proportion $\frac{1}{15} = \frac{x}{675}$ as "1 is to 15 as x is to 675."

The ratio of faculty members to students at a college is 1:15. There are 675 students. How many faculty members are there?

$$\frac{\text{faculty}}{\text{students}} \rightarrow \frac{1}{15}$$

Write a ratio comparing faculty to students.

$$\frac{1}{15} = \frac{x}{675}$$

Write a proportion. Let x be the number of faculty members.

$$675 \left(\frac{x}{675} \right) = 675 \left(\frac{1}{15} \right)$$

Since x is divided by 675, multiply both sides of the equation by 675.

$$x = 45$$

There are 45 faculty members.



- The ratio of games won to games lost for a baseball team is 3:2. The team won 18 games. How many games did the team lose?

A **rate** is a ratio of two quantities with different units, such as $\frac{34 \text{ mi}}{2 \text{ gal}}$. Rates are usually written as *unit rates*. A **unit rate** is a rate with a second quantity of 1 unit, such as $\frac{17 \text{ mi}}{1 \text{ gal}}$, or 17 mi/gal. You can convert any rate to a unit rate.

EXAMPLE 2 Finding Unit Rates

Takeru Kobayashi of Japan ate 53.5 hot dogs in 12 minutes to win a contest. Find the unit rate in hot dogs per minute. Round to the nearest hundredth.

$$\frac{53.5}{12} = \frac{x}{1}$$

Write a proportion to find an equivalent ratio with a second quantity of 1.

$$4.46 \approx x$$

Divide on the left side to find x .

The unit rate is approximately 4.46 hot dogs per minute.



- Cory earns \$52.50 in 7 hours. Find the unit rate in dollars per hour.

Dimensional analysis is a process that uses rates to convert measurements from one unit to another. A rate such as $\frac{12 \text{ in.}}{1 \text{ ft}}$, in which the two quantities are equal but use different units, is called a **conversion factor**. To convert from one set of units to another, multiply by a conversion factor.

EXAMPLE 3 Using Dimensional Analysis

Helpful Hint

In Example 3A Step 1, "pt" appears to divide out, leaving "qt," as the unit. Use this strategy of "dividing out" units when using dimensional analysis.

A A large adult male human has about 12 pints of blood. Use dimensional analysis to convert this quantity to gallons.

Step 1 Convert pints to quarts.

$$12 \text{ pt} \cdot \frac{1 \text{ qt}}{2 \text{ pt}} \quad \text{Multiply by a conversion factor whose first quantity is quarts and whose second quantity is pints.}$$

$$6 \text{ qt}$$

12 pints is 6 quarts.

Step 2 Convert quarts to gallons.

$$6 \text{ qt} \cdot \frac{1 \text{ gal}}{4 \text{ qt}} \quad \text{Multiply by a conversion factor whose first quantity is gallons and whose second quantity is quarts.}$$

$$\frac{6}{4} \text{ gal} = 1\frac{1}{2} \text{ gal}$$

A large adult male human has about $1\frac{1}{2}$ gallons of blood.

B The dwarf sea horse *Hippocampus zosterae* swims at a rate of 52.68 feet per hour. Use dimensional analysis to convert this speed to inches per minute.

Use the conversion factor $\frac{12 \text{ in.}}{1 \text{ ft}}$ to convert feet to inches, and use the conversion factor $\frac{1 \text{ h}}{60 \text{ min}}$ to convert hours to minutes.

$$\frac{52.68 \text{ ft}}{1 \text{ h}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} \cdot \frac{1 \text{ h}}{60 \text{ min}} = \frac{10.536 \text{ in.}}{1 \text{ min}}$$

The speed is 10.536 inches per minute.

Check that the answer is reasonable. The answer is about 10 in./min.

- There are 60 min in 1 h, so 10 in./min is $60(10) = 600$ in./h.
- There are 12 in. in 1 ft, so 600 in./h is $\frac{600}{12} = 50$ ft/h. This is close to the rate given in the problem, 52.68 ft/h.



Hippocampus zosterae



3. A cyclist travels 56 miles in 4 hours. Use dimensional analysis to convert the cyclist's speed to feet per second. Round your answer to the nearest tenth, and show that your answer is reasonable.

In the proportion $\frac{a}{b} = \frac{c}{d}$, the products $a \cdot d$ and $b \cdot c$ are called **cross products**. You can solve a proportion for a missing value by using the Cross Products Property.



Cross Products Property

WORDS

In a proportion, cross products are equal.

NUMBERS

$$\frac{2}{3} \neq \frac{4}{6}$$

$$2 \cdot 6 = 3 \cdot 4$$

ALGEBRA

If $\frac{a}{b} = \frac{c}{d}$ and $b \neq 0$
and $d \neq 0$,
then $ad = bc$.

EXAMPLE 4 Solving Proportions

Solve each proportion.

$$\text{A } \frac{5}{9} = \frac{3}{w}$$

$$\frac{\cancel{5} \times \cancel{3}}{\cancel{9} \times w}$$

$$5(w) = 9(3) \quad \text{Use cross products.}$$

$$5w = 27$$

$$\frac{5w}{5} = \frac{27}{5} \quad \text{Divide both sides by 5.}$$

$$w = \frac{27}{5}$$

$$\text{B } \frac{8}{x+10} = \frac{1}{12}$$

$$\frac{\cancel{8} \times \cancel{1}}{x+10 \times \cancel{12}}$$

$$8(12) = 1(x+10) \quad \text{Use cross products.}$$

$$96 = x + 10$$

$$\frac{-10}{-10} \quad \frac{-10}{-10} \quad \text{Subtract 10 from both sides.}$$

$$86 = x$$



Solve each proportion.

$$4\text{a. } \frac{-5}{2} = \frac{y}{8}$$

$$4\text{b. } \frac{g+3}{5} = \frac{7}{4}$$

A **scale** is a ratio between two sets of measurements, such as 1 in : 5 mi. A **scale drawing** or **scale model** uses a scale to represent an object as smaller or larger than the actual object. A map is an example of a scale drawing.

EXAMPLE 5 Scale Drawings and Scale Models

- A** On the map, the distance from Chicago to Evanston is 0.625 in. What is the actual distance?

$$\frac{\text{map}}{\text{actual}} \rightarrow \frac{1 \text{ in.}}{18 \text{ mi}}$$

Write the scale as a fraction.

$$\frac{1}{18} \times \frac{0.625}{x}$$

Let x be the actual distance.

$$x \cdot 1 = 18(0.625) \quad \text{Use cross products to solve.}$$

$$x = 11.25$$

The actual distance is 11.25 mi.

- B** The actual distance between North Chicago and Waukegan is 4 mi. What is this distance on the map? Round to the nearest tenth.

$$\frac{\text{map}}{\text{actual}} \rightarrow \frac{1 \text{ in.}}{18 \text{ mi}}$$

Write the scale as a fraction.

$$\frac{1}{18} \times \frac{x}{4}$$

Let x be the distance on the map.

$$4 = 18x$$

Use cross products to solve the proportion.

$$\frac{4}{18} = \frac{18x}{18}$$

Since x is multiplied by 18, divide both sides by 18 to undo the multiplication.

$$0.2 \approx x$$

Round to the nearest tenth.

The distance on the map is about 0.2 in.



Reading Math

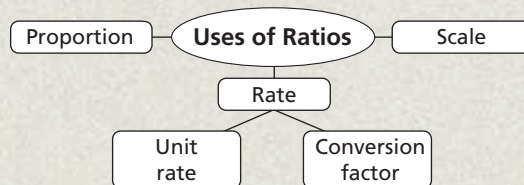
A scale written without units, such as 32:1, means that 32 units of any measure correspond to 1 unit of that same measure.



5. A scale model of a human heart is 16 ft long. The scale is 32:1. How many inches long is the actual heart it represents?

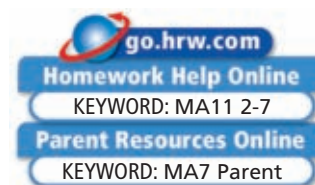
THINK AND DISCUSS

1. Explain two ways to solve the proportion $\frac{t}{4} = \frac{3}{5}$.
2. How could you show that the answer to Example 5A is reasonable?
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of each use of ratios.



2-7

Exercises



GUIDED PRACTICE

SEE EXAMPLE 1
p. 120

1. **Vocabulary** What does it mean when two ratios form a *proportion*?
2. The ratio of the sale price of a jacket to the original price is 3 : 4. The original price is \$64. What is the sale price?
3. **Chemistry** The ratio of hydrogen atoms to oxygen atoms in water is 2 : 1. If an amount of water contains 341 trillion atoms of oxygen, how many hydrogen atoms are there?

SEE EXAMPLE 2
p. 120

4. A computer's fan rotates 2000 times in 40 seconds. Find the unit rate in rotations per second.
5. Twelve cows produce 224,988 pounds of milk. Find the unit rate in pounds per cow.
6. A yellow jacket can fly 4.5 meters in 9 seconds. Find the unit rate in meters per second.

SEE EXAMPLE 3
p. 121

7. Lydia wrote $4\frac{1}{2}$ pages of her science report in one hour. What was her writing rate in pages per minute?
8. A model airplane flies 18 feet in 2 seconds. What is the airplane's speed in miles per hour? Round your answer to the nearest hundredth.
9. A vehicle uses 1 tablespoon of gasoline to drive 125 yards. How many miles can the vehicle travel per gallon? Round your answer to the nearest mile. (*Hint:* There are 256 tablespoons in a gallon.)

SEE EXAMPLE 4
p. 122

Solve each proportion.

10. $\frac{3}{z} = \frac{1}{8}$

11. $\frac{x}{3} = \frac{1}{5}$

12. $\frac{b}{4} = \frac{3}{2}$

13. $\frac{f+3}{12} = \frac{7}{2}$

14. $\frac{-1}{5} = \frac{3}{2d}$

15. $\frac{3}{14} = \frac{s-2}{21}$

16. $\frac{-4}{9} = \frac{7}{x}$

17. $\frac{3}{s-2} = \frac{1}{7}$

18. $\frac{10}{h} = \frac{52}{13}$

SEE EXAMPLE 5

p. 122

19. **Archaeology** Stonehenge II in Hunt, Texas, is a scale model of the ancient construction in Wiltshire, England. The scale of the model to the original is 3 : 5. The Altar Stone of the original construction is 4.9 meters tall. Write and solve a proportion to find the height of the model of the Altar Stone.



Alfred Sheppard, one of the builders of Stonehenge II.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
20–21	1
22–23	2
24–25	3
26–37	4
38	5

Extra Practice

Skills Practice p. 57

Application Practice p. 529

20. **Gardening** The ratio of the height of a bonsai ficus tree to the height of a full-size ficus tree is 1 : 9. The bonsai ficus is 6 inches tall. What is the height of a full-size ficus?
21. **Manufacturing** At one factory, the ratio of defective light bulbs produced to total light bulbs produced is about 3 : 500. How many light bulbs are expected to be defective when 12,000 are produced?
22. Four gallons of gasoline weigh 25 pounds. Find the unit rate in pounds per gallon.
23. Fifteen ounces of gold cost \$6058.50. Find the unit rate in dollars per ounce.
24. **Biology** The tropical giant bamboo can grow 11.9 feet in 3 days. What is this rate of growth in inches per hour? Round your answer to the nearest hundredth, and show that your answer is reasonable.
25. **Transportation** The maximum speed of the Tupolev Tu-144 airliner is 694 m/s. What is this speed in kilometers per hour?

Solve each proportion.

26. $\frac{v}{6} = \frac{1}{2}$

27. $\frac{2}{5} = \frac{4}{y}$

28. $\frac{2}{h} = \frac{-5}{6}$

29. $\frac{3}{10} = \frac{b+7}{20}$

30. $\frac{5t}{9} = \frac{1}{2}$

31. $\frac{2}{3} = \frac{6}{q-4}$

32. $\frac{x}{8} = \frac{7.5}{20}$

33. $\frac{3}{k} = \frac{45}{18}$

34. $\frac{6}{a} = \frac{15}{17}$

35. $\frac{9}{2} = \frac{5}{x+1}$

36. $\frac{3}{5} = \frac{x}{100}$

37. $\frac{38}{19} = \frac{n-5}{20}$

38. **Science** The image shows a dust mite as seen under a microscope. The scale of the drawing to the dust mite is 100:1. Use a ruler to measure the length of the dust mite in the image in millimeters. What is the actual length of the dust mite?



39. **Finance** On a certain day, the exchange rate was 60 U.S. dollars for 50 euro. How many U.S. dollars were 70 euro worth that day? Show that your answer is reasonable.

40. **Environmental Science** An environmental scientist wants to estimate the number of carp in a pond. He captures 100 carp, tags all of them, and releases them. A week later, he captures 85 carp and records how many have tags. His results are shown in the table. Write and solve a proportion to estimate the number of carp in the pond.

Status	Number Captured
Tagged	20
Not tagged	65

41. **/// ERROR ANALYSIS ///** Below is a bonus question that appeared on an algebra test and a student's response.

The ratio of junior varsity members to varsity members on the track team is 3:5. There are 24 members on the team. Write a proportion to find the number of junior varsity members.

$$\frac{3}{5} = \frac{x}{24}$$



Sports



The records for the women's 100-meter dash and the women's 200-meter dash were set by Florence Griffith-Joyner, known as "Flo Jo." She is still referred to as the world's fastest woman.

The student did not receive the bonus points. Why is this proportion incorrect?

42. **Sports** The table shows world record times for women's races of different distances.

World Records (Women)	
Distance (m)	Time (s)
100	10.5
200	21.3
800	113.3
5000	864.7

- Find the speed in meters per second for each race. Round your answers to the nearest hundredth.
- Which race has the fastest speed? the slowest?
- Critical Thinking** Give a possible reason why the speeds are different.

43. **Entertainment** Lynn, Faith, and Jeremy are film animators. In one 8-hour day, Lynn rendered 203 frames, Faith rendered 216 frames, and Jeremy rendered 227 frames. How many more frames per hour did Faith render than Lynn did?

Solve each proportion.

44. $\frac{x-1}{3} = \frac{x+1}{5}$ 45. $\frac{m}{3} = \frac{m+4}{7}$ 46. $\frac{1}{x-3} = \frac{3}{x-5}$ 47. $\frac{a}{2} = \frac{a-4}{30}$
 48. $\frac{3}{2y} = \frac{16}{y+2}$ 49. $\frac{n+3}{5} = \frac{n-1}{2}$ 50. $\frac{1}{y} = \frac{1}{6y-1}$ 51. $\frac{2}{n} = \frac{4}{n+3}$
 52. $\frac{5t-3}{-2} = \frac{t+3}{2}$ 53. $\frac{3}{d+3} = \frac{4}{d+12}$ 54. $\frac{3x+5}{14} = \frac{x}{3}$ 55. $\frac{5}{2n} = \frac{8}{3n-24}$

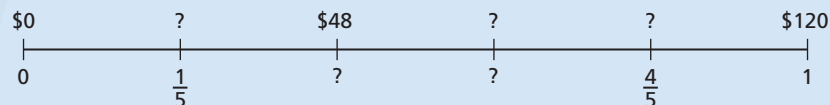
56. **Decorating** A particular shade of paint is made by mixing 5 parts red paint with 7 parts blue paint. To make this shade, Shannon mixed 12 quarts of blue paint with 8 quarts of red paint. Did Shannon mix the correct shade? Explain.
57. **Write About It** Give three examples of proportions. How do you know they are proportions? Then give three nonexamples of proportions. How do you know they are not proportions?

MULTI-STEP TEST PREP



58. This problem will prepare you for the Multi-Step Test Prep on page 152.

- Marcus is shopping for a new jacket. He finds one with a price tag of \$120. Above the rack is a sign that says that he can take off $\frac{1}{5}$. Find out how much Marcus can deduct from the price of the jacket.
- What price will Marcus pay for the jacket?
- Copy the model below. Complete it by placing numerical values on top and the corresponding fractional parts below.



- Explain how this model shows proportional relationships.

59. One day the U.S. dollar was worth approximately 100 yen. An exchange of 2500 yen was made that day. What was the value of the exchange in dollars?
 (A) \$25 (B) \$400 (C) \$2500 (D) \$40,000
60. Brett walks at a speed of 4 miles per hour. He walks for 20 minutes in a straight line at this rate. Approximately what distance does Brett walk?
 (F) 0.06 miles (G) 1.3 miles (H) 5 miles (J) 80 miles
61. A shampoo company conducted a survey and found that 3 out of 8 people use their brand of shampoo. Which proportion could be used to find the expected number of users n in a city of 75,000 people?
 (A) $\frac{3}{8} = \frac{75,000}{n}$ (B) $\frac{3}{75,000} = \frac{n}{8}$ (C) $\frac{8}{3} = \frac{n}{75,000}$ (D) $\frac{3}{8} = \frac{n}{75,000}$
62. A statue is 3 feet tall. The display case for a model of the statue can fit a model that is no more than 9 inches tall. Which of the scales below allows for the tallest model of the statue that will fit in the display case?
 (F) 2:1 (G) 1:1 (H) 1:3 (J) 1:4

CHALLENGE AND EXTEND



63. **Geometry** Complementary angles are two angles whose measures add up to 90° . The ratio of the measures of two complementary angles is 4:5. What are the measures of the angles?
64. A customer wanted 24 feet of rope. The clerk at the hardware store used what she thought was a yardstick to measure the rope, but the yardstick was actually 2 inches too short. How many inches were missing from the customer's piece of rope?
65. **Population** The population density of Jackson, Mississippi, is 672.2 people per square kilometer. What is the population density in people per square meter? Show that your answer is reasonable. (*Hint:* There are 1000 meters in 1 kilometer. How many square meters are in 1 square kilometer?)

SPIRAL REVIEW

Evaluate each expression. (*Lesson 1-4*)

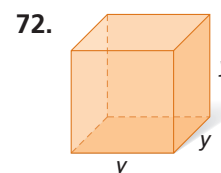
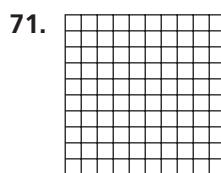
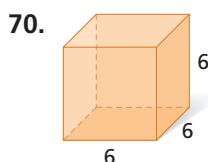
66. 8^2

67. $(-3)^3$

68. $(-3)^2$

69. $-\left(\frac{1}{2}\right)^5$

Write the power represented by each geometric model. (*Lesson 1-4*)



Solve each equation. Check your answer. (*Lesson 2-4*)

73. $2x - 12 = 5x + 3$

74. $3a - 4 = 6 - 7a$

75. $3x - 4 = 2x + 4$

Solve for the indicated variable. (*Lesson 2-5*)

76. $y = mx + b$ for b

77. $PV = nRT$ for V

78. $A = \frac{1}{2}bh$ for h

2-8

Applications of Proportions

Objectives

Use proportions to solve problems involving geometric figures.

Use proportions and similar figures to measure objects indirectly.

Vocabulary

similar
corresponding sides
corresponding angles
indirect measurement
scale factor

Why learn this?

Proportions can be used to find the heights of tall objects, such as totem poles, that would otherwise be difficult to measure. (See Example 2.)

Similar figures have exactly the same shape but not necessarily the same size.

Corresponding sides of two figures are in the same relative position, and **corresponding angles** are in the same relative position. Two figures are similar if and only if the lengths of corresponding sides are proportional and all pairs of corresponding angles have equal measures.



Reading Math

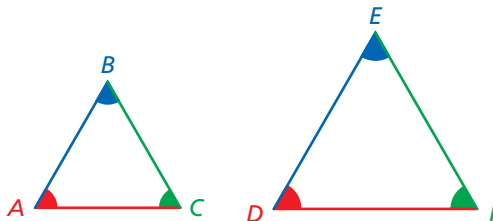
- \overline{AB} means segment AB . AB means the length of \overline{AB} .
- $\angle A$ means angle A . $m\angle A$ means the measure of angle A .

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$m\angle A = m\angle D$$

$$m\angle B = m\angle E$$

$$m\angle C = m\angle F$$



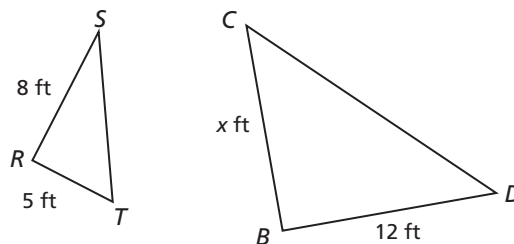
When stating that two figures are similar, use the symbol \sim . For the triangles above, you can write $\triangle ABC \sim \triangle DEF$. Make sure corresponding vertices are in the same order. It would be incorrect to write $\triangle ABC \sim \triangle EFD$.

You can use proportions to find missing lengths in similar figures.

EXAMPLE 1 Finding Missing Measures in Similar Figures

Find the value of x in each diagram.

A $\triangle RST \sim \triangle BCD$



R corresponds to B , S corresponds to C , and T corresponds to D .

$$\frac{5}{12} = \frac{8}{x}$$

$$5x = 96$$

$$\frac{5x}{5} = \frac{96}{5}$$

$$x = 19.2$$

The length of \overline{BC} is 19.2 ft.

$$\frac{RT}{BD} = \frac{RS}{BC}$$

Use cross products.

Since x is multiplied by 5, divide both sides by 5 to undo the multiplication.

Find the value of x in each diagram.

B $FGHJKL \sim MNPQRS$

$$\frac{6}{4} = \frac{x}{2}$$

$$4x = 12$$

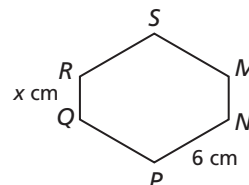
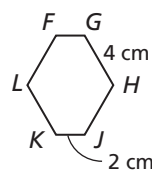
$$\frac{4x}{4} = \frac{12}{4}$$

$$x = 3$$

$$\frac{NP}{GH} = \frac{RQ}{KJ}$$

Use cross products.

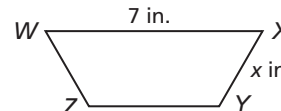
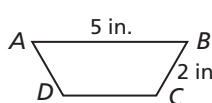
Since x is multiplied by 4, divide both sides by 4 to undo the multiplication.



The length of \overline{QR} is 3 cm.



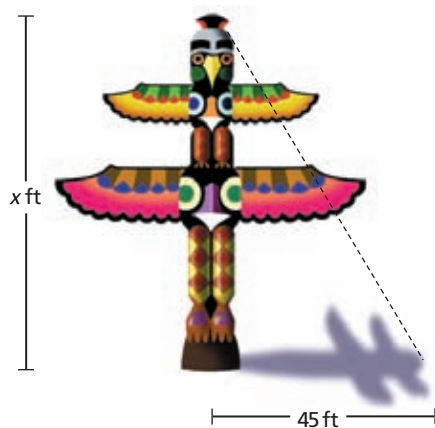
1. Find the value of x in the diagram if $ABCD \sim WXYZ$.



You can solve a proportion involving similar triangles to find a length that is not easily measured. This method of measurement is called **indirect measurement**. If two objects form right angles with the ground, you can apply indirect measurement using their shadows.

EXAMPLE 2 Measurement Application

A totem pole casts a shadow 45 feet long at the same time that a 6-foot-tall man casts a shadow that is 3 feet long. Write and solve a proportion to find the height of the totem pole.



Both the man and the totem pole form right angles with the ground, and their shadows are cast at the same angle. You can form two similar right triangles.



Helpful Hint

A height of 90 ft seems reasonable for a totem pole. If you got 900 or 9000 ft, that would not be reasonable, and you should check your work.

$$\frac{6}{x} = \frac{3}{45}$$

$$3x = 270$$

$$\frac{3x}{3} = \frac{270}{3}$$

$$x = 90$$

$$\frac{\text{man's height}}{\text{pole's height}} = \frac{\text{man's shadow}}{\text{pole's shadow}}$$

Use cross products. Since x is multiplied by 3, divide both sides by 3 to undo the multiplication.

The totem pole is 90 feet tall.

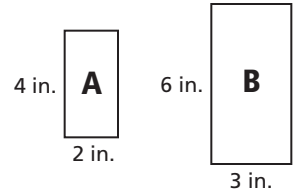


- 2a. A forest ranger who is 150 cm tall casts a shadow 45 cm long. At the same time, a nearby tree casts a shadow 195 cm long. Write and solve a proportion to find the height of the tree.
- 2b. A woman who is 5.5 feet tall casts a shadow 3.5 feet long. At the same time, a building casts a shadow 28 feet long. Write and solve a proportion to find the height of the building.

If every dimension of a figure is multiplied by the same number, the result is a similar figure. The multiplier is called a **scale factor**.

EXAMPLE 3 Changing Dimensions

A Every dimension of a 2-by-4-inch rectangle is multiplied by 1.5 to form a similar rectangle. How is the ratio of the perimeters related to the ratio of corresponding sides? How is the ratio of the areas related to the ratio of corresponding sides?



	Rectangle A	Rectangle B
$P = 2\ell + 2w$	$2(2) + 2(4) = 12$	$2(6) + 2(3) = 18$
$A = \ell w$	$4(2) = 8$	$6(3) = 18$

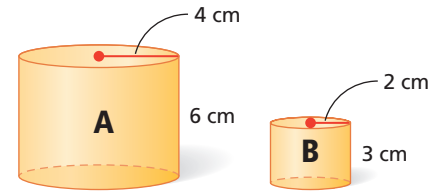
Sides: $\frac{4}{6} = \frac{2}{3}$ Perimeters: $\frac{12}{18} = \frac{2}{3}$ Areas: $\frac{8}{18} = \frac{4}{9} = \left(\frac{2}{3}\right)^2$

The ratio of the perimeters is equal to the ratio of corresponding sides. The ratio of the areas is the square of the ratio of corresponding sides.

Helpful Hint

A scale factor between 0 and 1 reduces a figure. A scale factor greater than 1 enlarges it.

B Every dimension of a cylinder with radius 4 cm and height 6 cm is multiplied by $\frac{1}{2}$ to form a similar cylinder. How is the ratio of the volumes related to the ratio of corresponding dimensions?



	Cylinder A	Cylinder B
$V = \pi r^2 h$	$\pi(4)^2(6) = 96\pi$	$\pi(2)^2(3) = 12\pi$

Radii: $\frac{4}{2} = \frac{2}{1} = 2$ Heights: $\frac{6}{3} = \frac{2}{1} = 2$ Volumes: $\frac{96\pi}{12\pi} = \frac{8}{1} = 8 = 2^3$

The ratio of the volumes is the cube of the ratio of corresponding dimensions.

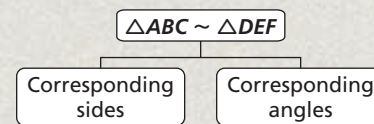


3. A rectangle has width 12 inches and length 3 inches. Every dimension of the rectangle is multiplied by $\frac{1}{3}$ to form a similar rectangle. How is the ratio of the perimeters related to the ratio of the corresponding sides?

THINK AND DISCUSS

1. Name some pairs of real-world items that appear to be similar figures.

2. **GET ORGANIZED** Copy and complete the graphic organizer. In the top box, sketch and label two similar triangles. Then list the corresponding sides and angles in the bottom boxes.



GUIDED PRACTICE

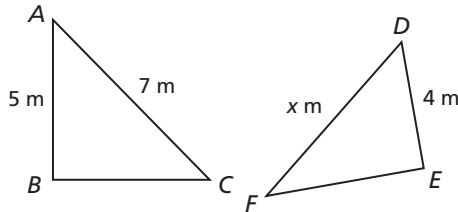
1. **Vocabulary** What does it mean for two figures to be *similar*?

SEE EXAMPLE 1

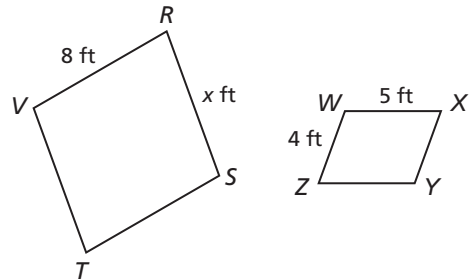
Find the value of x in each diagram.

p. 127

2. $\triangle ABC \sim \triangle DEF$



3. $RSTV \sim WXYZ$



SEE EXAMPLE 2

4. Roger is 5 feet tall and casts a shadow 3.5 feet long. At the same time, the flagpole outside his school casts a shadow 14 feet long. Write and solve a proportion to find the height of the flagpole.

p. 128

SEE EXAMPLE 3

5. A rectangle has length 12 feet and width 8 feet. Every dimension of the rectangle is multiplied by $\frac{3}{4}$ to form a similar rectangle. How is the ratio of the areas related to the ratio of corresponding sides?

p. 129

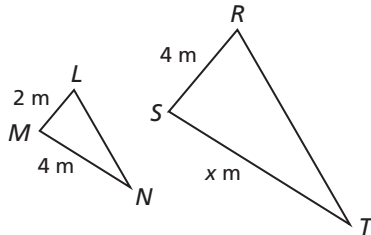
PRACTICE AND PROBLEM SOLVING

Find the value of x in each diagram.

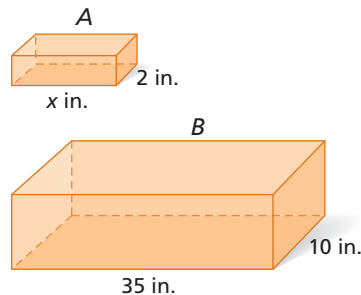
Independent Practice

For Exercises	See Example
6–7	1
8	2
9	3

6. $\triangle LMN \sim \triangle RST$



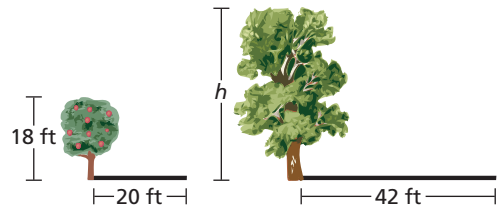
7. prism A \sim prism B



Extra Practice

Skills Practice p. S7
 Application Practice p. S29

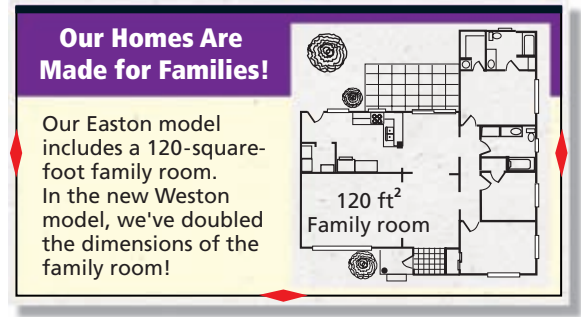
8. Write and solve a proportion to find the height of the taller tree in the diagram at right.



9. A triangle has side lengths of 5 inches, 12 inches, and 15 inches. Every dimension is multiplied by $\frac{1}{5}$ to form a new triangle. How is the ratio of the perimeters related to the ratio of corresponding sides?

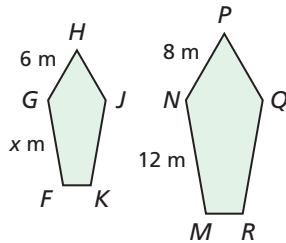
10. **Hobbies** For a baby shower gift, Heather crocheted a baby blanket whose length was $2\frac{1}{2}$ feet and whose width was 2 feet. She plans to crochet a proportionally larger similar blanket for the baby's mother. If she wants the length of the mother's blanket to be $6\frac{1}{4}$ feet, what should the width be? Show that your answer is reasonable.

11. **Real Estate** Refer to the home builder's advertisement. The family rooms in both models are rectangular. How much carpeting is needed to carpet the family room in the Weston model?
12. A rectangle has an area of 16 ft^2 . Every dimension is multiplied by a scale factor, and the new rectangle has an area of 64 ft^2 . What was the scale factor?
13. A cone has a volume of $98\pi \text{ cm}^3$. Every dimension is multiplied by a scale factor, and the new cone has a volume of $6272\pi \text{ cm}^3$. What was the scale factor?

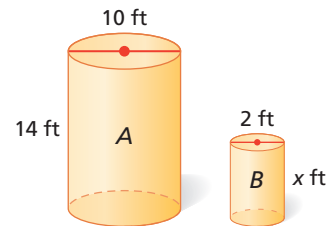


Find the value of x in each diagram.

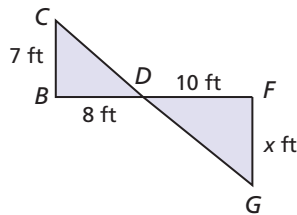
14. $FGHJK \sim MNPQR$



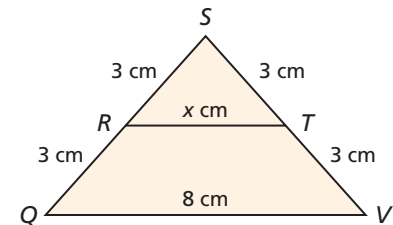
15. cylinder $A \sim$ cylinder B



16. $\triangle BCD \sim \triangle FGD$



17. $\triangle RST \sim \triangle QSV$



18. A tower casts a 450 ft shadow at the same time that a 4 ft child casts a 6 ft shadow. Write and solve a proportion to find the height of the tower.
19. **Write About It** At Pizza Palace, a pizza with a diameter of 8 inches costs \$6.00. The restaurant manager says that a 16-inch pizza should be priced at \$12.00 because it is twice as large. Do you agree? Explain why or why not.

**MULTI-STEP
TEST PREP**



20. This problem will prepare you for the Multi-Step Test Prep on page 152.

Another common application of proportion is *percents*. A percent is a ratio of a number to 100. For example, $80\% = \frac{80}{100}$.

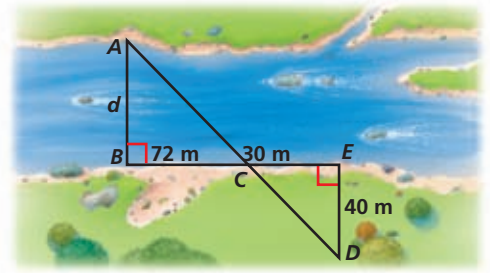
- Write 12%, 18%, 25%, 67%, and 98% as ratios.
- Percents can also be written as decimals. Write each of your ratios from part **a** as a decimal.
- What do you notice about a percent and its decimal equivalent?

You will learn more about percents and their connections to proportions in upcoming lessons.

21. A lighthouse casts a shadow that is 36 meters long. At the same time, a person who is 1.5 meters tall casts a shadow that is 4.5 meters long. Write and solve a proportion to find the height of the lighthouse.

22. In the diagram, $\triangle ABC \sim \triangle DEC$. What is the distance across the river from A to B ?

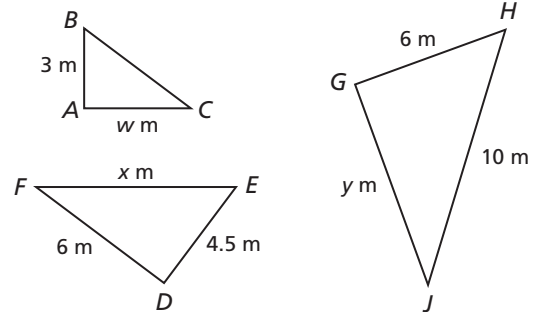
23. **Critical Thinking** If every dimension of a two-dimensional figure is multiplied by k , by what quantity is the area multiplied?



24. A beach ball holds 800 cubic inches of air. Another beach ball has a radius that is half that of the larger ball. How much air does the smaller ball hold?
- (A) 400 cubic inches (C) 100 cubic inches
(B) 200 cubic inches (D) 80 cubic inches
25. For two similar triangles, $\frac{SG}{MW} = \frac{GT}{WR} = \frac{TS}{RM}$. Which statement below is NOT correct?
- (F) $\triangle SGT \sim \triangle MWR$ (H) $\triangle TGS \sim \triangle RWM$
(G) $\triangle GST \sim \triangle MRW$ (J) $\triangle GTS \sim \triangle WRM$
26. **Gridded Response** A rectangle has length 5 centimeters and width 3 centimeters. A similar rectangle has length 7.25 centimeters. What is the width in centimeters of this rectangle?

CHALLENGE AND EXTEND

27. Find the values of w , x , and y given that $\triangle ABC \sim \triangle DEF \sim \triangle GHJ$.
28. $\triangle RST \sim \triangle VWX$ and $\frac{RT}{VX} = b$.
What is $\frac{\text{area of } \triangle RST}{\text{area of } \triangle VWX}$?
29. **Multi-Step** Rectangles A and B are similar. The area of A is 30.195 cm^2 . The length of B is 6.1 cm . Each dimension of B is $\frac{2}{3}$ the corresponding dimension of A . What is the perimeter of B ?



SPIRAL REVIEW

Add or subtract. (Lesson 1-2)

30. $-9 - 2$ 31. $-7 + (-5)$ 32. $12 - (-18)$ 33. $19 - 65$

Generate ordered pairs for each function for $x = -2, -1, 0, 1, 2$. (Lesson 1-8)

34. $y = 2x$ 35. $y = x^2$ 36. $y = 6 - x$ 37. $y = 3x - 1$

Solve each proportion. (Lesson 2-7)

38. $\frac{x}{8} = \frac{1}{4}$ 39. $\frac{6}{x} = \frac{3}{16}$ 40. $\frac{5}{12} = \frac{-4}{f}$ 41. $\frac{3}{10} = \frac{x+1}{15}$

2-9

Percents

Objective

Solve problems involving percents.

Vocabulary

percent

Who uses this?

Jewelers use percents to determine the purity of precious metals. (See Example 4.)



A **percent** is a ratio that compares a number to 100. For example, $25\% = \frac{25}{100}$.

To find the fraction equivalent of a percent, write the percent as a ratio with a denominator of 100. Then simplify.

$$25\% = \frac{25}{100} = \frac{1}{4}$$

To find the decimal equivalent of a percent, divide by 100.

$$25\% = \frac{25}{100} = 0.25$$



Some Common Equivalents											
Percent	10%	20%	25%	$33\frac{1}{3}\%$	40%	50%	60%	$66\frac{2}{3}\%$	75%	80%	100%
Fraction	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	1
Decimal	0.1	0.2	0.25	$0.\bar{3}$	0.4	0.5	0.6	$0.\bar{6}$	0.75	0.8	1.0

The greatest percent shown in the table is 100%, or 1. But percents can be greater than 100%. For example, $120\% = \frac{120}{100} = 1.2$. You can also find percents that are less than 1%. For example, $0.5\% = \frac{0.5}{100} = 0.005$.

You can use the proportion $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$ to find unknown values.

EXAMPLE 1 Finding the Part

A Find 50% of 20.

Method 1 Use a proportion.

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100} \quad \text{Use the percent proportion.}$$

$$\frac{x}{20} = \frac{50}{100} \quad \text{Let } x \text{ represent the part.}$$

$$100x = 1000 \quad \text{Find the cross products.}$$

$$x = 10 \quad \text{Since } x \text{ is multiplied by 100, divide both sides by 100 to undo the multiplication.}$$

50% of 20 is 10.

Check 50% is the same as $\frac{1}{2}$, and $\frac{1}{2}$ of 20 is 10. ✓

B Find 105% of 72.

Method 2 Use an equation.

$$x = 105\% \text{ of } 72 \quad \textit{Write an equation. Let } x \textit{ represent the part.}$$

$$x = 1.05(72) \quad \textit{Write the percent as a decimal and multiply.}$$

$$x = 75.6$$

$$105\% \text{ of } 72 \text{ is } 75.6.$$



1a. Find 20% of 60. **1b.** Find 210% of 8. **1c.** Find 4% of 36.

EXAMPLE 2 Finding the Percent

Helpful Hint

Before solving, decide what is a reasonable answer. For Example 2A, 50% of 60 is 30, so the answer will be less than 50% of 60.

A What percent of 60 is 15?

Method 1 Use a proportion.

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100} \quad \textit{Use the percent proportion.}$$

$$\frac{15}{60} = \frac{x}{100} \quad \textit{Let } x \textit{ represent the percent.}$$

$$60x = 1500 \quad \textit{Find the cross products.}$$

$$\frac{60x}{60} = \frac{1500}{60} \quad \textit{Since } x \textit{ is multiplied by } 60, \textit{ divide both sides by } 60 \textit{ to undo the multiplication.}$$

$$x = 25$$

$$15 \text{ is } 25\% \text{ of } 60.$$

B 440 is what percent of 400?

Method 2 Use an equation.

$$440 = x \cdot 400 \quad \textit{Write an equation. Let } x \textit{ represent the percent.}$$

$$440 = 400x \quad \textit{Since } x \textit{ is multiplied by } 400, \textit{ divide both sides by } 400 \textit{ to undo the multiplication.}$$

$$\frac{440}{400} = \frac{400x}{400}$$

$$1.1 = x \quad \textit{The answer is a decimal.}$$

$$110\% = x \quad \textit{Write the decimal as a percent. This answer is}$$

$$440 \text{ is } 110\% \text{ of } 400. \quad \textit{reasonable; } 440 \text{ is more than } 100\% \text{ of } 400.$$



2a. What percent of 35 is 7? **2b.** 27 is what percent of 9?

EXAMPLE 3 Finding the Whole

A 40% of what number is 14?

Method 1 Use a proportion.

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100} \quad \textit{Use the percent proportion.}$$

$$\frac{14}{x} = \frac{40}{100} \quad \textit{Let } x \textit{ represent the whole.}$$

$$40x = 1400 \quad \textit{Find the cross products.}$$

$$\frac{40x}{40} = \frac{1400}{40} \quad \textit{Since } x \textit{ is multiplied by } 40, \textit{ divide both sides by } 40 \textit{ to undo the multiplication.}$$

$$x = 35$$

$$40\% \text{ of } 35 \text{ is } 14.$$

B 40 is 0.8% of what number?

Method 2 Use an equation.

$$40 = 0.8\% \text{ of } x$$

$$40 = 0.008 \cdot x$$

$$\frac{40}{0.008} = \frac{0.008x}{0.008}$$

$$5000 = x$$

40 is 0.8% of 5000.

Write an equation. Let x represent the whole.

Write the percent as a decimal.

Since x is multiplied by 0.008, divide both sides by 0.008 to undo the multiplication.



3a. 120% of what number is 90?

3b. 48 is 15% of what number?

EXAMPLE 4 Career Application

Jewelers use the karat system to determine the amount of pure gold in jewelry. Pure gold is 24 karat, meaning the item is 100% gold. A 14-karat gold ring contains 14 parts gold and 10 parts other metal. What percent of the ring is gold? Round your answer to the nearest percent.



$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$$

Use the percent proportion.

$$\frac{14}{24} = \frac{x}{100}$$

Let x represent the percent.

$$24x = 1400$$

Find the cross products.

$$\frac{24x}{24} = \frac{1400}{24}$$

Since x is multiplied by 24, divide both sides by 24 to undo the multiplication.

$$x = 58.\bar{3}$$

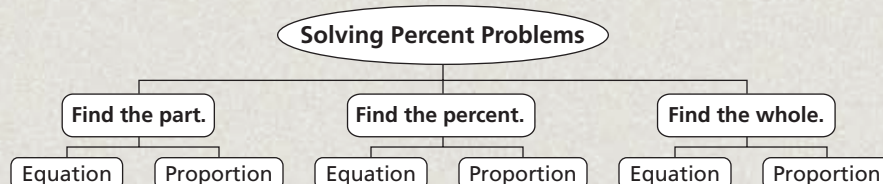
A 14-karat gold ring is approximately 58% gold.



4. Use the information above to find the number of karats in a bracelet that is 42% gold. Round your answer to the nearest whole number.

THINK AND DISCUSS

- Describe the numerical value of the percent when the part is greater than the whole. Give an example.
- 64% of a number is 32. Is the number greater than or less than 32? Explain.
- GET ORGANIZED** Copy and complete the graphic organizer. In each box, write and solve an example using the given method.



GUIDED PRACTICE

1. **Vocabulary** In your own words, write a definition of *percent*.

SEE EXAMPLE 1

p. 133

2. Find 75% of 40.

3. Find $12\frac{1}{2}\%$ of 168.

4. Find 115% of 57.

5. Find 70% of 8.

SEE EXAMPLE 2

p. 134

6. What percent of 40 is 25?

7. What percent of 225 is 180?

8. 57 is what percent of 30?

9. 1 is what percent of 8?

SEE EXAMPLE 3

p. 134

10. 28 is 32% of what number?

11. 4% of what number is 7?

12. 16 is 10% of what number?

13. 105% of what number is 37.8?

SEE EXAMPLE 4

p. 135

14. **Nutrition** A certain granola bar has 2 grams of fiber. This is 8% of the recommended daily value. How many grams of fiber are recommended daily?

PRACTICE AND PROBLEM SOLVING

Find each value. Round to the nearest tenth if necessary.

15. 60% of 80

16. 35% of 90

17. $\frac{1}{2}\%$ of 500

18. 210% of 30

19. What percent of 52 is 13?

20. What percent of 9 is 27?

21. 11 is what percent of 22?

22. 5 is what percent of 67?

23. 36 is 90% of what number?

24. 8.2 is 2% of what number?

25. $4\frac{1}{2}\%$ of what number is 23?

26. 16% of what number is 94?

27. **Nutrition** A certain can of iced tea contains 4% of the recommended daily allowance of sodium. The recommended daily allowance is 2500 milligrams. How many milligrams of sodium are in the can of iced tea?

Write each decimal or fraction as a percent.

28. $\frac{5}{4}$

29. 0.02

30. 0.27

31. $\frac{2}{25}$

32. $\frac{7}{7}$

33. 0.64

34. $\frac{31}{100}$

35. 0.85

36. 0.003

37. $\frac{17}{20}$

Write each percent as a decimal and as a fraction.

38. 23%

39. 52%

40. 12.5%

41. 90%

42. 87.2%

43. 112%

44. 29%

45. 6%

46. 1.5%

47. $\frac{3}{5}\%$

48. **Estimation** To estimate 26% of 400, think:

$$26\% \text{ is close to } 25\% \text{ and } 25\% = \frac{1}{4}$$

$$\frac{1}{4} \text{ of } 400 = 100.$$

Therefore, 26% of 400 is about 100.

Use a similar method to estimate 48% of 610 and 73% of 820. Then check your estimates by finding each percent.

Independent Practice

For Exercises	See Example
15–18	1
19–22	2
23–26	3
27	4

Extra Practice

Skills Practice p. S7

Application Practice p. S29

49. **Critical Thinking** Which is greater, 0.5 or $\frac{1}{2}\%$? Explain.

Write each list in order from least to greatest.

50. $\frac{1}{20}$, 5.3%, 5.1, 0.005, $\frac{1}{2}$

51. 1.1, 11%, $\frac{1}{10}$, 0.001, 1%

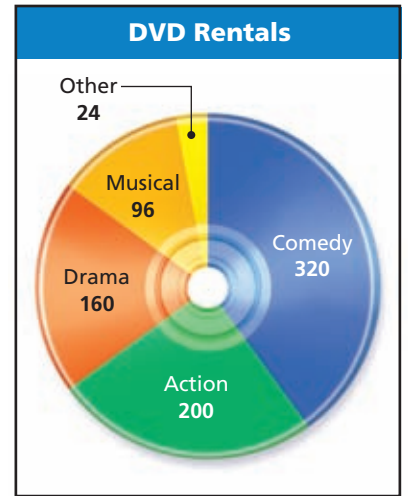
52. $\frac{3}{8}$, 29%, $\frac{2}{5}$, 0.25, 38%

53. 0.49, 82%, 0.94, $\frac{4}{5}$, $\frac{5}{9}$

54. **Biology** On average, sloths spend 16.5 hours per day sleeping. What percent of the day do sloths spend sleeping? Round your answer to the nearest percent.

55. **Entertainment** The numbers of various types of movies rented over a period of time are indicated in the graph.

- What percent of the movies rented were comedies?
- What type of movie made up 25% of the rentals?
- What percent of the movies rented were in the "other" category?
- What if...?** If 25 of the comedy rentals had instead been action rentals, what percent of the movies rented would have been comedies? Round your answer to the nearest tenth.



56. **Multi-Step** According to the 2000 U.S. Census, 138,053,563 Americans are male, and 143,368,343 Americans are female. About what percent of the population is male? female? Round your answers to the nearest percent.

57. Complete each statement in the table below. Describe any patterns you see in the completed table.

1% of 400 is 4.	100% of <input type="text"/> is 12.	<input type="text"/> % of 80 is 20.
2% of <input type="text"/> is 4.	50% of <input type="text"/> is 12.	<input type="text"/> % of 40 is 20.
4% of <input type="text"/> is 4.	25% of <input type="text"/> is 12.	<input type="text"/> % of 20 is 20.
8% of <input type="text"/> is 4.	12.5% of <input type="text"/> is 12.	<input type="text"/> % of 10 is 20.



58. **Write About It** Explain the advantages of using the proportion method to solve percent problems. Then, explain the advantages of using the equation method to solve percent problems.

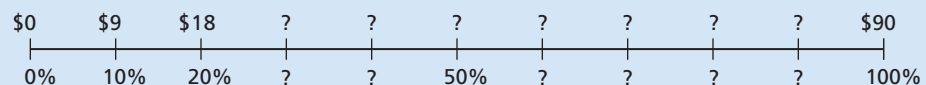
**MULTI-STEP
TEST PREP**



59. This problem will prepare you for the Multi-Step Test Prep on page 152.

Kathryn found a new dress at the mall. The price tag reads \$90. The sign above the rack of dresses says that all items on the rack are 40% off.

- Set up a proportion to find 40% of \$90. This is the amount of the discount.
- How much will Kathryn pay for the dress? Show that your answer is reasonable.
- Copy and complete the model below. Explain how this model can help you answer the above questions in another way.



60. Which proportion can be used to find 14% of 60?
- (A) $\frac{x}{100} = \frac{60}{14}$ (B) $\frac{14}{100} = \frac{60}{x}$ (C) $\frac{x}{100} = \frac{14}{60}$ (D) $\frac{14}{100} = \frac{x}{60}$
61. Raul surveyed 35 students about their preferred lunch. Fourteen preferred chicken. Half of those students preferred chicken with barbecue sauce. What percent should Raul report as preferring chicken with barbecue sauce?
- (F) 20% (G) 40% (H) 50% (J) 80%
62. After an election in a small town, the newspaper reported that 42% of the registered voters actually voted. If 12,000 people voted, which equation can be used to find the number of registered voters?
- (A) $v = 42 \cdot 12,000$ (B) $v = 0.42 \cdot 12,000$ (C) $42v = 12,000$ (D) $0.42v = 12,000$
63. Which list is in order from least to greatest?
- (F) $\frac{1}{2}$, 20%, 33%, 0.625, $\frac{1}{8}$, 1 (H) $\frac{1}{8}$, $\frac{1}{2}$, 0.625, 1, 20%, 30%
- (G) $\frac{1}{8}$, 20%, 33%, $\frac{1}{2}$, 0.625, 1 (J) 0.625, $\frac{1}{8}$, $\frac{1}{2}$, 1, 20%, 30%
64. Moises saves 8% of his weekly paycheck in his savings account. He deposited \$18.80 from his last paycheck into his savings account. Which is the best estimate of the total amount of Moises's last paycheck?
- (A) \$26 (B) \$100 (C) \$160 (D) \$200

CHALLENGE AND EXTEND

Find each value. Round to the nearest tenth if necessary.

65. What percent of 16 is 2.75? 66. 22 is 73.5% of what number?
67. 121.3% of 73 is what number? 68. What percent of 8000 is 6525?
69. Find 10% of 8 and 8% of 10. What do you notice? Try this with several other pairs of numbers. Do you think this relationship will be true for all pairs of numbers? Why or why not?
70. **Chemistry** A chemist has 20 milliliters of a solution that is 40% acid. She wants to increase the acid content of the solution to make it a 50%-acid solution. How many milliliters of pure acid should she add to the solution? (*Hint: Begin by finding the number of milliliters of acid in the original solution.*)

SPIRAL REVIEW

Simplify each expression. (*Lesson 1-7*)

71. $32 + 47 + 28 + 13$ 72. $4 \cdot 23 \cdot 25$
73. $8 \cdot 4 \cdot 5$ 74. $44 + 27 + 56$
75. A picture has a width of 4 in. and a length of 6 in. It is enlarged on a copier, and the new length is 9 in. What is the new width? (*Lesson 2-8*)
76. A picture has a width of 4 in. and a length of 6 in. It is reduced on a copier, and the new length is 4.8 in. What is the new width? (*Lesson 2-8*)
77. A rectangle has an area of 9 ft^2 . Every dimension is multiplied by a scale factor, and the new rectangle has an area of 81 ft^2 . What was the scale factor? (*Lesson 2-8*)

2-10

Applications of Percents



Objectives

Use common applications of percents.

Estimate with percents.

Vocabulary

commission principal
interest tip
sales tax

Who uses this?

Sales representatives use percents to calculate their total pay.

A **commission** is money paid to a person or a company for making a sale. Usually the commission is a percent of the sale amount.

EXAMPLE 1 Business Application

Caution!

You must convert a percent to a decimal or a fraction before doing any calculations with it.

Ms. Barns earns a base salary of \$42,000 plus a 1.5% commission on sales. Her total sales one year were \$700,000. Find her total pay for the year.

$$\begin{aligned}
 \text{total pay} &= \text{base salary} + \text{commission} && \text{Write the formula for total pay.} \\
 &= \text{base salary} + \% \text{ of total sales} && \text{Write the formula for commission.} \\
 &= 42,000 + 1.5\% \text{ of } 700,000 && \text{Substitute values given in the problem.} \\
 &= 42,000 + (0.015)(700,000) && \text{Write the percent as a decimal.} \\
 &= 42,000 + 10,500 && \text{Multiply.} \\
 &= 52,500 && \text{Add.}
 \end{aligned}$$

Ms. Barns's total pay was \$52,500.



1. A telemarketer earns \$350 per week plus a 12% commission on sales. Find her total pay for a week in which her sales are \$940.

Interest is the amount of money charged for borrowing money, or the amount of money earned when saving or investing money. **Principal** is the amount borrowed or invested. Simple interest is interest paid only on the principal.

Simple Interest Paid Annually

$$\text{Simple interest} \rightarrow I = Prt \leftarrow \text{Time in years}$$

↑ Principal ↑ Interest rate per year as a decimal

EXAMPLE 2 Finance Application

- A** Find the simple interest paid annually for 2 years on a \$900 loan at 16% per year.

$$\begin{aligned}
 I &= Prt && \text{Write the formula for simple interest.} \\
 I &= (900)(0.16)(2) && \text{Substitute known values. Write the interest rate as a decimal.} \\
 I &= 288
 \end{aligned}$$

The amount of interest is \$288.

Helpful Hint

When you are using the formula $I = Prt$ to find simple interest paid annually, t represents time in years. One month is $\frac{1}{12}$ year. In Example 2B, $t = \frac{3}{12}$.

- B** After 3 months the simple interest earned annually on an investment of \$7000 was \$63. Find the interest rate.

$$I = Prt$$

Write the formula for simple interest.

$$63 = (7000)(r)\left(\frac{3}{12}\right)$$

Substitute the given values.

$$63 = 1750r$$

Multiply $7000\left(\frac{3}{12}\right)$. Since r is multiplied by

$$\frac{63}{1750} = \frac{1750r}{1750}$$

1750, divide both sides by 1750 to undo the multiplication.

$$0.036 = r$$

The interest rate is 3.6%.



- 2a.** Find the simple interest earned after 2 years on an investment of \$3000 at 4.5% interest earned annually.
- 2b.** The simple interest paid on a loan after 6 months was \$306. The annual interest rate was 8%. Find the principal.

A **tip** is an amount of money added to a bill for service. It is usually a percent of the bill before *sales tax* is added. **Sales tax** is a percent of an item's cost.

Sales tax and tips are sometimes estimated instead of calculated exactly. When estimating percents, use percents that you can calculate mentally.

- Find 10% of a number by moving the decimal point one place to the left.
- Find 1% of a number by moving the decimal point two places to the left.
- Find 5% of a number by finding $\frac{1}{2}$ of 10% of the number.

EXAMPLE 3 Estimating with Percents

- A** The dinner check for Maria's family is \$67.95. Estimate a 15% tip.

Step 1 First round \$67.95 to \$70.

Step 2 Think: $15\% = 10\% + 5\%$

10% of \$70 = \$7.00 *Move the decimal point one place left.*

Step 3 Think: $5\% = 10\% \div 2$

$$= \$7.00 \div 2 = \$3.50$$

Step 4 $15\% = 10\% + 5\%$

$$= \$7.00 + \$3.50 = \$10.50$$

The tip should be about \$10.50.

- B** The sales tax rate is 6.25%. Estimate the sales tax on a shirt that costs \$29.50.

Step 1 First round 6.25% to 6% and \$29.50 to \$30.

Step 2 Think: $6\% = 6(1\%)$

1% of \$30 = \$0.30 *Move the decimal point 2 places left.*

Step 3 $6\% = 6(1\%)$

$$= 6(\$0.30) = \$1.80$$

The sales tax is about \$1.80.

Helpful Hint

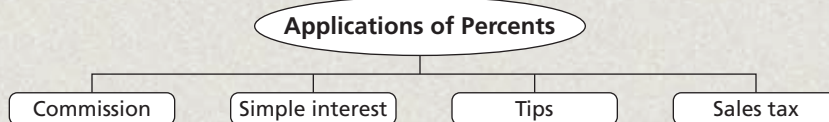
A 6.25% sales tax rate means that for every \$100 you spend, you would pay \$6.25 in sales tax.



- 3a.** Estimate a 15% tip on a check for \$21.98.
- 3b.** Estimate the tax on shoes that cost \$68.50 when the sales tax rate is 8.25%

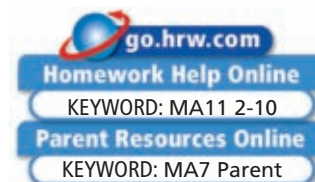
THINK AND DISCUSS

1. Explain how commission, interest, sales tax, and tips are alike.
2. When the sales tax rate is 8.25%, the tax on a \$10 purchase is \$0.83. Is the tax on \$20 twice as much? Explain.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of each type of application and find the answer.



2-10

Exercises



GUIDED PRACTICE

1. **Vocabulary** How are *commission* and *tips* alike? How are they different?

SEE EXAMPLE 1
p. 139

2. A sales representative earns a 2.5% commission on sales. Find the commission earned when the total sales are \$80,700.
3. Karen earns a salary of \$28,600 per year plus a 4.25% commission on sales. Find her total earnings for a year when the sales are \$310,000.

SEE EXAMPLE 2
p. 139

4. Find the amount of simple interest earned after 2 years on \$480 invested at a 7% annual interest rate.
5. Find the number of years it would take for \$1200 to earn simple interest of \$324 at an annual interest rate of 6% per year.
6. Find the total amount owed after 6 months on a loan of \$900 at an annual simple interest rate of 8.5%.

SEE EXAMPLE 3
p. 140

7. Estimate a 15% tip on a \$42.65 check.
8. Estimate the tax on a \$198 stereo when the sales tax is 5.25%.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
9–10	1
11–13	2
14–15	3

9. A boat salesperson earns a 2.5% commission on the sale of each boat. Find the commission earned on a boat that sells for \$18,500.
10. A cell phone distributor earns a yearly salary of \$28,000 plus a 17.5% commission on sales. Find the total earnings for a year when the sales are \$38,000.
11. Find the simple interest paid after 3 months on a loan of \$9700 borrowed at an annual interest rate of 11%.
12. After 8 months, \$750 simple interest was owed on a loan of \$9000. Find the annual interest rate.

Extra Practice

Skills Practice p. S7
Application Practice p. S29

13. How long will it take \$680 to earn \$102 in simple interest at an annual interest rate of 3%?
14. Estimate the tip on a \$19.65 check using a tip rate of 15%.
15. Estimate the tax on tires that cost \$498 with a 6.25% sales tax.

Use $I = Prt$ to complete the table. All interest rates are annual.

	I	$=$	P	\times	r	\times	t
16.	■		\$8275		13%		3 years
17.	\$3969		■		10.5%		9 months
18.	\$23.75		\$950		■		6 months
19.	\$380		\$4750		4%		■



Technology



Koopa is a Gulf Coast box turtle from Hartford, Connecticut. His paintings have sold for hundreds of dollars at online auctions.

20. Chris has \$21.50. He wants a book for \$5.85 and a CD for \$14.99. The sales tax is 6.25%.
 - a. How could Chris estimate whether he has enough to buy the book and the CD?
 - b. Does he have enough money to buy the book and the CD?
21. **Technology** An online auction company charges sellers a commission fee of 5.25% of an item's final selling price. If you sell an item for \$55, what fee will you pay to the auction company? Show that your answer is reasonable.
22. **Business** Sometimes business partners do not share the ownership of a business equally. Instead, they each own a percent of the business, and each receives that percent of the profits. Alvarez, Brown, and Chow are partners in a business that earned \$500,000. Alvarez owns 40% of the business. Chow received \$175,000.
 - a. How much money did Alvarez and Brown each receive?
 - b. What percent of the business is owned by Brown? by Chow?
23. **Write About It** Lewis invested \$1000 at 3% annual simple interest for 4 years. Lisa invested \$1000 at 4% annual simple interest for 3 years. Explain why Lewis and Lisa earned the same amount of interest.
24. **Critical Thinking** To estimate a tip of 15%, Amy tips \$1.00 for every \$6.00 in the total bill. Is this method reasonable? Why or why not?
25. **ERROR ANALYSIS** Which solution is incorrect? Explain the error.

A

4% of 240
$0.4 \cdot 240$
96

B

4% of 240
$0.04 \cdot 240$
9.6

**MULTI-STEP
TEST PREP**



26. This problem will prepare you for the Multi-Step Test Prep on page 152.
 - a. Juan is shopping for a new CD player. He finds one he likes for \$225. The sales tax is 7.5%. What will be the total cost of the CD player?
 - b. The salesperson tells Juan that a sale starts tomorrow, and the CD player will be reduced to \$157.50. What is the total cost, including tax, that Juan will pay if he buys the CD player tomorrow?
 - c. How much will Juan save if he buys the CD player tomorrow?

27. Which account earns the most simple interest after 1 year? Assume that interest is paid annually.
- (A) \$5000 at 8% per year (C) \$8000 at 4.8% per year
 (B) \$10,000 at 4% per year (D) \$4000 at 10.2% per year
28. Craig earns \$200 per week plus 8% commission on sales. Joan earns \$150 per week plus 12% commission on sales. Last week, both had sales of \$1500. Who earned more money?
- (F) Craig earned more than Joan. (H) Both earned the same amount.
 (G) Joan earned more than Craig. (J) Cannot be determined
29. **Short Response** If 2% of a number is 300, what is 6% of the number? Explain how you got your answer.

CHALLENGE AND EXTEND

30. **Multi-Step** A lunch check for Mark and a friend was \$19.50 before the 6% sales tax was added. Mark wants to leave a tip of at least 20%. He has no coins, and he does not want to wait for change. What is the least amount he should leave to pay the check, tax, and tip?
31. The final cost of an item was \$50. This included 6% sales tax. What was the price of the item before tax?

Finance A stockbroker earns a commission based on the amount of a transaction according to the table below.

Stockbroker's Commissions				
Transaction	\$0–\$10,000	For each additional dollar up to \$20,000	For each additional dollar up to \$40,000	For each additional dollar over \$40,000
Commission	0.5%	0.4%	0.3%	0.2%

For example, the commission on a transaction of \$11,000 is 0.5% of \$10,000 plus 0.4% of \$1000. Use the table for Exercises 32–34.

32. Find the amount of commission earned on a transaction of \$15,000.
33. Find the amount of commission earned on a transaction of \$21,000.
34. Find the amount of commission earned on a transaction of \$100,000.

SPIRAL REVIEW

Compare. Write $<$, $>$, or $=$. (Lesson 1-5)

35. $\sqrt{65}$ \square 8 36. 7 \square $\sqrt{51}$ 37. 9 \square $\sqrt{80}$ 38. $\sqrt{196}$ \square 14

Solve each equation. Check your answer. (Lesson 2-6)

39. $|x - 7| = 5$ 40. $|3x| + 5 = 14$ 41. $8|3x - 10| - 12 = 20$

Find each value. Round to the nearest tenth if necessary. (Lesson 2-9)

42. 40% of 60 is what number? 43. What percent of 26 is 13?
 44. 22 is what percent of 99? 45. 80% of what number is 64?

2-11

Percent Increase and Decrease



"May I suggest you read the sign again, Sir."

Objective

Find percent increase and decrease.

Vocabulary

percent change
percent increase
percent decrease
discount
markup

Who uses this?

Consumers can use percent change to determine how much money they can save. (See Example 3.)

A **percent change** is an increase or decrease given as a percent of the original amount. **Percent increase** describes an amount that has grown and **percent decrease** describes an amount that has been reduced.



Percent Change

percent change = $\frac{\text{amount of increase or decrease}}{\text{original amount}}$, expressed as a percent

EXAMPLE 1 Finding Percent Increase or Decrease

Find each percent change. Tell whether it is a percent increase or decrease.

A from 25 to 49

$$\begin{aligned} \text{percent change} &= \frac{\text{amount of increase}}{\text{original amount}} \\ &= \frac{49 - 25}{25} \\ &= \frac{24}{25} && \text{Simplify the numerator.} \\ &= 0.96 \\ &= 96\% && \text{Write the answer as a percent.} \end{aligned}$$

25 to 49 is an increase, so a change from 25 to 49 is a 96% increase.

B from 50 to 45

$$\begin{aligned} \text{percent change} &= \frac{\text{amount of decrease}}{\text{original amount}} \\ &= \frac{50 - 45}{50} \\ &= \frac{5}{50} && \text{Simplify the numerator.} \\ &= \frac{1}{10} && \text{Simplify the fraction.} \\ &= 10\% && \text{Write the answer as a percent.} \end{aligned}$$

50 to 45 is a decrease, so a change from 50 to 45 is a 10% decrease.

Helpful Hint

Before solving, decide what is a reasonable answer. For Example 1A, 25 to 50 would be a 100% increase. So 25 to 49 should be slightly less than 100%.



Find each percent change. Tell whether it is a percent increase or decrease.

- 1a. from 200 to 110 1b. from 25 to 30 1c. from 80 to 115

EXAMPLE 2 Finding the Result of a Percent Increase or Decrease

A Find the result when 30 is increased by 20%.

$$0.20(30) = 6 \quad \textit{Find 20\% of 30. This is the amount of the increase.}$$

$$30 + 6 = 36 \quad \textit{It is a percent increase, so add 6 to the original amount.}$$

30 increased by 20% is 36.

B Find the result when 65 is decreased by 80%.

$$0.80(65) = 52 \quad \textit{Find 80\% of 65. This is the amount of the decrease.}$$

$$65 - 52 = 13 \quad \textit{It is a percent decrease, so subtract 52 from 65.}$$

65 decreased by 80% is 13.



2a. Find the result when 72 is increased by 25%.

2b. Find the result when 10 is decreased by 40%.

Common applications of percent change are *discounts* and *markups*.

A **discount** is an amount by which an original price is reduced.

$$\text{discount} = \% \text{ of original price}$$

$$\text{final price} = \text{original price} - \text{discount}$$

A **markup** is an amount by which a wholesale cost is increased.

$$\text{markup} = \% \text{ of wholesale cost}$$

$$\text{final price} = \text{wholesale cost} + \text{markup}$$

EXAMPLE 3 Discounts

Helpful Hint

Before solving, decide what is a reasonable answer. For Example 3A, a 25% discount is \$2 off. So a 15% discount will be less than \$2 off.

A Admission to the museum is \$8. Students receive a 15% discount. How much is the discount? How much do students pay?

Method 1 A discount is a percent decrease. So find \$8 decreased by 15%.

$$0.15(8) = 1.20 \quad \textit{Find 15\% of 8. This is the amount of the discount.}$$

$$8 - 1.20 = 6.80 \quad \textit{Subtract 1.20 from 8. This is the student price.}$$

Method 2 Subtract percent discount from 100%.

$$100\% - 15\% = 85\% \quad \textit{Students pay 85\% of the regular price, \$8.}$$

$$0.85(8) = 6.80 \quad \textit{Find 85\% of 8. This is the student price.}$$

$$8 - 6.80 = 1.20 \quad \textit{Subtract 6.80 from 8. This is the amount of the discount.}$$

By either method, the discount is \$1.20. Students pay \$6.80.

B Christo used a coupon and paid \$7.35 for a pizza that normally costs \$10.50. Find the percent discount.

$$\$10.50 - \$7.35 = \$3.15$$

$$3.15 = x(10.50)$$

$$\frac{3.15}{10.50} = \frac{x \cdot 10.50}{10.50}$$

$$0.3 = x$$

$$30\% = x$$

The discount is 30%.

Think: 3.15 is what percent of 10.50? Let x represent the percent.

Since x is multiplied by 10.50, divide both sides by 10.50 to undo the multiplication.

Write the answer as a percent.



3a. A \$220 bicycle was on sale for 60% off. Find the sale price.

3b. Ray paid \$12 for a \$15 T-shirt. What was the percent discount?

EXAMPLE 4 Markups

- A** Kaleb buys necklaces at a wholesale cost of \$48 each. He then marks up the price by 75% and sells the necklaces. What is the amount of the markup? What is the selling price?

Method 1

A markup is a percent increase. So find \$48 increased by 75%.

$$0.75(48) = 36 \quad \textit{Find 75\% of 48. This is the amount of the markup.}$$

$$48 + 36 = 84 \quad \textit{Add to 48. This is the selling price.}$$

Method 2

Add percent markup to 100%.

$$100\% + 75\% = 175\% \quad \textit{The selling price is 175\% of the wholesale price, \$48.}$$

$$1.75(48) = 84 \quad \textit{Find 175\% of 48. This is the selling price.}$$

$$84 - 48 = 36 \quad \textit{Subtract from 84. This is the amount of the markup.}$$

By either method, the amount of the markup is \$36. The selling price is \$84.

- B** Lars purchased a daily planner for \$32. The wholesale cost was \$25. What was the percent markup?

$$32 - 25 = 7 \quad \textit{Find the amount of the markup.}$$

$$7 = x(25) \quad \textit{Think: 7 is what percent of 25? Let } x \text{ represent the percent.}$$

$$\frac{7}{25} = \frac{25x}{25} \quad \textit{Since } x \text{ is multiplied by 25, divide both sides by 25 to undo the multiplication.}$$

$$0.28 = x$$

$$28\% = x \quad \textit{Write the answer as a percent.}$$

The markup was 28%.



- 4a.** A video game has a 70% markup. The wholesale cost is \$9. What is the selling price?

- 4b.** What is the percent markup on a car selling for \$21,850 that had a wholesale cost of \$9500?

THINK AND DISCUSS

- 80% of a number is the same as a ?% decrease from that number. A 30% increase from a number is the same as ?% of that number.
- A markup of 200% will result in a final cost that is how many times the wholesale cost?
- What information would you need to find the percent change in your school's population over the last ten years?
- GET ORGANIZED** Copy and complete the graphic organizer. In each box, write and solve an example of the given type of problem.



Percent Increase	Percent Decrease	Discount	Markup

GUIDED PRACTICE

1. **Vocabulary** Compare *percent increase* and *percent decrease*.

SEE EXAMPLE 1

p. 144

Find each percent change. Tell whether it is a percent increase or decrease

2. 25 to 45 3. 10 to 8 4. 400 to 300
 5. 16 to 18 6. 40 to 50 7. 50 to 40

SEE EXAMPLE 2

p. 144

8. Find the result when 40 is increased by 85%.
 9. Find the result when 60 is increased by 3%.
 10. Find the result when 350 is decreased by 10%.
 11. Find the result when 16 is decreased by 50%.

SEE EXAMPLE 3

p. 145

12. What is the final price on a \$185 leather jacket that is on sale for 40% off?
 13. Neal bought a book on sale for \$3.60. It was originally priced at \$12. What was Neal's discount as a percent?

SEE EXAMPLE 4

p. 146

14. Yolanda bought a video that was priced at a 65% markup over the manufacturer's cost of \$12. What was Yolanda's cost?
 15. Randy sells hats for \$12.35. The wholesale cost of each hat is \$6.50. What is Randy's markup as a percent?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
16–27	1
28–31	2
32–33	3
34–35	4

Find each percent change. Tell whether it is a percent increase or decrease.

16. 50 to 60 17. 4 to 3 18. 96 to 84 19. 9 to 45
 20. 32 to 30 21. 15 to 19.5 22. 150 to 180 23. 17 to 14.45
 24. 20 to 15 25. 265 to 318 26. 35 to 105 27. 300 to 275
 28. Find the result when 24 is increased by 75%.

29. Find the result when 240 is increased by 5%.
 30. Find the result when 30 is decreased by 85%.
 31. Find the result when 8 is decreased by 5%.
 32. The cost of Hisako's school supplies was \$49.80. She had a coupon for 30% off the entire purchase. What was the final price?
 33. With the purchase of 10 greeting cards, Addie received a discount. She paid \$26.35 for the cards that would normally have cost \$31. What percent discount did Addie receive?
 34. Irma sells boxing gloves in her sporting goods store for a 9% markup over the manufacturer's cost of \$40. What is the selling price of the gloves?
 35. Bottled water in a certain vending machine costs \$1.50. This price is a markup from the wholesale cost of \$0.20. What is the markup as a percent?
 36. **Critical Thinking** Is the percent increase from 50 to 80 the same as the percent decrease from 80 to 50? Why or why not?

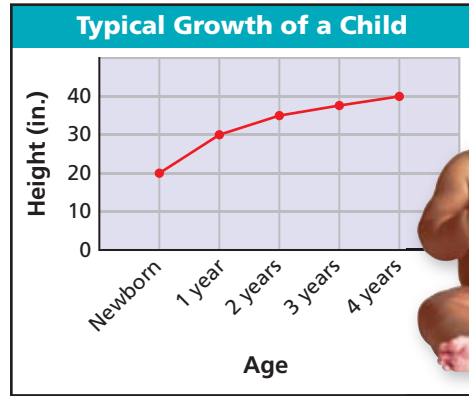
Extra Practice

Skills Practice p. S7

Application Practice p. S29

Multi-Step The graph shows the average height of a child from birth to age 4. Use the graph for Exercises 37–39.

37. By what percent does a child's height increase from birth to age 1 year?
38. By what percent does a child's height increase from birth to age 4 years?
39. **Estimation** Estimate the amount and percent of increase in a child's height from age 2 to age 3. Show that your estimate is reasonable.



Source: www.kidshealth.org

40. **Employment** Last summer, Duncan charged \$20 to mow a lawn in his neighborhood. This summer, he'll charge \$23. What is the percent increase in Duncan's price? Show that your answer is reasonable.

Copy and complete the table.

	Original Amount	New Amount	Percent Change
41.	12	■	50% increase
42.	\$48	\$42.24	■
43.	$4\frac{1}{2}$	$13\frac{1}{2}$	■
44.	8525	■	20% decrease

Find each missing number.

45. 20 increased by ■% is 24.
46. 80 decreased by ■% is 76.
47. 120 decreased by 50% is ■.
48. 200 increased by ■% is 210.
49. **Nutrition** A can of soup had 480 mg of sodium per serving. The sodium was reduced to 360 mg per serving so that the soup could be advertised as "low sodium." What was the percent change in sodium content?

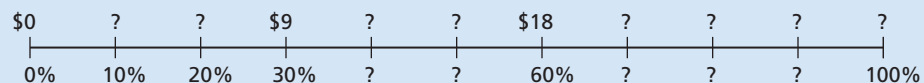


50. **Write About It** Dana is shopping for shoes at a store advertising "everything 45% off." Describe a method she could use to estimate the discount and final price on a pair of shoes.

**MULTI-STEP
TEST PREP**



51. This problem will prepare you for the Multi-Step Test Prep on page 152.
- Robert finds a shirt on a sale rack. All items on the rack are 40% off. The price on his shirt is missing. When the clerk scans the bar code, he tells Robert that the sale price of the shirt is \$18. What percent of the original price is \$18?
 - Set up a proportion to find the original price of the shirt.
 - Copy and complete the model below and explain how it helps you to solve this problem in another way.



52. Lucia gets film developed at Photo King, where 24 prints cost \$7.80. This week, Photo King is having a sale, and 24 prints cost \$6.63. What percent of the regular cost will Lucia save?
- (A) 15% (B) 17% (C) 25% (D) 85%
53. Which of these does NOT represent “200 decreased by 45%”?
- (F) $200(0.55)$ (G) 110 (H) $200 - 0.45$ (J) $200(1 - 0.45)$
54. Which of these represents a 15% increase?
- (A) A price is marked up from \$12.50 to \$15.
 (B) Joanna’s bank account balance grew from \$127.50 to \$150.
 (C) A baseball card’s value rose from \$6 to \$6.15.
 (D) Luis’s hourly wage was raised from \$8 to \$9.20.
55. The original price of an item was \$199. During a sale, the price was reduced by 45%. Then, during a clearance sale, the price was reduced an additional 20%. What was the final price?
- (F) \$71.91 (G) \$87.56 (H) \$98.97 (J) \$101.89
56. **Gridded Response** A skateboard that sells for \$65 is on sale for 15% off. What is the sale price in dollars?

CHALLENGE AND EXTEND

Find each missing number.

57. ■ increased by 15% is 230. 58. ■ increased by 50% is 48.
 59. ■ decreased by 20% is 500. 60. ■ decreased by 70% is 4.35.
61. The label on a bottle of orange juice says “now 25% more.” The bottle has 80 fluid ounces of juice. What was the original volume? Show that your answer is reasonable.
62. Angelina paid \$21 for a backpack that was 30% off. What was the original price? Show that your answer is reasonable.
63. **Multi-Step** Mr. Hansen owns a bookstore. He buys used books at 25% of the cover price and sells them at a 45% markup of what he paid. Jerry sold Mr. Hansen three books with cover prices of \$7.95, \$5.95, and \$12.10. If Jerry bought back his books, how much would he pay?

SPIRAL REVIEW

Find the complement and the supplement of each angle. (*Previous course*)

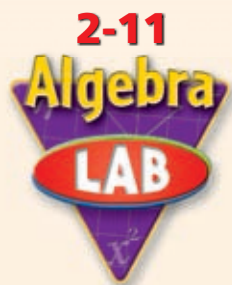
64. 65° 65. 10° 66. 45° 67. 30°

Solve each equation. Check your answer. (*Lessons 2-1 and 2-2*)

68. $-15 + x = -3$ 69. $x + 16 = -4$ 70. $r - 3 = 6$ 71. $n - (-10) = 67$
 72. $98 = 7z$ 73. $\frac{x}{3} = 12$ 74. $-x = 4$ 75. $\frac{x}{5} = -20$

Estimate each amount. (*Lesson 2-10*)

76. the tip on a \$60.65 check using a tip rate of 15%
 77. the tax on a \$70 DVD player when the sales tax is 6%



Explore Changes in Population

You can use percents to describe changes in populations. A population may grow by a certain percent or decrease by a certain percent. Explore changing populations in these activities.

Use with Lesson 2-11

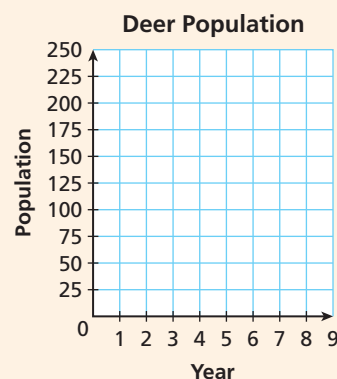


Activity 1

A team of biologists is studying a population of deer. There are 32 deer in the first year of the study. Due to a lack of predators, the biologists find that the herd grows by 50% every year.

1 Copy and complete the table. The first two rows have been completed for you.

Year	Percent Increase	Amount of Increase	Population
1			32
2	50%	$0.50 \cdot 32 = 16$	$32 + 16 = 48$
3	50%	■	■
4	50%	■	■
5	50%	■	■
6	50%	■	■



- Describe the percent increase from year to year.
- Describe the amount of increase from year to year.
- Copy the grid above onto graph paper. Plot the year and the population of deer on the graph as six ordered pairs (year, population). Connect the points with a smooth curve.
- Describe the shape of your graph.

Try This

A researcher places 10 bacteria on a dish. This species increases by 100% every hour.

1. Copy and complete the table below.

Hour	0	1	2	3	4	5
Amount of Increase		■	■	■	■	■
Bacteria	10	■	■	■	■	■

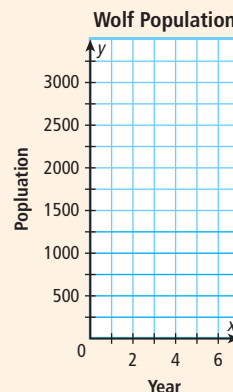
- Graph points from the table as (hour, bacteria). Connect the points with a smooth curve.
- Compare this graph with the graph of the deer population.
- Why does the amount of increase change when the percent of increase stays the same?

Activity 2

A second team of biologists is studying a population of wolves. There are 3125 wolves in the first year. The biologists find that this population decreases by 40% every year.

- 1 Make a Prediction** Based on your results in Activity 1, what do you think will happen to the amount of decrease each year?
- 2** Copy and complete the table below. The first two rows have been completed for you.

Year	Percent Decrease	Amount of Decrease	Population
1			3125
2	40%	$0.40 \cdot 3125 = 1250$	$3125 - 1250 = 1875$
3	40%	■	■
4	40%	■	■
5	40%	■	■
6	40%	■	■



- 3** What happens to the amount of decrease in the wolf population from year to year? Was your prediction from Problem 1 correct?
- 4 Make a Prediction** Copy the grid above onto graph paper. Based on your results in Activity 1, what do you think the graph of ordered pairs (year, population) will look like?
- 5** Plot the year and the population of wolves on the graph as six ordered pairs (year, population). Connect the points with a smooth curve.
- 6** Describe the shape of your graph. Was your prediction from Problem 4 correct?

Try This

A half-life is the amount of time it takes half of an amount of radioactive substance to decay into another substance. Tritium is a radioactive form of hydrogen with a half-life of 12.3 years. In other words, after one half-life of 12.3 years, an amount of tritium will have decreased by 50%.

- 5.** Suppose you start with 128 grams of tritium. Copy and complete the table below.
- 6.** Make a graph that shows how much tritium is left after 0, 1, 2, 3, 4, and 5 half-lives.

Half-lives	0	1	2	3	4	5
Percent Decrease	0	50%	50%	50%	50%	50%
Amount of Decrease (g)		■	■	■	■	■
Tritium Remaining (g)	128	■	■	■	■	■

- 7.** Compare this graph with the graph of the wolf population.
- 8.** Describe the graph of a population that increases by a fixed percent. Why does the graph have this shape?
- 9.** Describe the graph of a population that decreases by a fixed percent. Why does the graph have this shape?

MULTI-STEP TEST PREP



Percentages

Bargain Hunters Maria is on her high school's lacrosse team, and her friend Paula is on the softball team. The girls notice an advertisement in the newspaper for a clearance sale at their favorite sporting goods store. The ad shows an additional $\frac{1}{4}$ off the already reduced prices of 60% off. Maria and Paula head to the store to shop for bargains.

1. Maria finds a lacrosse stick with a regular price of \$65. Find the sale price of the lacrosse stick prior to the additional $\frac{1}{4}$ off.
2. Find the sale price of Maria's lacrosse stick with the additional $\frac{1}{4}$ off.
3. Paula says that with the extra $\frac{1}{4}$ off, the total discount is 85% off. Maria thinks the discount is less than that. Who is correct? Explain your reasoning.
4. Paula finds a softball glove with a price tag that is not readable. The sales clerk scans the bar code and says the sale price, including the extra $\frac{1}{4}$ off, is \$16.50. What was the original price of the softball glove? Show your reasoning.
5. Sales tax is 7.8%. Find the total amount that the girls will pay for the lacrosse stick and the softball glove together, including tax.



Quiz for Lessons 2-7 Through 2-11

2-7 Rates, Ratios, and Proportions

- Last week, the ratio of laptops to desktops sold at a computer store was 2:3. Eighteen desktop models were sold. How many laptop models were sold?
- Anita read 150 pages in 5 hours. What is her reading rate in pages per minute?
- Twenty-six crackers contain 156 Calories. Find the unit rate in Calories per cracker.
- A store developed 1024 photographs in 8 hours. Find the unit rate in photographs per hour.

Solve each proportion.

5. $\frac{-18}{n} = \frac{9}{2}$

6. $\frac{d}{5} = \frac{2}{4}$

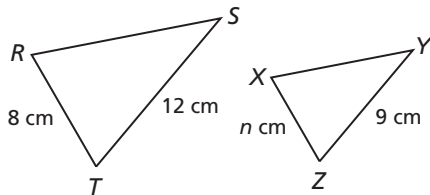
7. $\frac{4}{12} = \frac{r+2}{16}$

8. $\frac{-3}{7} = \frac{6}{x+6}$

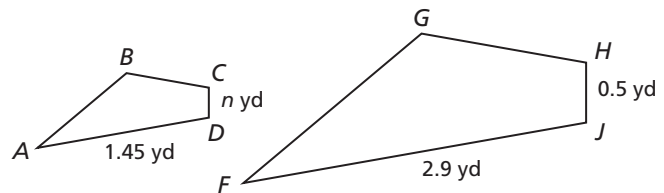
2-8 Applications of Proportions

Find the value of n in each diagram.

9. $\triangle RST \sim \triangle XYZ$



10. $ABCD \sim FGHI$



2-9 Percents

- Find 40% of 25.
- Find 130% of 9.
- 35 is what percent of 70?
- What percent of 400 is 640?
- 16 is 80% of what number?
- 200% of what number is 28?
- A volunteer at the zoo is responsible for feeding the animals in 15 exhibits in the reptile house. This represents 20% of the total exhibits in the reptile house. How many exhibits are in the reptile house?

2-10 Applications of Percents

- Peter earns \$32,000 per year plus a 2.5% commission on his jewelry sales. Find Peter's total salary for the year when his sales are valued at \$420,000.
- Estimate the tax on a \$21,899 car when the tax rate is 5%.

2-11 Percent Increase and Decrease

Find each percent change. Tell whether it is a percent increase or decrease.

- from 60 to 66
- from 48 to 12
- from 200 to 80
- from 9.8 to 14.7
- Andrea purchased a picture frame for \$14.56. This price was a 30% markup from the wholesale cost. What was the wholesale cost?

Vocabulary

commission	139	interest	139	sales tax	140
conversion factor	121	literal equation	108	scale	122
corresponding angles	127	markup	145	scale drawing	122
corresponding sides	127	percent	133	scale factor	129
cross products	121	percent change	144	scale model	122
dimensional analysis	121	percent decrease	144	similar	127
discount	145	percent increase	144	solution of an equation	77
equation	77	principal	139	tip	140
formula	107	proportion	120	unit rate	120
identity	101	rate	120		
indirect measurement	128	ratio	120		

Complete the sentences below with vocabulary words from the list above.

- A formula is a type of a(n) _____?
- A(n) _____ is used to compare two quantities by division.

2-1 Solving Equations by Adding or Subtracting (pp. 77–82)**EXAMPLES**

Solve each equation. Check your answer.

$$\blacksquare x - 12 = -8.3 \quad \blacksquare -7.8 = 5 + t$$

$$\begin{array}{r} +12 \quad +12 \\ x = 3.7 \end{array}$$

$$\begin{array}{r} -5 \quad -5 \\ -12.8 = t \end{array}$$

Check $x - 12 = -8.3$ Check $-7.8 = 5 + t$

$3.7 - 12$	-8.3	-7.8	$5 + (-12.8)$
-8.3	$-8.3 \checkmark$	-7.8	$-7.8 \checkmark$

EXERCISES

Solve each equation. Check your answer.

- $b - 16 = 20$
- $4 + x = 2$
- $9 + a = -12$
- $-7 + y = 11$
- $z - \frac{1}{4} = \frac{7}{8}$
- $w + \frac{2}{3} = 3$

- Robin needs 108 signatures for her petition. So far, she has 27. Write and solve an equation to determine how many more signatures she needs.

2-2 Solving Equations by Multiplying or Dividing (pp. 84–90)**EXAMPLES**

Solve each equation.

$$\blacksquare \frac{z}{2.4} = 12 \quad \blacksquare -8x = 148$$

$$\begin{array}{r} (2.4) \frac{z}{2.4} = (2.4) 12 \\ z = 28.8 \end{array}$$

$$\begin{array}{r} -8x = 148 \\ -8 \quad -8 \\ x = -18.5 \end{array}$$

EXERCISES

Solve each equation. Check your answer.

- $35 = 5x$
- $-3n = 10$
- $-30 = \frac{n}{3}$
- $\frac{x}{-5} = -2.6$
- $5y = 0$
- $-4.6r = 9.2$

2-3 Solving Two-Step and Multi-Step Equations (pp. 92–98)

EXAMPLE

■ Solve $\frac{3x}{5} - \frac{x}{4} + \frac{1}{2} = \frac{6}{5}$.

$$\frac{3x}{5} - \frac{x}{4} + \frac{1}{2} = \frac{6}{5}$$

$$20\left(\frac{3x}{5} - \frac{x}{4} + \frac{1}{2}\right) = 20\left(\frac{6}{5}\right)$$

$$12x - 5x + 10 = 24$$

$$7x + 10 = 24$$

$$\begin{array}{r} -10 \\ \hline 7x \end{array} = \begin{array}{r} -10 \\ \hline 14 \end{array}$$

$$\frac{7x}{7} = \frac{14}{7}$$

$$x = 2$$

Multiply by
the LCD.

Combine like
terms.

EXERCISES

Solve each equation. Check your answer.

16. $4t - 13 = 57$

17. $5 - 2y = 15$

18. $\frac{k}{5} - 6 = 2$

19. $\frac{5}{6}f - \frac{3}{4}f + \frac{3}{4} = \frac{1}{2}$

20. $7x - 19x = 6$

21. $4 + 3a - 6 = 43$

22. If $8n + 22 = 70$, find the value of $3n$.

23. If $0 = 6n - 36$, find the value of $n - 5$.

24. The sum of the measures of two angles is 180° . One angle measures $3a$ and the other angle measures $2a - 25$. Find a . Then find the measure of each angle.

2-4 Solving Equations with Variables on Both Sides (pp. 100–106)

EXAMPLE

■ Solve $x + 7 = 12 + 3x - 7x$.

$$x + 7 = 12 + 3x - 7x$$

$$x + 7 = 12 - 4x$$

$$\begin{array}{r} +4x \\ \hline 5x + 7 = 12 \end{array}$$

$$\begin{array}{r} -7 \\ \hline 5x = 5 \end{array}$$

$$\frac{5x}{5} = \frac{5}{5}$$

$$x = 1$$

Combine like
terms.

EXERCISES

Solve each equation. Check your answer.

25. $4x + 2 = 3x$

26. $-3r - 8 = -5r - 12$

27. $-a - 3 + 7 = 3a$

28. $-(x - 4) = 2x + 6$

29. $\frac{2}{3}n = 4n - \frac{10}{3}n - \frac{1}{2}$

30. $0.2(7 + 2t) = 0.4t + 1.4$

31. One photo shop charges \$0.36 per print. Another photo shop charges \$2.52 plus \$0.08 per print. Juan finds that the cost of printing his photos is the same at either shop. How many photos does Juan have to print?

2-5 Solving for a Variable (pp. 107–111)

EXAMPLE

■ Solve $A = P + Prt$ for r .

$$A = P + Prt$$

$$\begin{array}{r} -P \\ \hline A - P = \quad Prt \end{array}$$

$$\frac{A - P}{Pt} = \frac{Prt}{Pt}$$

$$\frac{A - P}{Pt} = r$$

EXERCISES

Solve for the indicated variable.

32. $C = \frac{360}{n}$ for n

33. $S = \frac{n}{2}(a + \ell)$ for a

34. The formula $a = \frac{d}{g}$ gives the average gas mileage a of a vehicle that uses g gallons of gas to travel d miles. Use the formula to find how many gallons of gas a vehicle with an average gas mileage of 20.2 miles per gallon will use to travel 75 miles. Round your answer to the nearest tenth.

2-6 Solving Absolute-Value Equations (pp. 112–117)

EXAMPLES

- Solve $3|y + 4| = 30$.

$$\frac{3|y + 4|}{3} = \frac{|30|}{3} \quad \text{Divide both sides by 3.}$$

$$|y + 4| = 10$$

Case 1

$$y + 4 = 10$$

$$\frac{-4}{-4} = \frac{-4}{-4}$$

$$y = 6$$

Case 2

$$y + 4 = -10$$

$$\frac{-4}{-4} = \frac{-4}{-4}$$

$$y = -14$$

EXERCISES

Solve each equation. Check your answer.

35. $|x + 6| = 21$ 36. $7|y - 5| = 14$

37. $3|y| + 4 = 31$ 38. $12 = |x - 5.4|$

39. $|g + 6| + 12 = 14$ 40. $|x| = \frac{5}{7}$

41. Jason is driving his car at 55 mi/h. He needs to keep his car within 5 mi/h of his current speed. Write and solve an absolute-value equation to find Jason's maximum and minimum speeds.

2-7 Rates, Ratios, and Proportions (pp. 120–126)

- The ratio of skateboarders to bikers in an extreme sports contest is 7:2. There are 91 skateboarders. How many bikers are there?

$$\frac{\text{skateboarders}}{\text{bikers}} \rightarrow \frac{7}{2} \quad \text{Write a proportion.}$$

$$\frac{7}{2} \times \frac{91}{x} \quad \text{Let } x \text{ be the number of bikers.}$$

$$7 \cdot x = 2 \cdot 91 \quad \text{Use cross products.}$$

$$7x = 182$$

$$\frac{7x}{7} = \frac{182}{7} \quad \text{Solve for } x.$$

$$x = 26$$

There are 26 bikers.

42. A recipe for a casserole calls for 2 cups of rice. The recipe makes 6 servings of casserole. How many cups of rice will you need to make 10 servings of casserole?

Use dimensional analysis to convert each rate. Round your answer to the nearest hundredth if necessary.

43. 30 cm/s to m/h

44. 75 ft/s to mi/min

Solve each proportion. Check your answer.

45. $\frac{n}{8} = \frac{2}{10}$

46. $\frac{2}{9} = \frac{12}{x}$

47. $\frac{3}{k} = \frac{9}{15}$

48. $\frac{1}{3} = \frac{x}{x - 6}$

2-8 Applications of Proportions (pp. 127–132)

- When Janelle stood next to the Washington Monument, she cast a 1.2-foot-long shadow, and the monument cast a 111-foot-long shadow. Janelle is 6 feet tall. How tall is the monument?

$$\frac{x}{111} = \frac{6}{1.2} \quad \text{Write a proportion.}$$

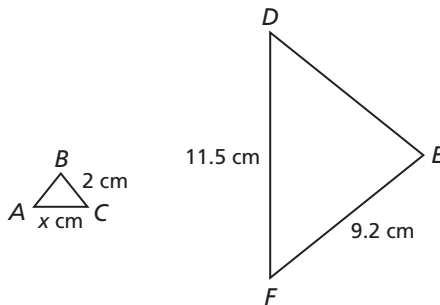
$$1.2x = 666 \quad \text{Use cross products.}$$

$$\frac{1.2x}{1.2} = \frac{666}{1.2} \quad \text{Solve for } x.$$

The monument is 555 feet tall.

49. Find the value of x in the diagram.

$$\triangle ABC \sim \triangle DEF$$



50. A tree casts a shadow that is 14 ft long at the same time that a nearby 2-foot-tall pole casts a shadow that is 1.75 ft long. How tall is the tree?
51. A circle has a radius of 9 inches. The radius is multiplied by $\frac{2}{3}$ to form a second circle. How is the ratio of the areas related to the ratio of the radii?

2-9 Percents (pp. 133–138)

EXAMPLES

- Earth's surface area is about 197 million square miles. About 58 million square miles is land. What percent is water?

197 million – 58 million = 139 million

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$$

$$\frac{139}{197} = \frac{n}{100}$$

$$197n = 13,900 \quad \text{Use cross products.}$$

$$n = 70.56 \quad \text{Solve for } n.$$

About 71% of Earth's surface area is water.

EXERCISES

- Find 2.3% of 230.
- Find 115% of 2700.
- What percent of 18 is 12? Round your answer to the nearest tenth of a percent.
- What percent of 14 is 56?
- 90% of what number is 120? Round the number to the nearest tenth.
- 90 is 37.5% of what number?
- A student answered 32 questions correctly and 8 incorrectly. What percent of the questions were answered correctly?

2-10 Applications of Percents (pp. 139–143)

- After 10 months, the simple interest earned on \$3000 was \$52.50. Find the interest rate.

197 million – 58 million = 139 million

$$I = Prt$$

$$52.5 = 3000(r)\left(\frac{10}{12}\right) \quad \text{Substitute.}$$

$$52.5 = 2500r \quad \text{Multiply.}$$

$$\frac{52.5}{2500} = \frac{2500r}{2500} \quad \text{Solve for } r.$$

$$0.021 = r$$

The interest rate is 2.1%. Write as a percent.

- A salesperson earns a base salary of \$36,000 plus $2\frac{1}{2}\%$ commission on sales. His total sales for one year was \$500,000. Find the salesperson's total pay for that year.
- Find the simple interest paid for 10 years on a \$10,000 loan at 9% per year.
- The sales tax rate is 8%. Estimate the tax on a jacket that costs \$69.95.
- Estimate the tip on a \$10.75 check using a tip rate of 20%.

2-11 Percent Increase and Decrease (pp. 144–149)

Find each percent change. Tell whether it is a percent increase or decrease.

- from 50 to 56

$$\begin{aligned} \text{percent change} &= \frac{\text{amount of increase}}{\text{original amount}} \\ &= \frac{56 - 50}{50} = \frac{6}{50} = 0.12 = 12\% \end{aligned}$$

The change is a 12% increase.

- from 120 to 72

$$\begin{aligned} \text{percent change} &= \frac{\text{amount of decrease}}{\text{original amount}} \\ &= \frac{120 - 72}{120} = \frac{48}{120} = 0.40 = 40\% \end{aligned}$$

The change is a 40% decrease.

Find each percent change. Tell whether it is a percent increase or decrease. Round your answer to the nearest percent.

- from 19 to 26
- from 42 to 28
- Find the result when 65 is increased by 40%.
- Find the result when 150 is decreased by 15%.
- Tom sells sunglasses that he buys wholesale for \$2.50 each. He then marks up the price 150%. What is the amount of the markup? What is the selling price?
- The original price of a shirt was \$79.99. It is on sale for \$49.99. What is the percent discount? Round to the nearest tenth.

Solve each equation.

1. $y - 7 = 2$
2. $x + 12 = 19$
3. $-5 + z = 8$
4. $9x = 72$
5. $\frac{m}{-8} = -2.5$
6. $\frac{7}{8}a = 42$
7. $15 = 3 - 4x$
8. $\frac{2a}{3} + \frac{1}{5} = \frac{7}{6}$
9. $8 - (b - 2) = 11$
10. $-2x + 4 = 5 - 3x$
11. $3(q - 2) + 2 = 5q - 7 - 2q$
12. $5z = -3(z + 7)$

Solve for the indicated variable.

13. $r - 2s = 14$ for s
14. $V = \frac{1}{3}bh$ for b
15. $P = 2(\ell + w)$ for ℓ

Solve each equation.

16. $|x - 14| = 21$
17. $13 = |y + 2| - 3$
18. $4|z| = 20$
19. $3|x| + 5 = 8$
20. $3|g + 1| + 5 = 7$
21. $|2v| = 6$

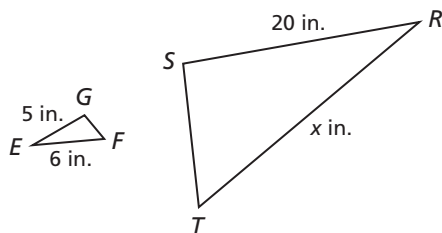
22. A store sells 3 videotapes for \$4.99. Find the unit rate in dollars per videotape. Round to the nearest cent.
23. Twenty-five students use 120 sheets of paper. Find the unit rate in sheets per student.
24. Nutritionists recommend that teenagers consume 1300 milligrams of calcium per day. Use dimensional analysis to convert this rate to grams per year.

Solve each proportion.

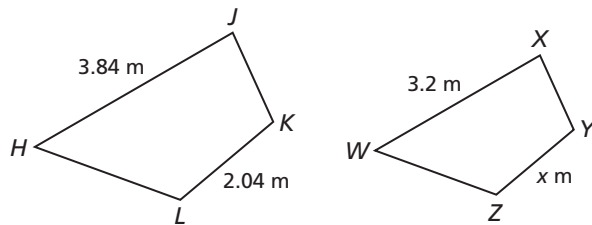
25. $\frac{5}{4} = \frac{x}{12}$
26. $\frac{8}{2z} = \frac{15}{60}$
27. $\frac{x + 10}{10} = \frac{18}{12}$
28. The scale on a map is 1 inch : 500 miles. If two cities are 875 miles apart, how far apart are they on the map?

Find the value of x in each diagram. Round your answer to the nearest tenth.

29. $\triangle EFG \sim \triangle RTS$



30. $HJKL \sim WXYZ$



31. What is 23% of 46?
32. 37.5 is 60% of what number?
33. What percent of 175 is 35?
34. Find the simple interest earned after 5 years on an investment of \$2000 at 3.2% per year.
35. A lunch check is \$27.95. Estimate a 15% tip.

Find each percent change. Tell whether it is a percent increase or decrease.

36. from 180 to 234
37. from 12 to 48
38. from 56 to 21

COLLEGE ENTRANCE EXAM PRACTICE



FOCUS ON ACT

The ACT Mathematics Test is one of four tests in the ACT. You have 60 minutes to answer 60 multiple-choice questions. The questions cover material typically taught through the end of eleventh grade. You will need to know some basic formulas.



There is no penalty for incorrect answers on the ACT. If you are unsure of the correct answer, eliminate as many answer choices as possible. Then make your best guess. Be sure you have marked an answer for every question before time runs out.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

1. At a certain high school, the ratio of left-handed to right-handed basketball players is 1:4. If there are a total of 20 players on the team, how many players are right-handed?
- (A) 1
(B) 4
(C) 5
(D) 12
(E) 16

2. If $y - 3 = \frac{2}{5}(x + 1)$, then $x = ?$

(F) $\frac{5(y - 3) - 2}{2}$

(G) $y - \frac{22}{5}$

(H) $\frac{2(y - 3)}{5} - 1$

(J) $\frac{2(y + 1) + 15}{5}$

(K) $\frac{5}{2}y - 4$

3. What is $\frac{1}{5}\%$ of 20?

- (A) 0.004
(B) 0.04
(C) 0.4
(D) 4
(E) 100

4. If $x - 3 = 4 - 2(x + 5)$, then $x = ?$

(F) -3

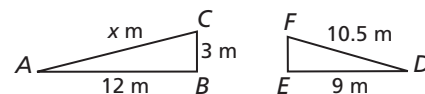
(G) -1

(H) 1

(J) $\frac{3}{2}$

(K) $\frac{11}{3}$

5. If $\triangle ABC \sim \triangle DEF$, what is the length of \overline{AC} ?



- (A) 2.6 meters
(B) 3.5 meters
(C) 7 meters
(D) 14 meters
(E) 15 meters

6. A movie theater makes 30% of its revenue from concession sales. If concession sales were \$174,000, what was the total revenue?

- (F) \$52,200
(G) \$121,800
(H) \$248,570
(J) \$580,000
(K) \$746,000



Multiple Choice: Eliminate Answer Choices

You can answer some problems without doing many calculations. Use logic to eliminate answer choices and save time.

EXAMPLE 1

Which number is the square of 123,765?

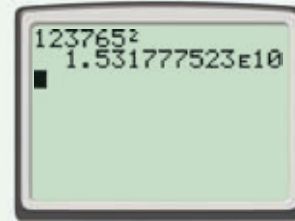
- (A) 15,317,775,225 (C) 15,317,775,230
 (B) 15,317,775,233 (D) 15,317,775,227

Your calculator will not help you on this question. Due to rounding, any of the answer choices are possible.

But you can use this fact to eliminate three of the answer choices:

The square of any number ending in 5 is also a number ending in 5.

The only answer choice that ends in 5 is A, 15,317,775,225.



$$\begin{array}{r} \square \square \square \square 5 \\ \times \square \square \square \square 5 \\ \hline \square \square \square \square 5 \end{array}$$

EXAMPLE 2

What is a possible area of the wooden triangle shown?

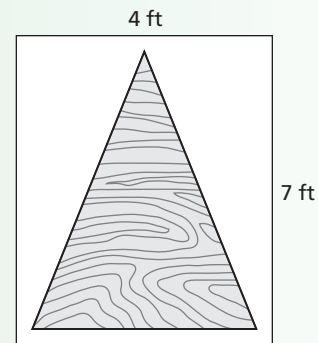
- (F) 11 square feet (H) 14 square feet
 (G) 20 square feet (J) 24 square feet

The triangle is inside a rectangle with an area of $7 \times 4 = 28$ square feet.

If the triangle had the same base and height as the rectangle, its area would be half the area of the rectangle, 14 square feet.

However, the triangle fits inside the rectangle, so its area must be less than 14 square feet.

The only answer choice that is less than 14 square feet is F, 11 square feet.





Try to eliminate unreasonable answer choices. Some choices may be too great or too small, have incorrect units, or not be divisible by a necessary number.

Read each test item and answer the questions that follow.

Item A

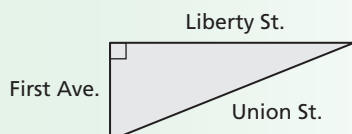
The top speed of a three-toed sloth is 0.12 miles per hour. About how many feet can a sloth travel in an hour?

- (A) 0.12 feet (C) 2.27 feet
(B) 600 feet (D) 7500 inches

1. Are there any answer choices you can eliminate immediately? If so, which choices and why?
2. Describe how you can use estimation to find the correct answer.

Item B

A city park is shaped like a triangle. The Liberty Street side of the park is 120 feet long, and the First Avenue side is 50 feet long.



What is the approximate length of the side of the park that faces Union Street?

- (F) 25 feet (H) 65 feet
(G) 110 inches (J) 130 feet

3. Can any of the answer choices be eliminated immediately? If so, which choices and why?
4. Are there any properties you can use to solve this problem? If so, what are they?
5. Describe how to find the correct answer without doing any calculations.

Item C

Approximately how long will the average 18-year-old have slept in his lifetime?

- (A) 6 weeks (C) 6 years
(B) 6 months (D) 6 decades

6. Which answer choice can be eliminated immediately? Why?
7. Explain how to use mental math to solve this problem.

Item D

Sheila's paychecks for February and March were equal. If she worked every day during both months, for which month was her daily pay lower?

- (F) February
(G) March
(H) Her daily pay did not change.
(J) Cannot be determined

8. What do you need to know to solve this problem?
9. Describe how you can find the correct answer.

Item E

Greg tripled the number of baseball cards he had last week. Which of these could be the number of cards Greg has now?

- (A) 100 (C) 150
(B) 200 (D) 250

10. The number of cards that Greg has now must be divisible by what number? How can you tell if a number is divisible by this number?
11. Describe how to find the answer to this problem.

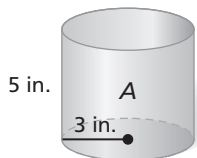


STANDARDIZED TEST PREP

CUMULATIVE ASSESSMENT, CHAPTERS 1–2

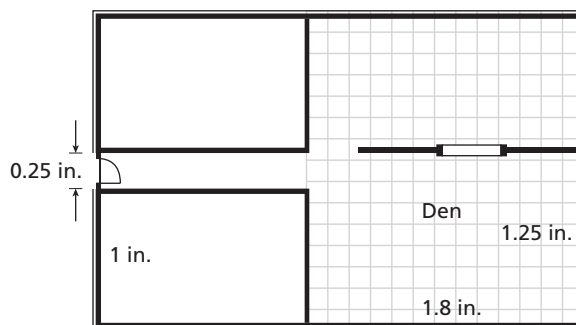
Multiple Choice

- What operation does \diamond represent if $x \diamond 2.2 = 4.5$ when $x = 9.9$?
 - Addition
 - Subtraction
 - Multiplication
 - Division
- A couple earns \$4819.25 a month. They pay 9.5% of their monthly income as the monthly payment on their car. To the nearest dollar, how much does the couple pay for their monthly car payment?
 - \$458
 - \$507
 - \$4578
 - \$4810
- Every dimension of cylinder *A* is multiplied by 4 to make cylinder *B*. What is the ratio of the volume of cylinder *A* to the volume of cylinder *B*?



- $\frac{1}{64}$
 - $\frac{1}{16}$
 - $\frac{1}{4}$
 - $\frac{1}{3}$
- A clock loses 5 minutes every day. How much time will it lose in 2 hours?
 - 0.417 second
 - 25 seconds
 - 240 seconds
 - 600 seconds
 - A statue is 8 feet tall. The display case for a model of the statue is 18 inches tall. Which scale allows for the tallest model of the statue that will fit in the display case?
 - 1 inch:2 inches
 - 1 inch:7 inches
 - 1 inch:5 inches
 - 1 inch:10 inches
 - What is the value of $-|6^2|$?
 - 36
 - 12
 - 8
 - 3

- Mr. Phillips wants to install hardwood flooring in his den. The flooring costs \$25.86 per square yard. The blueprint below shows his house. What other information do you need in order to find the total cost of the flooring?



- The lengths and widths of the adjoining rooms in the blueprint
 - The total area of the blueprint
 - The scale of inches in the blueprint to yards in the house
 - The width of the den
- What value of n makes the equation below have no solution?

$$2x + 2 = nx - 3$$
 - 2
 - 0
 - 2
 - 3
 - Which of the equations below represents the second step of the solution process?

Step 1: $3(5x - 2) + 27 = -24$

Step 2:

Step 3: $15x + 21 = -24$

Step 4: $15x = -45$

Step 5: $x = -3$

 - $3(5x + 27) - 2 = -24$
 - $3(5x + 25) = -24$
 - $15x - 2 + 27 = -24$
 - $15x - 6 + 27 = -24$



If you are stuck on a problem, skip it and come back later. Another problem might remind you of something that will help. If you feel yourself become tense, take a few deep breaths to relax.

10. Cass drove 3 miles to school, and then she drove m miles to a friend's house. The total mileage for these two trips was 8 miles. Which equation CANNOT be used to determine the number of miles Cass drove?

- (F) $3 + m = 8$
- (G) $3 - m = 8$
- (H) $8 - 3 = m$
- (J) $8 - m = 3$

11. If $\frac{20}{x} = \frac{4}{x-5}$, which of the following is a true statement?

- (A) $x(x-5) = 80$
- (B) $20x = 4(x-5)$
- (C) $20(x-5) = 4x$
- (D) $24 = 2x - 5$

Gridded Response

12. Four times a number is two less than six times the same number minus ten. What is the number?
13. Melissa invested her savings in a retirement account that pays simple interest. A portion of her account record is shown below. What is the interest rate on Melissa's account? Write your answer as a decimal.

Date	Transaction	Amount	Balance
8/1	Beginning deposit	\$6000.00	\$6000.00
8/31	Interest payment	\$192.00	\$6192.00
9/1	Withdrawal	\$1000.00	\$5192.00
9/30	Interest payment	\$166.14	\$5358.14

14. At 2:45 P.M. you are 112 miles from Dallas. You want to be in Dallas at 4:30 P.M. What is the average number of miles per hour you must travel to be on time?
15. A cyclist travels 45 miles in 4 hours. How many feet does she travel in one second?
16. A bike rental shop charges a one-time charge of \$8 plus an hourly fee to rent a bike. Dan paid \$24.50 to rent a bike for $5\frac{1}{2}$ hours. Find the bike shop's hourly fee in dollars.

Short Response

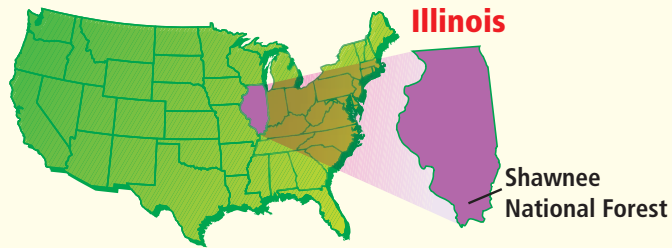
17. Alex buys 5 calendars to give as gifts. Each calendar has the same price. When the cashier rings up Alex's calendars, the total cost before tax is \$58.75.
- a. Write and solve an equation to find the cost of each calendar.
 - b. The total cost of Alex's calendars after tax is \$63.45. Find the percent sales tax. Show your work and explain in words how you found your answer.
 - c. Alex's friend Keisha buys some calendars for the same price. She uses her 15% discount card. The total cost before tax is \$39.95. How many calendars did Keisha buy? Show your work and explain in words how you found your answer.
18. Is the percent increase from 50 to 100 the same as the percent decrease from 100 to 50? Explain your reasoning in words.

Extended Response

19. Korena is putting a decorative border around her rectangular flower garden. The total perimeter of the garden is 200 feet.
- a. Draw three different rectangles that could represent Korena's flower garden. Label the dimensions of your rectangles.
 - b. Use the table to show the lengths and widths of five different rectangles that could represent Korena's flower garden. Do not use any of your rectangles from part a.

Possible Dimensions of Korena's Garden		
Length (ℓ)	Width (w)	Perimeter (P)
■	■	■
■	■	■
■	■	■
■	■	■
■	■	■

- c. The length of Korena's garden is 4 times its width. Explain how to use the perimeter formula $P = 2\ell + 2w$ to find the dimensions of Korena's garden.
- d. Find the dimensions of Korena's garden.



★ Bullfrogs

The bullfrog is the largest frog in Illinois. Adult bullfrogs are usually between $3\frac{1}{2}$ and 6 inches in length, but some have been known to grow even larger. The largest recorded bullfrog found in Illinois was 8 inches long and weighed more than a pound!



Choose one or more strategies to solve each problem.

1. A bullfrog jumping up a hill is 8 feet from the top. Each time it jumps, it moves forward 18 inches. However, mud on the hill causes it to slide backward 2 inches after each jump. How many jumps will it take the frog to reach the top of the hill?
2. A certain pond is home to 120 bullfrog tadpoles. This is 90 more than 3 times the number of adult bullfrogs that live in a second pond. How many adult bullfrogs live in the second pond?

For 3–5, use the table.

3. How far could the largest recorded bullfrog in Illinois jump?
4. If a bullfrog is $4\frac{1}{2}$ inches long, what is its maximum jumping distance?
5. A certain adult bullfrog can jump $8\frac{3}{4}$ feet. This bullfrog must be at least how long?

Jumping Distances of Bullfrogs	
Length of Bullfrog (in.)	Maximum Jumping Distance (in.)
4	60
5	75
6	90



★ The Great Snake Migration

When temperatures drop, large numbers of snakes leave the swamps of southern Illinois. Some travel just a few hundred feet, while others travel several miles, to Snake Road, a 3-mile-long road on the western edge of the Shawnee Forest. In this area, you can see as many as 40 to 50 snakes in a day, including cottonmouth water moccasins, rattlesnakes, and copperheads.

Choose one or more strategies to solve each problem.

- In one hour, a researcher saw six snakes crossing Snake Road—three moccasins, a rattlesnake, a garter snake, and a diamondback water snake, though not in that order. The second snake to cross was a moccasin, and the other two moccasins crossed one after the other. The diamondback crossed before the garter snake. The first snake to cross was not the garter snake or the diamondback. A moccasin crossed last. In what order did the snakes cross Snake Road?



For 2, use the table.

In 2001–2002, several Illinois hospitals were asked how many vials of antivenom they had available to treat snake bites. The results are shown in the table. For example, 37 hospitals reported that they had between 1 and 4 vials of antivenom available.

Antivenom Survey Results						
Vials	0	1–4	5–9	10–14	15–19	20+
Hospitals	■	37	14	6	0	10

- The number of hospitals with no antivenom available to treat snake bites was 6 less than 3 times the number of hospitals that had 1–4 vials. How many hospitals had no antivenom with which to treat snake bites? What percent of the total number of hospitals surveyed does this represent? Round your answer to the nearest percent.

