

Functions

4A Function Concepts

- 4-1 Graphing Relationships
- 4-2 Relations and Functions
- Lab The Vertical-Line Test
- Lab Model Variable Relationships
- 4-3 Writing Functions
- 4-4 Graphing Functions
- Lab Connect Function Rules, Tables, and Graphs

4B Applying Functions

- 4-5 Scatter Plots and Trend Lines
- Lab Interpret Scatter Plots and Trend Lines
- 4-6 Arithmetic Sequences

Chapter Focus

- Use tables, diagrams, graphs, and equations to describe functions.
- Translate among representations of functions.
- Use functions to represent, analyze, and solve problems.

Is That *Your* Foot?

Criminologists use measurements, such as the size of footprints, and functions to help them identify criminals.



Chapter Project Online

KEYWORD: MA7 ChProj



ARE YOU READY?

✓ Vocabulary

Match each term on the left with a definition on the right.

- | | |
|-------------------------|---|
| 1. absolute value | A. a letter used to represent a value that can change |
| 2. algebraic expression | B. the value generated for y |
| 3. input | C. a group of numbers, symbols, and variables with one or more operations |
| 4. output | D. the distance of a number from zero on the number line |
| 5. x -axis | E. the horizontal number line in the coordinate plane |
| | F. a value substituted for x |

✓ Ordered Pairs

Graph each point on the same coordinate plane.

- | | | | |
|---------------|----------------|---------------|---------------|
| 6. $(-2, 4)$ | 7. $(0, -5)$ | 8. $(1, -3)$ | 9. $(4, 2)$ |
| 10. $(3, -2)$ | 11. $(-1, -2)$ | 12. $(-1, 3)$ | 13. $(-4, 0)$ |

✓ Function Tables

Generate ordered pairs for each function for $x = -2, -1, 0, 1, 2$.

- | | | |
|----------------------------|---------------------|---------------------|
| 14. $y = -2x - 1$ | 15. $y = x + 1$ | 16. $y = -x^2$ |
| 17. $y = \frac{1}{2}x + 2$ | 18. $y = (x + 1)^2$ | 19. $y = (x - 1)^2$ |

✓ Solve Multi-Step Equations

Solve each equation. Check your answer.

- | | | |
|-------------------------|--------------------|----------------------------|
| 20. $17x - 15 = 12$ | 21. $-7 + 2t = 7$ | 22. $-6 = \frac{p}{3} + 9$ |
| 23. $5n - 10 = 35$ | 24. $3r - 14 = 7$ | 25. $9 = \frac{x}{2} + 1$ |
| 26. $-2.4 + 1.6g = 5.6$ | 27. $34 - 2x = 12$ | 28. $2(x + 5) = -8$ |

✓ Solve for a Variable

Solve each equation for the indicated variable.

- | | | |
|--------------------------|---------------------------|---------------------------------------|
| 29. $A = \ell w$ for w | 30. $V = \ell wh$ for w | 31. $A = bh$ for h |
| 32. $C = 2\pi r$ for r | 33. $I = Prt$ for P | 34. $V = \frac{1}{3} \ell wh$ for h |

Where You've Been

Previously, you

- were introduced to functions when you generated and graphed ordered pairs.
- stated rules for relationships among values.
- represented and interpreted data using bar graphs and circle graphs.

In This Chapter

You will study

- relationships between variables and determine whether a relation is a function.
- relationships in function notation.
- how trend lines on scatter plots can help you make predictions.

Where You're Going

You can use the skills in this chapter

- to find values of a function from a graph.
- to analyze data and make predictions in other courses, such as Chemistry.
- to calculate total earnings for a certain hourly rate.

Key Vocabulary/Vocabulario

arithmetic sequence	sucesión aritmética
common difference	diferencia común
correlation	correlación
dependent variable	variable dependiente
domain	dominio
function	función
function notation	notación de función
independent variable	variable independiente
no correlation	sin correlación
range	rango
relation	relación
scatter plot	diagrama de dispersión
sequence	sucesión

Vocabulary Connections

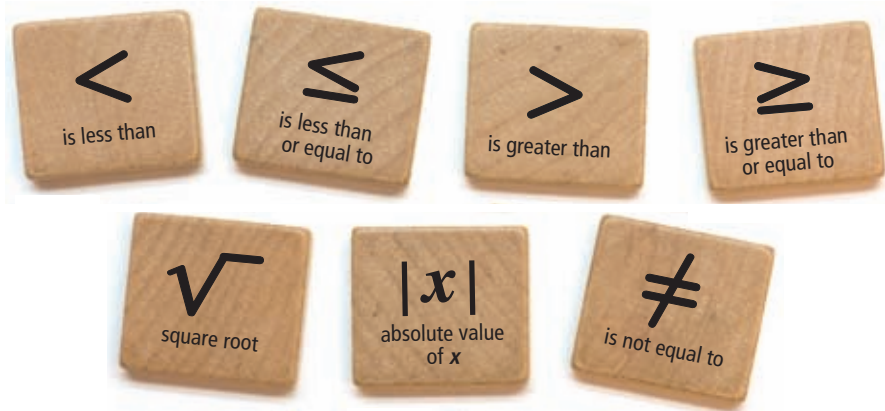
To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. What does the word *dependent* mean? What do you think is true about the value of a **dependent variable**?
2. A *function* is a special type of relation and *notation* is a method of writing. What do you suppose is meant by **function notation**?
3. The word *correlation* means “relationship.” What might it mean if two sets of data have **no correlation**?
4. What does it mean when someone says that two people have something in *common*? If *difference* is the answer to a subtraction problem, what might it mean for a list of numbers to have a **common difference**?

Reading Strategy: Read and Interpret Math Symbols

It is essential that as you read through each lesson of the textbook, you can interpret mathematical symbols.

Common Math Symbols



You must be able to translate symbols into words . . .

Using Symbols	Using Words
$3\left(\frac{x}{12}\right) - 1 = 21$	Three times the quotient of x and 12, minus 1 equals 21.
$25x + 6 \geq 17$	Twenty-five times x plus 6 is greater than or equal to 17.
$ x > 14$	The absolute value of x is greater than 14.
$\sqrt{60 + x} \leq 40$	The square root of the sum of 60 and x is less than or equal to 40.

. . . and words into symbols.

Using Words	Using Symbols
The height of the shed is at least 9 feet.	$h \geq 9$ ft
The distance is at most one tenth of a mile.	$d \leq 0.1$ mi
The silo contains more than 600 cubic feet of corn.	$c > 600$ ft ³

Try This

Translate the symbols into words.

- $x \leq \sqrt{10}$
- $|x| + 2 > 45$
- $-5 \leq x < 8$
- $-6 - \frac{1}{5}x = -32$

Translate the words into symbols.

- There are less than 15 seconds remaining.
- The tax rate is 8.25 percent of the cost.
- Ann counted over 100 pennies.
- Joe can spend at least \$22 but no more than \$30.

4-1

Graphing Relationships



Objectives

Match simple graphs with situations.

Graph a relationship.

Vocabulary

continuous graph
discrete graph

Who uses this?

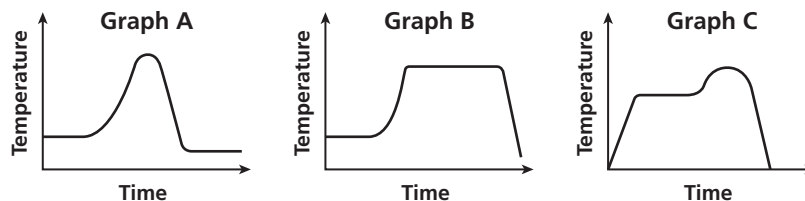
Cardiologists can use graphs to analyze their patients' heartbeats. (See Example 2.)

Graphs can be used to illustrate many different situations. For example, trends shown on a cardiograph can help a doctor see how the patient's heart is functioning.

To relate a graph to a given situation, use key words in the description.

EXAMPLE 1 Relating Graphs to Situations

The air temperature was constant for several hours at the beginning of the day and then rose steadily for several hours. It stayed the same temperature for most of the day before dropping sharply at sundown. Choose the graph that best represents this situation.



Step 1 Read the graphs from left to right to show time passing.

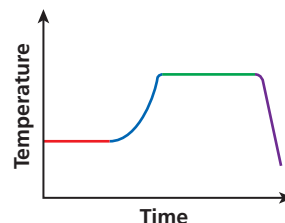
Step 2 List key words in order and decide which graph shows them.

Key Words	Segment Description	Graphs
Was constant	Horizontal	Graphs A and B
Rose steadily	Slanting upward	Graphs A and B
Stayed the same	Horizontal	Graph B
Dropped sharply	Slanting downward	Graph B

Step 3 Pick the graph that shows all the key phrases in order.

horizontal, slanting upward,
horizontal, slanting downward

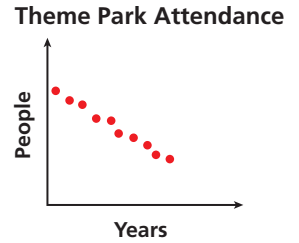
The correct graph is B.



- The air temperature increased steadily for several hours and then remained constant. At the end of the day, the temperature increased slightly again before dropping sharply. Choose the graph above that best represents this situation.

As seen in Example 1, some graphs are connected lines or curves called **continuous graphs**. Some graphs are only distinct points. These are called **discrete graphs**.

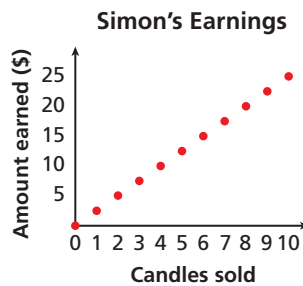
The graph on theme-park attendance is an example of a discrete graph. It consists of distinct points because each year is distinct and people are counted in whole numbers only. The values between the whole numbers are not included, since they have no meaning for the situation.



EXAMPLE 2 Sketching Graphs for Situations

Sketch a graph for each situation. Tell whether the graph is continuous or discrete.

A Simon is selling candles to raise money for the school dance. For each candle he sells, the school will get \$2.50. He has 10 candles that he can sell.



The amount earned (y -axis) increases by \$2.50 for each candle Simon sells (x -axis).

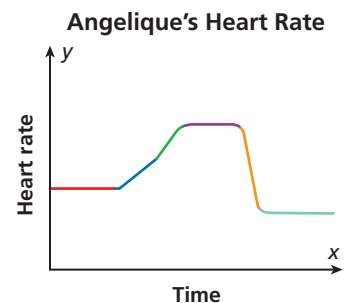
Since Simon can only sell whole numbers of candles, the graph is 11 distinct points.

The graph is discrete.

B Angelique's heart rate is being monitored while she exercises on a treadmill. While walking, her heart rate remains the same. As she increases her pace, her heart rate rises at a steady rate. When she begins to run, her heart rate increases more rapidly and then remains high while she runs. As she decreases her pace, her heart rate slows down and returns to her normal rate.

As time passes during her workout (moving left to right along the x -axis), her heart rate (y -axis) does the following:

- remains the same,
- rises at a steady rate,
- increases more rapidly (steeper than previous segment),
- remains high,
- slows down,
- and then returns to her normal rate.



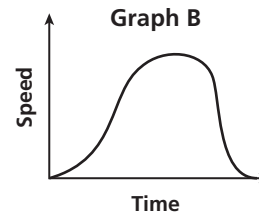
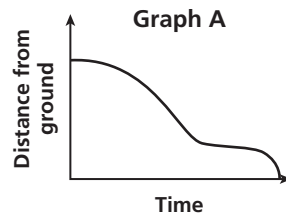
The graph is continuous.



Sketch a graph for each situation. Tell whether the graph is continuous or discrete.

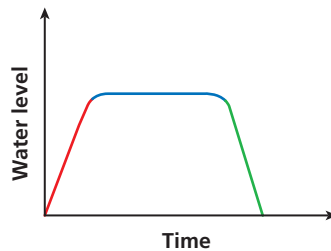
- 2a.** Jamie is taking an 8-week keyboarding class. At the end of each week, she takes a test to find the number of words she can type per minute. She improves each week.
- 2b.** Henry begins to drain a water tank by opening a valve. Then he opens another valve. Then he closes the first valve. He leaves the second valve open until the tank is empty.

When sketching or interpreting a graph, pay close attention to the labels on each axis. Both graphs below show a relationship about a child going down a slide. **Graph A** represents the child's *distance from the ground* over time. **Graph B** represents the child's *speed* over time.



EXAMPLE 3 Writing Situations for Graphs

Write a possible situation for the given graph.



Step 1 Identify labels.

x -axis: time y -axis: water level

Step 2 Analyze sections.

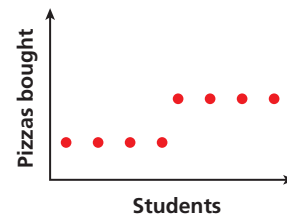
Over time, the water level

- **increases steadily,**
- **remains unchanged,**
- and then **decreases steadily.**

Possible Situation: A watering can is **filled with water**. It **sits for a while** until some flowers are planted. The water is then **emptied** on top of the planted flowers.



3. Write a possible situation for the given graph.

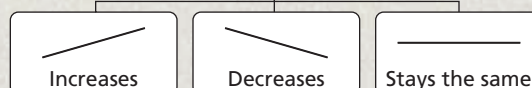


THINK AND DISCUSS

1. Should a graph of age related to height be a continuous graph or a discrete graph? Explain.
2. Give an example of a situation that, when graphed, would include a horizontal segment.
3. **GET ORGANIZED** Copy and complete the graphic organizer. Write an example of key words that suggest the given segments on a graph. One example for each segment is given for you.



Key Words for Graph Segments



GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

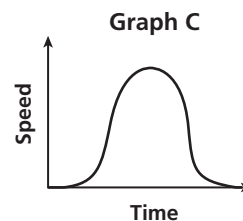
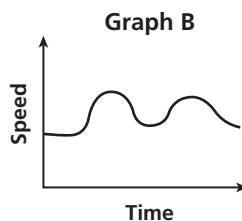
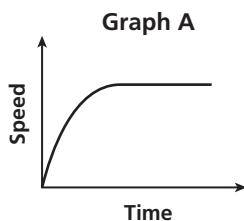
1. A ___?___ graph is made of connected lines or curves. (*continuous* or *discrete*)
2. A ___?___ graph is made of only distinct points. (*continuous* or *discrete*)

SEE EXAMPLE 1

p. 234

Choose the graph that best represents each situation.

3. A person alternates between running and walking.
4. A person gradually speeds up to a constant running pace.
5. A person walks, gradually speeds up to a run, and then slows back down to a walk.



SEE EXAMPLE 2

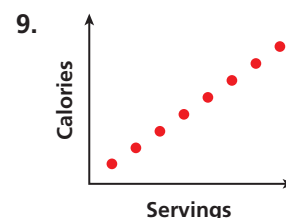
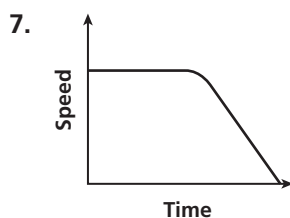
p. 235

6. Maxine is buying extra pages for her photo album. Each page holds exactly 8 photos. Sketch a graph to show the maximum number of photos she can add to her album if she buys 1, 2, 3, or 4 extra pages. Tell whether the graph is continuous or discrete.

SEE EXAMPLE 3

p. 236

Write a possible situation for each graph.



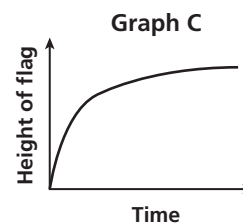
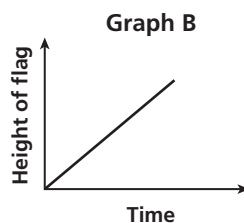
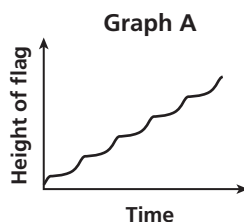
PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
10–12	1
13	2
14–16	3

Choose the graph that best represents each situation.

10. A flag is raised up a flagpole quickly at the beginning and then more slowly near the top.
11. A flag is raised up a flagpole in a jerky motion, using a hand-over-hand method.
12. A flag is raised up a flagpole at a constant rate of speed.



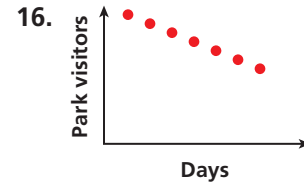
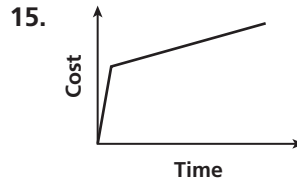
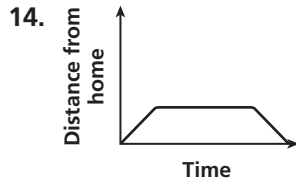
Extra Practice

Skills Practice p. S10

Application Practice p. S31

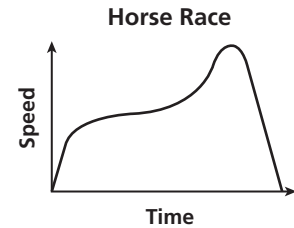
13. For six months, a puppy gained weight at a steady rate. Sketch a graph to illustrate the weight of the puppy during that time period. Tell whether the graph is continuous or discrete.

Write a possible situation for each graph.



17. **Data Collection** Use a graphing calculator and motion detector for the following.
- On a coordinate plane, draw a graph relating distance from a starting point walking at various speeds and time.
 - Using the motion detector as the starting point, walk away from the motion detector to make a graph on the graphing calculator that matches the one you drew.
 - Compare your walking speeds to each change in steepness on the graph.

18. **Sports** The graph shows the speed of a horse during and after a race. Use it to describe the changing pace of the horse during the race.



19. **Recreation** You hike up a mountain path starting at 10 A.M. You camp overnight and then walk back down the same path at the same pace at 10 A.M. the next morning. On the same set of axes, graph the relationship between distance from the top of the mountain and the time of day for both the hike up and the hike down. What does the point of intersection of the graphs represent?
20. **Critical Thinking** Suppose that you sketched a graph of speed related to time for a brick that fell from the top of a building. Then you sketched a graph for speed related to time for a ball that was rolled down a hill and then came to rest. How would the graphs be the same? How would they be different?

21. **Write About It** Describe a real-life situation that could be represented by a distinct graph. Then describe a real-life situation that could be represented by a continuous graph.

LINK

Sports



On November 1, 1938, the underdog Seabiscuit beat the heavily favored Triple-Crown winner War Admiral in a historic horse race at Pimlico Race Course in Baltimore, Maryland.

MULTI-STEP TEST PREP

22. This problem will prepare you for the Multi-Step Test Prep on page 264.

A rectangular pool that is 4 feet deep at all places is being filled at a constant rate.

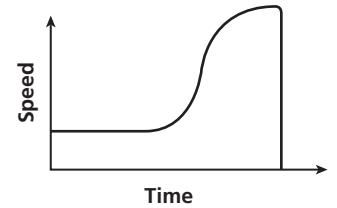
- Sketch a graph to show the depth of the water as it increases over time.
- The side view of another swimming pool is shown. If the pool is being filled at a constant rate, sketch a graph to show the depth of the water as it increases over time.



23. Which situation would NOT be represented by a distinct graph?
- (A) Amount of money earned based on the number of cereal bars sold
 - (B) Number of visitors to a grocery store per day for one week
 - (C) The amount of iced tea in a pitcher at a restaurant during the lunch hour
 - (D) The total cost of buying 1, 2, or 3 CDs at the music store

24. Which situation is best represented by the graph?

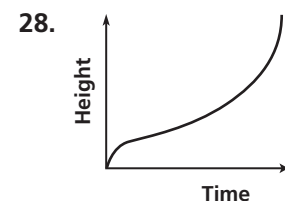
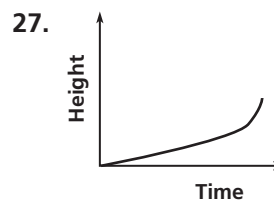
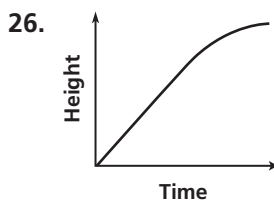
- (F) A snowboarder starts at the bottom of the hill and takes a ski lift to the top.
- (G) A cruise boat travels at a steady pace from the port to its destination.
- (H) An object falls from the top of a building and gains speed at a rapid pace before hitting the ground.
- (J) A marathon runner starts at a steady pace and then runs faster at the end of the race before stopping at the finish line.



25. **Short Response** Marla participates in a triathlon consisting of swimming, biking, and running. Would a graph of Marla's speed during the triathlon be a continuous graph or a distinct graph? Explain.

CHALLENGE AND EXTEND

Pictured are three vases and graphs representing the height of water as it is poured into each of the vases at a constant rate. Match each vase with the correct graph.



SPIRAL REVIEW

Evaluate each expression. (*Lesson 1-4*)

29. -2^3

30. 4^4

31. $\left(\frac{1}{3}\right)^2$

Generate ordered pairs for each function for $x = -2, -1, 0, 1,$ and 2 . Graph the ordered pairs and describe the pattern. (*Lesson 1-8*)

32. $y = x - 2$

33. $2x + y = 1$

34. $y = |x - 1|$

35. $y = x^2 + 2$

Write and solve an equation to represent each relationship. (*Lesson 2-1*)

36. A number increased by 11 is equal to 3. 37. Five less than a number is equal to -2 .

4-2

Relations and Functions



Objectives

Identify functions.
Find the domain and range of relations and functions.

Vocabulary

relation
domain
range
function

Why learn this?

You can use a relation to show finishing positions and scores in a track meet.

In Lesson 4-1, you saw relationships represented by graphs. Relationships can also be represented by a set of ordered pairs, called a **relation**.

In the scoring system of some track meets, **first place** is worth **5** points, **second place** is worth **3** points, **third place** is worth **2** points, and **fourth place** is worth **1** point. This scoring system is a relation, so it can be shown as ordered pairs, $\{(1, 5), (2, 3), (3, 2), (4, 1)\}$. You can also show relations in other ways, such as tables, graphs, or *mapping diagrams*.

EXAMPLE 1 Showing Multiple Representations of Relations

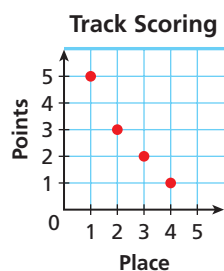
Express the relation for the track meet scoring system, $\{(1, 5), (2, 3), (3, 2), (4, 1)\}$, as a table, as a graph, and as a mapping diagram.

Table

Track Scoring	
Place	Points
1	5
2	3
3	2
4	1

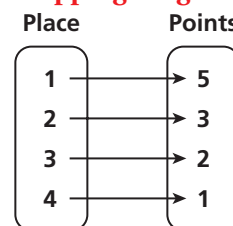
Write all *x*-values under "Place" and all *y*-values under "Points."

Graph



Use the *x*- and *y*-values to plot the ordered pairs.

Mapping Diagram



Write all *x*-values under "Place" and all *y*-values under "Points." Draw an arrow from each *x*-value to its corresponding *y*-value.

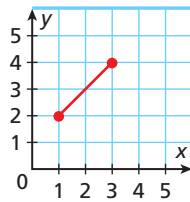


- Express the relation $\{(1, 3), (2, 4), (3, 5)\}$ as a table, as a graph, and as a mapping diagram.

The **domain** of a relation is the set of first coordinates (or *x*-values) of the ordered pairs. The **range** of a relation is the set of second coordinates (or *y*-values) of the ordered pairs. The domain of the track meet scoring system is $\{1, 2, 3, 4\}$. The range is $\{5, 3, 2, 1\}$.

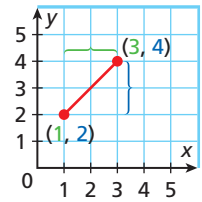
EXAMPLE 2 Finding the Domain and Range of a Relation

Give the domain and range of the relation.



The domain is all x -values from 1 through 3, inclusive.

The range is all y -values from 2 through 4, inclusive.

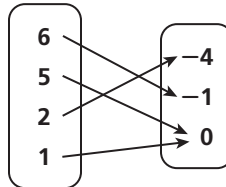


$$D: 1 \leq x \leq 3 \quad R: 2 \leq y \leq 4$$



Give the domain and range of each relation.

2a.



2b.

x	y
1	1
4	4
8	1

A **function** is a special type of relation that pairs each domain value with exactly one range value.

EXAMPLE 3 Identifying Functions

Give the domain and range of each relation. Tell whether the relation is a function. Explain.

A

Field Trip	
Students x	Buses y
75	2
68	2
125	3

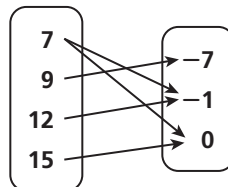
$$D: \{75, 68, 125\}$$

$$R: \{2, 3\}$$

Even though 2 appears twice in the table, it is written only once when writing the range.

This relation is a function. Each domain value is paired with exactly one range value.

B



$$D: \{7, 9, 12, 15\}$$

$$R: \{-7, -1, 0\}$$

Use the arrows to determine which domain values correspond to each range value.

This relation is not a function. Each domain value does not have exactly one range value. The domain value 7 is paired with the range values -1 and 0 .

Writing Math

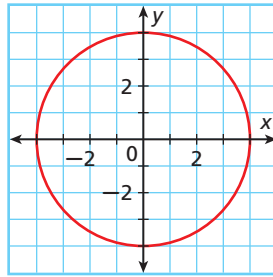
When there is a finite number of values in a domain or range, list the values inside braces.

Helpful Hint

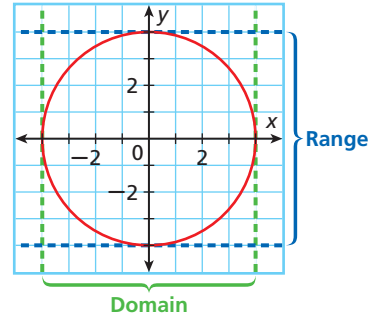
To find the domain and range of a graph, it may help to draw lines to see the x - and y -values.

Give the domain and range of each relation. Tell whether the relation is a function. Explain.

C



Draw lines to see the domain and range values.



D: $-4 \leq x \leq 4$ R: $-4 \leq y \leq 4$

x	4	0	0	-4
y	0	4	-4	0

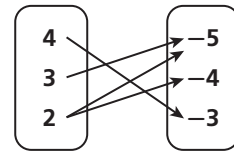
To compare domain and range values, make a table using points from the graph.

This relation is not a function because there are several domain values that have more than one range value. For example, the domain value 0 is paired with both 4 and -4.



Give the domain and range of each relation. Tell whether the relation is a function. Explain.

3a. $\{(8, 2), (-4, 1), (-6, 2), (1, 9)\}$ 3b.



Student to Student

Functions



Eric Dawson
Boone High School

I decide whether a list of ordered pairs is a function by looking at the x -values. If they're all different, then it's a function.

$(1, 6), (2, 5), (6, 5), (0, 8)$
All different x -values
Function

$(5, 6), (7, 2), (5, 8), (6, 3)$
Same x -value (with different y -values)
Not a function

THINK AND DISCUSS

- Describe how to tell whether a set of ordered pairs is a function.
- Can the graph of a vertical line segment represent a function? Explain.



- GET ORGANIZED** Copy and complete the graphic organizer by explaining when a relation is a function and when it is not a function.

A relation is...	
A function if...	Not a function if...

GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

- Use a mapping diagram to show a relation that is not a *function*.
- The set of x -values for a relation is also called the ?. (*domain* or *range*)

SEE EXAMPLE 1

p. 240

Express each relation as a table, as a graph, and as a mapping diagram.

- $\{(1, 1), (1, 2)\}$
- $\{(-1, 1), (-2, \frac{1}{2}), (-3, \frac{1}{3}), (-4, \frac{1}{4})\}$
- $\{(-1, 1), (-3, 3), (5, -5), (-7, 7)\}$
- $\{(0, 0), (2, -4), (2, -2)\}$

SEE EXAMPLE 2

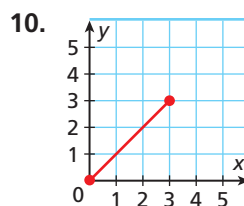
p. 241

Give the domain and range of each relation.

- $\{(-5, 7), (0, 0), (2, -8), (5, -20)\}$
- $\{(1, 2), (2, 4), (3, 6), (4, 8), (5, 10)\}$

9.

x	3	5	2	8	6
y	9	25	4	81	36



SEE EXAMPLE 3

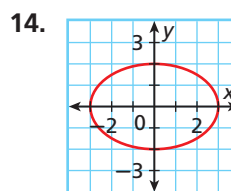
p. 241

Multi-Step Give the domain and range of each relation. Tell whether the relation is a function. Explain.

- $\{(1, 3), (1, 0), (1, -2), (1, 8)\}$
- $\{(-2, 1), (-1, 2), (0, 3), (1, 4)\}$

13.

x	-2	-1	0	1	2
y	1	1	1	1	1



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
15–16	1
17–18	2
19–20	3

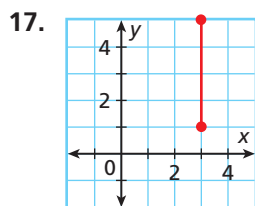
Express each relation as a table, as a graph, and as a mapping diagram.

- $\{(-2, -4), (-1, -1), (0, 0), (1, -1), (2, -4)\}$
- $\{(2, 1), (2, \frac{1}{2}), (2, 2), (2, 2\frac{1}{2})\}$

Extra Practice

Skills Practice p. S10
Application Practice p. S31

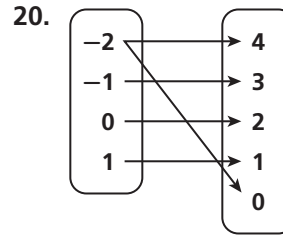
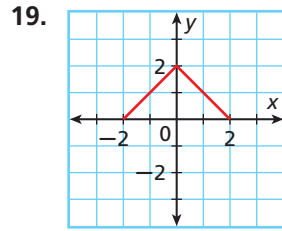
Give the domain and range of each relation.



18.

x	y
4	4
5	5
6	6
7	7
8	8

Multi-Step Give the domain and range of each relation. Tell whether the relation is a function. Explain.



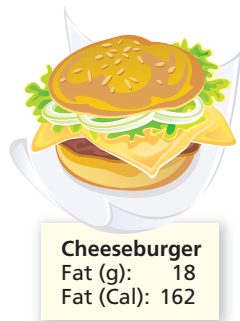
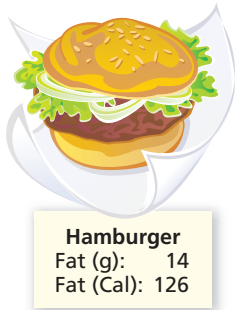
21. **Consumer Application** An electrician charges a base fee of \$75 plus \$50 for each hour of work. Create a table that shows the amount the electrician charges for 1, 2, 3, and 4 hours of work. Let x represent the number of hours and y represent the amount charged for x hours. Is this relation a function? Explain.

22. **Geometry** Write a relation as a set of ordered pairs in which the x -value represents the side length of a square and the y -value represents the area of that square. Use a domain of 2, 4, 6, 9, and 11.

23. **Multi-Step** Create a mapping diagram to display the numbers of days in 1, 2, 3, and 4 weeks. Is this relation a function? Explain.

24. **Nutrition** The illustrations list the number of grams of fat and the number of Calories from fat for selected foods.

- Create a graph for the relation between grams of fat and Calories from fat.
- Is this relation a function? Explain.



25. **Recreation** A shop rents canoes for a \$7 equipment fee plus \$2 per hour, with a maximum cost of \$15 per day. Express the number of hours x and the cost y as a relation in table form, and find the cost to rent a canoe for 1, 2, 3, 4, and 5 hours. Is this relation a function? Explain.

26. **Health** You can burn about 6 Calories per minute bicycling. Let x represent the number of minutes bicycled, and let y represent the number of Calories burned.

- Write ordered pairs to show the number of Calories burned by bicycling for 60, 120, 180, 240, or 300 minutes. Graph the ordered pairs.

- Find the domain and range of the relation.
- Does this graph represent a function? Explain.

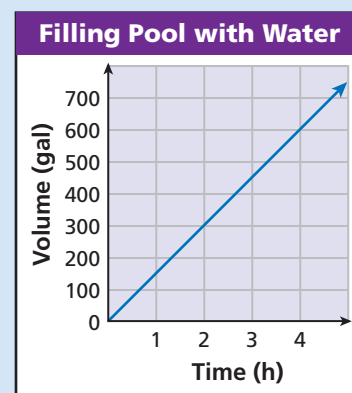
27. **Critical Thinking** For a function, can the number of elements in the range be greater than the number of elements in the domain? Explain.

28. **Critical Thinking** Tell whether each statement is true or false. If false, explain why.

- All relations are functions.
- All functions are relations.



29. This problem will prepare you for the Multi-Step Test Prep on page 264.
- The graph shows the amount of water being pumped into a pool over a 5-hour time period. Find the domain and range.
 - Does the graph represent a function? Explain.
 - Give the time and volume as ordered pairs at 2 hours and at 3 hours 30 minutes.



30. **/// ERROR ANALYSIS ///** When asked whether the relation $\{(-4, 16), (-2, 4), (0, 0), (2, 4)\}$ is a function, a student stated that the relation is not a function because 4 appears twice. What error did the student make? How would you explain to the student why this relation is a function?
31. **Write About It** Describe a real-world situation for a relation that is NOT a function. Create a mapping diagram to show why the relation is not a function.

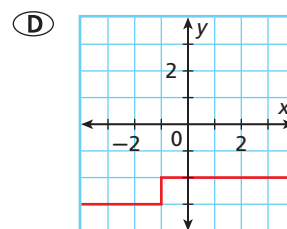
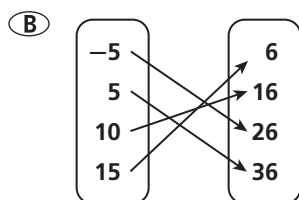


32. Which of the following relations is NOT a function?

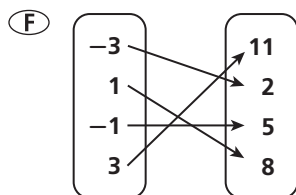
(A) $\{(6, 2), (-1, 2), (-3, 2), (-5, 2)\}$

(C)

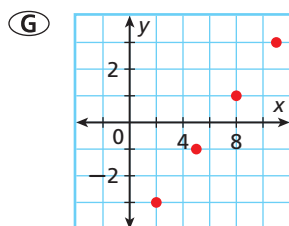
x	3	5	7
y	1	15	30



33. Which is NOT a correct way to describe the function $\{(-3, 2), (1, 8), (-1, 5), (3, 11)\}$?



(H) Domain: $\{-3, 1, -1, 3\}$
Range: $\{2, 8, 5, 11\}$

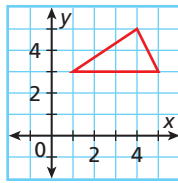


(J)

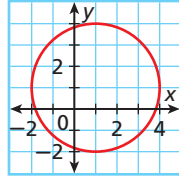
x	y
-3	2
-1	5
1	8
3	11

34. Which graph represents a function?

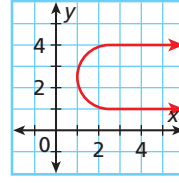
(A)



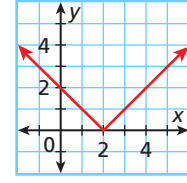
(B)



(C)



(D)



35. **Extended Response** Use the table for the following.

x	-3	-1	0	1	3
y	5	7	9	11	13

- Express the relation as ordered pairs.
- Give the domain and range of the relation.
- Does the relation represent a function? Explain your answer.

CHALLENGE AND EXTEND

- What values of a make the relation $\{(a, 1), (2, 3), (4, 5)\}$ a function? Explain.
- What values of b make the relation $\{(5, 6), (7, 8), (9, b)\}$ a function? Explain.
- The *inverse* of a relation is created by interchanging the x - and y - coordinates of each ordered pair in the relation.
 - Find the inverse of the following relation: $\{(-2, 5), (0, 4), (3, -8), (7, 5)\}$.
 - Is the original relation a function? Why or why not? Is the inverse of the relation a function? Why or why not?
 - The statement “If a relation is a function, then the inverse of the relation is also a function” is sometimes true. Give an example of a relation and its inverse that are both functions. Then give an example of a relation and its inverse that are both not functions.

SPIRAL REVIEW

- The ratio of a rectangle’s width to its length is 3:4. The length of the rectangle is 36 cm. Write and solve a proportion to find the rectangle’s width. (*Lesson 2-7*)
- A scale drawing of a house is drawn with a scale of 1 in.:16 ft. Find the actual length of a hallway that is $\frac{5}{8}$ in. on the scale drawing. (*Lesson 2-7*)
- Penny wants to drink at least 64 ounces of water today. She has consumed 45 ounces of water so far. Write, solve, and graph an inequality to determine how many more ounces of water Penny must drink to reach her goal. (*Lesson 3-2*)
- The local pizza parlor sold the following number of pizzas over 10 days. Sketch a graph for the situation. Tell whether the graph is continuous or discrete. (*Lesson 4-1*)

Time (days)	1	2	3	4	5	6	7	8	9	10
Pizzas Sold	5	11	2	4	8	10	3	6	12	1



The Vertical-Line Test

The *vertical-line test* can be used to visually determine whether a graphed relation is a function.

Use with Lesson 4-2

Activity

- 1 Look at the values in Table 1. Is every x -value paired with exactly one y -value? If not, what x -value(s) are paired with more than one y -value?
- 2 Is the relation a function? Explain.
- 3 Graph the points from the Table 1. Draw a vertical line through each point of the graph. Does any vertical line touch more than one point?

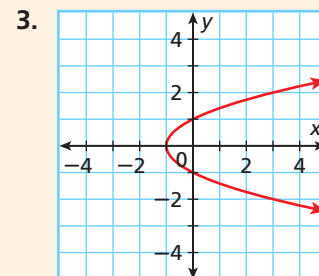
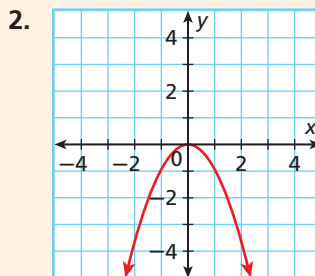
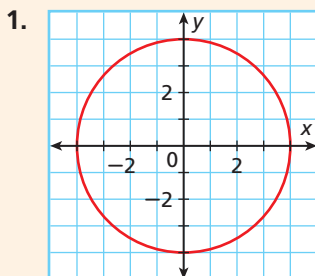
x	y
-2	-5
-1	-3
0	-1
1	1
2	3
3	5

- 4 Look at the values in Table 2. Is every x -value paired with exactly one y -value? If not, what x -value(s) are paired with more than one y -value?
- 5 Is the relation a function? Explain.
- 6 Graph the points from the Table 2. Draw a vertical line through each point of the graph. Does any vertical line touch more than one point?
- 7 What is the x -value of the two points that are on the same vertical line? Is that x -value paired with more than one y -value?
- 8 Write a statement describing how to use a vertical line to tell if a relation is a function. This is called the vertical-line test.
- 9 Why does the vertical-line test work?

x	y
-2	-3
1	4
0	5
1	2
2	3
3	5

Try This

Use the vertical-line test to determine whether each relation is a function. If a relation is not a function, list two ordered pairs that show the same x -value with two different y -values.





Model Variable Relationships

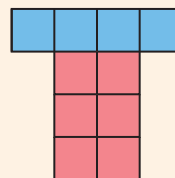
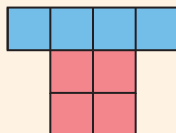
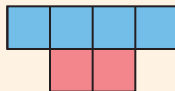
You can use models to represent an algebraic relationship. Using these models, you can write an algebraic expression to help describe and extend patterns.

Use with Lesson 4-3

The diagrams below represent the side views of tables. Each has a tabletop and a base. Copy and complete the chart using the pattern shown in the diagrams.

Tabletop →

Base →



TERM NUMBER	FIGURE	DESCRIPTION OF FIGURE	EXPRESSION FOR NUMBER OF BLOCKS	VALUE OF TERM (NUMBER OF BLOCKS)	ORDERED PAIR
1		length of tabletop = 4 height of base = 1	$4 + (2)1$	6	(1, 6)
2		length of tabletop = 4 height of base = 2	■	8	■
3		length of tabletop = 4 height of base = 3	■	10	■
4	■	■	■	■	■
5	■	■	■	■	■
n		■	■		■

Try This

1. Explain why you must multiply the height of the base by 2.
2. What does the ordered pair (1, 6) mean?
3. Does the ordered pair (10, 24) belong in this pattern? Why or why not?
4. Which expression from the table describes how you would find the total number of blocks for any term number n ?
5. Use your rule to find the 25th term in this pattern.

4-3

Writing Functions



Objectives

Identify independent and dependent variables.

Write an equation in function notation and evaluate a function for given input values.

Vocabulary

independent variable
dependent variable
function rule
function notation

Why learn this?

You can use a function rule to calculate how much money you will earn for working specific amounts of time.

Suppose Tasha baby-sits and charges \$5 per hour.

Time Worked (h) x	1	2	3	4
Amount Earned (\$) y	5	10	15	20

The amount of money Tasha earns is \$5 times the number of hours she works. Write an equation using two different variables to show this relationship.

Amount earned is \$5 times the number of hours worked.

$$y = 5 \cdot x$$

Tasha can use this equation to find how much money she will earn for any number of hours she works.

EXAMPLE 1 Using a Table to Write an Equation

Determine a relationship between the x - and y -values. Write an equation.

x	1	2	3	4
y	-2	-1	0	1

Step 1 List possible relationships between the first x - and y -values.

$$1 - 3 = -2 \text{ or } 1(-2) = -2$$

Step 2 Determine if one relationship works for the remaining values.

$$2 - 3 = -1 \checkmark \quad 2(-2) \neq -1 \times$$

$$3 - 3 = 0 \checkmark \quad 3(-2) \neq 0 \times$$

$$4 - 3 = 1 \checkmark \quad 4(-2) \neq 1 \times$$

The first relationship works. The value of y is 3 less than x .

Step 3 Write an equation.

$$y = x - 3 \quad \text{The value of } y \text{ is 3 less than } x.$$



- Determine a relationship between the x - and y -values in the relation $\{(1, 3), (2, 6), (3, 9), (4, 12)\}$. Write an equation.

The equation in Example 1 describes a function because for each x -value (input), there is only one y -value (output).

The **input** of a function is the **independent variable**. The **output** of a function is the **dependent variable**. The value of the dependent variable *depends* on, or is a function of, the value of the independent variable. For Tasha, the amount she earns depends on, or is a function of, the amount of time she works.

EXAMPLE 2 Identifying Independent and Dependent Variables

Helpful Hint

There are several different ways to describe the variables of a function.

Independent Variable	Dependent Variable
x-values	y-values
Domain	Range
Input	Output
x	$f(x)$

Identify the independent and dependent variables in each situation.

- A** In the winter, more electricity is used when the temperature goes down, and less is used when the temperature rises.

The **amount of electricity** used *depends on* the **temperature**.

Dependent: **amount of electricity** Independent: **temperature**

- B** The cost of shipping a package is based on its weight.

The **cost** of shipping a package *depends on* its **weight**.

Dependent: **cost** Independent: **weight**

- C** The faster Ron walks, the quicker he gets home.

The **time** it takes Ron to get home *depends on* the **speed** he walks.

Dependent: **time** Independent: **speed**



Identify the independent and dependent variables in each situation.

2a. A company charges \$10 per hour to rent a jackhammer.

2b. Apples cost \$0.99 per pound.

An algebraic expression that defines a function is a **function rule**. $5 \cdot x$ in the equation about Tasha's earnings is a function rule.

If x is the independent variable and y is the dependent variable, then **function notation** for y is $f(x)$, read “ f of x ,” where f names the function. When an equation in two variables describes a function, you can use function notation to write it.

The dependent variable **is a function of the independent variable**.

y is a function of x .

$y = f(x)$

Since $y = f(x)$, Tasha's earnings, $y = 5x$, can be rewritten in function notation by substituting $f(x)$ for y : $f(x) = 5x$. Sometimes functions are written using y , and sometimes functions are written using $f(x)$.

EXAMPLE 3 Writing Functions

Identify the independent and dependent variables. Write an equation in function notation for each situation.

- A** A lawyer's fee is \$200 per hour for her services.

The **fee** for the lawyer depends on how many **hours** she works.

Dependent: **fee** Independent: **hours**

Let h represent the number of hours the lawyer works.

The function for the lawyer's fee is $f(h) = 200h$.

Identify the independent and dependent variables. Write an equation in function notation for each situation.

- B** The admission fee to a local carnival is \$8. Each ride costs \$1.50. The **total cost** depends on **the number of rides** ridden, plus \$8.
 Dependent: **total cost** Independent: **number of rides**
 Let r represent the number of rides ridden.
 The function for the total cost of the carnival is $f(r) = 1.50r + 8$.

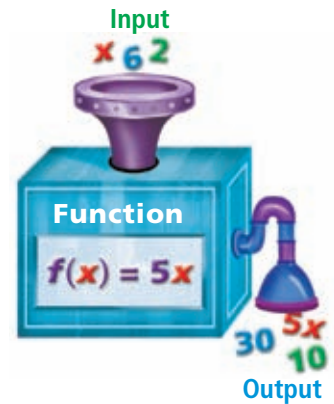


Identify the independent and dependent variables. Write an equation in function notation for each situation.

- 3a.** Steven buys lettuce that costs \$1.69/lb.
3b. An amusement park charges a \$6.00 parking fee plus \$29.99 per person.

You can think of a function as an **input-output** machine. For Tasha's earnings, $f(x) = 5x$, if you input a value x , the output is $5x$.

If Tasha wanted to know how much money she would earn by working 6 hours, she could input 6 for x and find the output. This is called *evaluating the function*.



EXAMPLE 4 Evaluating Functions

Reading Math

Functions can be named with any letter; f , g , and h are the most common. You read $f(6)$ as “ f of 6,” and $g(2)$ as “ g of 2.”

Evaluate each function for the given input values.

- A** For $f(x) = 5x$, find $f(x)$ when $x = 6$ and when $x = 7.5$.
- | | |
|---|---|
| $f(x) = 5x$ | $f(x) = 5x$ |
| $f(6) = 5(6)$ <i>Substitute 6 for x.</i> | $f(7.5) = 5(7.5)$ <i>Substitute 7.5 for x.</i> |
| $= 30$ <i>Simplify.</i> | $= 37.5$ <i>Simplify.</i> |
- B** For $g(t) = 2.30t + 10$, find $g(t)$ when $t = 2$ and when $t = -5$.
- | | |
|-----------------------|-------------------------|
| $g(t) = 2.30t + 10$ | $g(t) = 2.30t + 10$ |
| $g(2) = 2.30(2) + 10$ | $g(-5) = 2.30(-5) + 10$ |
| $= 4.6 + 10$ | $= -11.5 + 10$ |
| $= 14.6$ | $= -1.5$ |
- C** For $h(x) = \frac{1}{2}x - 3$, find $h(x)$ when $x = 12$ and when $x = -8$.
- | | |
|-------------------------------|-------------------------------|
| $h(x) = \frac{1}{2}x - 3$ | $h(x) = \frac{1}{2}x - 3$ |
| $h(12) = \frac{1}{2}(12) - 3$ | $h(-8) = \frac{1}{2}(-8) - 3$ |
| $= 6 - 3$ | $= -4 - 3$ |
| $= 3$ | $= -7$ |



Evaluate each function for the given input values.

- 4a.** For $h(c) = 2c - 1$, find $h(c)$ when $c = 1$ and $c = -3$.
4b. For $g(t) = \frac{1}{4}t + 1$, find $g(t)$ when $t = -24$ and $t = 400$.

When a function describes a real-world situation, every real number is not always reasonable for the domain and range. For example, a number representing the length of an object cannot be negative, and only whole numbers can represent a number of people.

EXAMPLE 5 Finding the Reasonable Domain and Range of a Function

Manuel has already sold \$20 worth of tickets to the school play. He has 4 tickets left to sell at \$2.50 per ticket. Write a function to describe how much money Manuel can collect from selling tickets. Find the reasonable domain and range for the function.

Money collected from ticket sales is \$2.50 per ticket plus the \$20 already sold.

$$f(x) = 2.50 \cdot x + 20$$

If he sells x more tickets, he will have collected $f(x) = 2.50x + 20$ dollars.

Manuel has only 4 tickets left to sell, so he could sell 0, 1, 2, 3, or 4 tickets. A reasonable domain is $\{0, 1, 2, 3, 4\}$.

Substitute these values into the function rule to find the range values.

x	0	1	2	3	4
$f(x)$	$2.50(0) + 20$ $= 20$	$2.50(1) + 20$ $= 22.50$	$2.50(2) + 20$ $= 25$	$2.50(3) + 20$ $= 27.50$	$2.50(4) + 20$ $= 30$

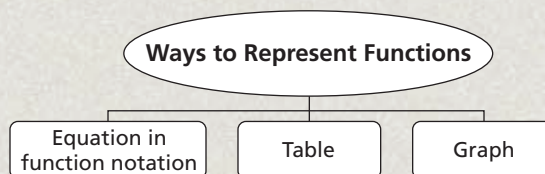
The reasonable range for this situation is $\{\$20, \$22.50, \$25, \$27.50, \$30\}$.



5. The settings on a space heater are the whole numbers from 0 to 3. The total number of watts used for each setting is 500 times the setting number. Write a function to describe the number of watts used for each setting. Find the reasonable domain and range for the function.

THINK AND DISCUSS

- When you input water into an ice machine, the output is ice cubes. Name another real-world object that has an input and an output.
- How do you identify the independent and dependent variables in a situation?
- Explain how to find reasonable domain values for a function.
- GET ORGANIZED** Copy and complete the graphic organizer. Use the function $y = x + 3$ and the domain $\{-2, -1, 0, 1, 2\}$.



GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

- The output of a function is the ___?___ variable. (*independent* or *dependent*)
- An algebraic expression that defines a function is a ___?___. (*function rule* or *function notation*)

SEE EXAMPLE 1

p. 249

Determine a relationship between the x - and y -values. Write an equation.

x	1	2	3	4
y	-1	0	1	2

3.

x	1	2	3	4
y	-1	0	1	2
4. $\{(1, 4), (2, 7), (3, 10), (4, 13)\}$

SEE EXAMPLE 2

p. 250

Identify the independent and dependent variables in each situation.

- A small-size bottle of water costs \$1.99 and a large-size bottle of water costs \$3.49.
- An employee receives 2 vacation days for every month worked.

SEE EXAMPLE 3

p. 250

Identify the independent and dependent variables. Write an equation in function notation for each situation.

- An air-conditioning technician charges customers \$75 per hour.
- An ice rink charges \$3.50 for skates and \$1.25 per hour.

SEE EXAMPLE 4

p. 251

Evaluate each function for the given input values.

- For $f(x) = 7x + 2$, find $f(x)$ when $x = 0$ and when $x = 1$.
- For $g(x) = 4x - 9$, find $g(x)$ when $x = 3$ and when $x = 5$.
- For $h(t) = \frac{1}{3}t - 10$, find $h(t)$ when $t = 27$ and when $t = -15$.

SEE EXAMPLE 5

p. 252

- A construction company uses beams that are 2, 3, or 4 meters long. The measure of each beam must be converted to centimeters. Write a function to describe the situation. Find the reasonable domain and range for the function. (*Hint: 1 m = 100 cm*)

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
13–14	1
15–16	2
17–19	3
20–22	4
23	5

Extra Practice

Skills Practice p. S10

Application Practice p. S31

Determine a relationship between the x - and y -values. Write an equation.

13.

x	1	2	3	4
y	-2	-4	-6	-8

14. $\{(1, -1), (2, -2), (3, -3), (4, -4)\}$

Identify the independent and dependent variables in each situation.

- Gardeners buy fertilizer according to the size of a lawn.
- The cost to gift wrap an order is \$3 plus \$1 per item wrapped.

Identify the independent and dependent variables. Write an equation in function notation for each situation.

- To rent a DVD, a customer must pay \$3.99 plus \$0.99 for every day that it is late.
- Stephen charges \$25 for each lawn he mows.
- A car can travel 28 miles per gallon of gas.

Evaluate each function for the given input values.

20. For $f(x) = x^2 - 5$, find $f(x)$ when $x = 0$ and when $x = 3$.

21. For $g(x) = x^2 + 6$, find $g(x)$ when $x = 1$ and when $x = 2$.

22. For $f(x) = \frac{2}{3}x + 3$, find $f(x)$ when $x = 9$ and when $x = -3$.

23. A mail-order company charges \$5 per order plus \$2 per item in the order, up to a maximum of 4 items. Write a function to describe the situation. Find the reasonable domain and range for the function.

24. **Transportation** Air Force One can travel 630 miles per hour. Let h be the number of hours traveled. The function $d = 630h$ gives the distance d in miles that Air Force One travels in h hours.

- Identify the independent and dependent variables. Write $d = 630h$ using function notation.
- What are reasonable values for the domain and range in the situation described?
- How far can Air Force One travel in 12 hours?

25. Complete the table for $g(z) = 2z - 5$. 26. Complete the table for $h(x) = x^2 + x$.

z	1	2	3	4
$g(z)$	■	■	■	■

x	0	1	2	3
$h(x)$	■	■	■	■

27. **Estimation** For $f(x) = 3x + 5$, estimate the output when $x = -6.89$, $x = 1.01$, and $x = 4.67$.

28. **Transportation** A car can travel 30 miles on a gallon of gas and has a 20-gallon gas tank. Let g be the number of gallons of gas the car has in its tank. The function $d = 30g$ gives the distance d in miles that the car travels on g gallons.

- What are reasonable values for the domain and range in the situation described?
- How far can the car travel on 12 gallons of gas?

29. **Critical Thinking** Give an example of a real-life situation for which the reasonable domain consists of 1, 2, 3, and 4 and the reasonable range consists of 2, 4, 6, and 8.

30. **ERROR ANALYSIS** Rashid saves \$150 each month. He wants to know how much he will have saved in 2 years. He writes the rule $s = m + 150$ to help him figure out how much he will save, where s is the amount saved and m is the number of months he saves. Explain why his rule is incorrect.



31. **Write About It** Give a real-life situation that can be described by a function. Identify the independent variable and the dependent variable.

32. This problem will prepare you for the Multi-Step Test Prep on page 264.

The table shows the volume v of water pumped into a pool after t hours.

- Determine a relationship between the time and the volume of water and write an equation.
- Identify the independent and dependent variables.
- If the pool holds 10,000 gallons, how long will it take to fill?

Amount of Water in Pool	
Time (h)	Volume (gal)
0	0
1	1250
2	2500
3	3750
4	5000



Transportation



Air Force One refers to two specially configured Boeing 747-200B airplanes. The radio call sign when the president is aboard either aircraft or any Air Force aircraft is "Air Force One."

MULTI-STEP TEST PREP



33. Marsha buys x pens at \$0.70 per pen and one pencil for \$0.10. Which function gives the total amount Marsha spends?

- (A) $c(x) = 0.70x + 0.10x$ (C) $c(x) = (0.70 + 0.10)x$
 (B) $c(x) = 0.70x + 1$ (D) $c(x) = 0.70x + 0.10$

34. Belle is buying pizzas for her daughter's birthday party, using the prices in the table. Which equation best describes the relationship between the total cost c and the number of pizzas p ?

Pizzas	Total Cost (\$)
5	26.25
10	52.50
15	78.75

- (F) $c = 26.25p$ (H) $c = p + 26.25$
 (G) $c = 5.25p$ (J) $c = 6p - 3.75$

35. **Gridded Response** What is the value of $f(x) = 5 - \frac{1}{2}x$ when $x = 3$?

CHALLENGE AND EXTEND

36. The formula to convert a temperature that is in degrees Celsius x to degrees Fahrenheit $f(x)$ is $f(x) = \frac{9}{5}x + 32$. What are reasonable values for the domain and range when you convert to Fahrenheit the temperature of water as it rises from 0° to 100° Celsius?

37. **Math History** In his studies of the motion of free-falling objects, Galileo Galilei found that regardless of its mass, an object will fall a distance d that is related to the square of its travel time t in seconds. The modern formula that describes free-fall motion is $d = \frac{1}{2}gt^2$, where g is the acceleration due to gravity and t is the length of time in seconds the object falls. Find the distance an object falls in 3 seconds. (*Hint:* Research to find acceleration due to gravity in meters per second squared.)

SPIRAL REVIEW

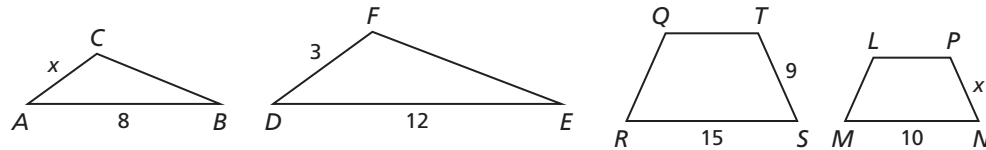
Solve each equation. Check your answer. (Lesson 2-3)

38. $5x + 2 - 7x = -10$ 39. $3(2 - y) = 15$ 40. $\frac{2}{3}p - \frac{1}{2} = \frac{1}{6}$

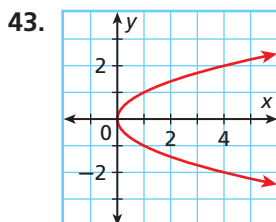
Find the value of x in each diagram. (Lesson 2-8)

41. $\triangle ABC \sim \triangle DEF$

42. $QRST \sim LMNP$



Give the domain and range of each relation. Tell whether the relation is a function. Explain. (Lesson 4-2)



44.

x	y
-3	4
-1	2
0	0
1	2
3	-4

4-4

Graphing Functions

Objectives

Graph functions given a limited domain.

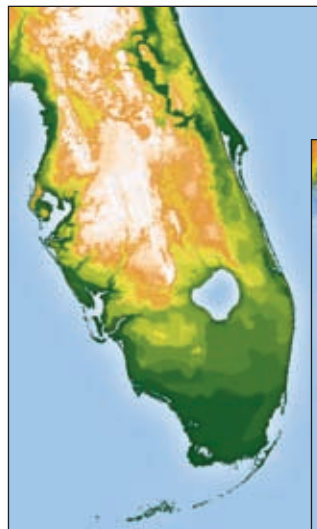
Graph functions given a domain of all real numbers.

Who uses this?

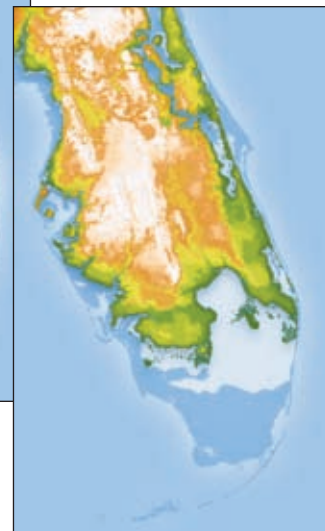
Scientists can use a function to make conclusions about rising sea level.

Sea level is rising at an approximate rate of 2.5 millimeters per year. If this rate continues, the function $y = 2.5x$ can describe how many millimeters y sea level will rise in the next x years.

One way to understand functions such as the one above is to graph them. You can graph a function by finding ordered pairs that satisfy the function.



Current Florida coastline.



Possible Florida coastline in 2400 years.

EXAMPLE 1 Graphing Solutions Given a Domain

Graph each function for the given domain.

A $-x + 2y = 6$; D: $\{-4, -2, 0, 2\}$

Step 1 Solve for y since you are given values of the domain, or x .

$$-x + 2y = 6$$

$$\begin{array}{r} +x \\ \hline 2y = x + 6 \end{array}$$

Add x to both sides.

$$\frac{2y}{2} = \frac{x + 6}{2}$$

Since y is multiplied by 2, divide both sides by 2.

$$y = \frac{x}{2} + \frac{6}{2}$$

Rewrite $\frac{x+6}{2}$ as two separate fractions.

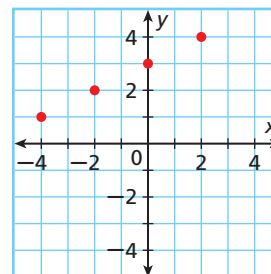
$$y = \frac{1}{2}x + 3$$

Simplify.

Step 2 Substitute the given values of the domain for x and find values of y .

Step 3 Graph the ordered pairs.

x	$y = \frac{1}{2}x + 3$	(x, y)
-4	$y = \frac{1}{2}(-4) + 3 = 1$	$(-4, 1)$
-2	$y = \frac{1}{2}(-2) + 3 = 2$	$(-2, 2)$
0	$y = \frac{1}{2}(0) + 3 = 3$	$(0, 3)$
2	$y = \frac{1}{2}(2) + 3 = 4$	$(2, 4)$



Helpful Hint

Sometimes solving for y first makes it easier to substitute values of x and find an ordered pair.

To review solving for a variable, see Lesson 2-5.

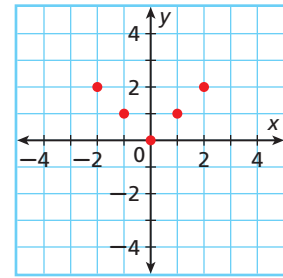
Graph each function for the given domain.

B $f(x) = |x|$; D: $\{-2, -1, 0, 1, 2\}$

Step 1 Use the given values of the **domain** to find values of $f(x)$.

x	$f(x) = x $	$(x, f(x))$
-2	$f(x) = -2 = 2$	$(-2, 2)$
-1	$f(x) = -1 = 1$	$(-1, 1)$
0	$f(x) = 0 = 0$	$(0, 0)$
1	$f(x) = 1 = 1$	$(1, 1)$
2	$f(x) = 2 = 2$	$(2, 2)$

Step 2 Graph the ordered pairs.



Graph each function for the given domain.

1a. $-2x + y = 3$; D: $\{-5, -3, 1, 4\}$

1b. $f(x) = x^2 + 2$; D: $\{-3, -1, 0, 1, 3\}$

If the domain of a function is all real numbers, any number can be used as an input value. This process will produce an infinite number of ordered pairs that satisfy the function. Therefore, arrowheads are drawn at both “ends” of a smooth line or curve to represent the infinite number of ordered pairs. If a domain is not given, assume that the domain is all real numbers.



Graphing Functions Using a Domain of All Real Numbers

Step 1	Use the function to generate ordered pairs by choosing several values for x .
Step 2	Plot enough points to see a pattern for the graph.
Step 3	Connect the points with a line or smooth curve.

EXAMPLE 2 Graphing Functions

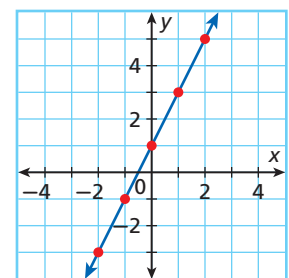
Graph each function.

A $2x + 1 = y$

Step 1 **Choose several values of x** and generate ordered pairs.

x	$2x + 1 = y$	(x, y)
-3	$2(-3) + 1 = -5$	$(-3, -5)$
-2	$2(-2) + 1 = -3$	$(-2, -3)$
-1	$2(-1) + 1 = -1$	$(-1, -1)$
0	$2(0) + 1 = 1$	$(0, 1)$
1	$2(1) + 1 = 3$	$(1, 3)$
2	$2(2) + 1 = 5$	$(2, 5)$
3	$2(3) + 1 = 7$	$(3, 7)$

Step 2 **Plot enough points** to see a pattern.



Helpful Hint

When choosing values of x , be sure to choose both positive and negative values. You may not need to graph all the points to see the pattern.

Step 3 The ordered pairs appear to form a line. **Draw a line** through all the points to show all the ordered pairs that satisfy the function. Draw arrowheads on both “ends” of the line.

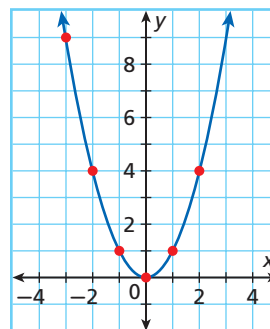
Graph each function.

B $y = x^2$

Step 1 Choose several values of x and generate ordered pairs.

x	$y = x^2$	(x, y)
-3	$y = (-3)^2 = 9$	$(-3, 9)$
-2	$y = (-2)^2 = 4$	$(-2, 4)$
-1	$y = (-1)^2 = 1$	$(-1, 1)$
0	$y = (0)^2 = 0$	$(0, 0)$
1	$y = (1)^2 = 1$	$(1, 1)$
2	$y = (2)^2 = 4$	$(2, 4)$

Step 2 Plot enough points to see a pattern.



Step 3 The ordered pairs appear to form an almost U-shaped graph. Draw a smooth curve through the points to show all the ordered pairs that satisfy the function. Draw arrowheads on the “ends” of the curve.

Check If the graph is correct, any point on it will satisfy the function. Choose an ordered pair on the graph that was not in your table, such as $(3, 9)$. Check whether it satisfies $y = x^2$.

$$\begin{array}{r|l} y = x^2 & \\ 9 & 3^2 \\ 9 & 9 \checkmark \end{array}$$

Substitute the values for x and y into the function. Simplify.
The ordered pair $(3, 9)$ satisfies the function.



Graph each function.

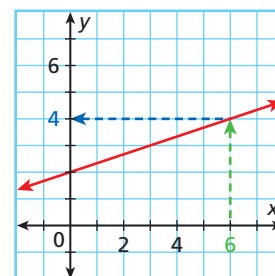
2a. $f(x) = 3x - 2$

2b. $y = |x - 1|$

EXAMPLE 3 Finding Values Using Graphs

Use a graph of the function $f(x) = \frac{1}{3}x + 2$ to find the value of $f(x)$ when $x = 6$. Check your answer.

Locate 6 on the x -axis. Move **up** to the graph of the function. Then move **left** to the y -axis to find the corresponding value of y .



$f(x) = 4$

Check Use substitution.

$$\begin{array}{r|l} f(x) = \frac{1}{3}x + 2 & \\ 4 & \frac{1}{3}(6) + 2 \\ 4 & 2 + 2 \\ 4 & 4 \checkmark \end{array}$$

Substitute the values for x and y into the function.
Simplify.
The ordered pair $(4, 6)$ satisfies the function.



3. Use the graph above to find the value of x when $f(x) = 3$. Check your answer.

Writing Math

“The value of y is 4 when $x = 6$ ” can also be written as $f(6) = 4$.

Recall that in real-world situations you may have to limit the domain to make answers reasonable. For example, quantities such as time, distance, and number of people can be represented using only nonnegative values. When both the domain and the range are limited to nonnegative values, the function is graphed only in Quadrant I.

EXAMPLE 4 Problem-Solving Application



The function $y = 2.5x$ describes how many millimeters sea level y rises in x years. Graph the function. Use the graph to estimate how many millimeters sea level will rise in 3.5 years.

1 Understand the Problem

The **answer** is a graph that can be used to find the value of y when x is 3.5.

List the important information:

- The function $y = 2.5x$ describes how many millimeters sea level rises.

2 Make a Plan

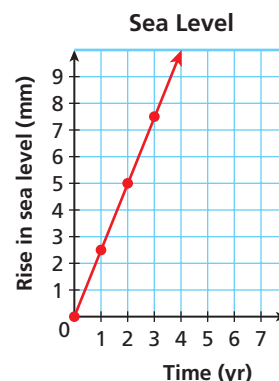
Think: What values should I use to graph this function? Both, the number of years sea level has risen and the distance sea level rises, cannot be negative. Use only nonnegative values for both the domain and the range. The function will be graphed in Quadrant I.

3 Solve

Choose several nonnegative values of x to find values of y . Then graph the ordered pairs.

x	$y = 2.5x$	(x, y)
0	$y = 2.5(0) = 0$	$(0, 0)$
1	$y = 2.5(1) = 2.5$	$(1, 2.5)$
2	$y = 2.5(2) = 5$	$(2, 5)$
3	$y = 2.5(3) = 7.5$	$(3, 7.5)$
4	$y = 2.5(4) = 10$	$(4, 10)$

Draw a line through the points to show all the ordered pairs that satisfy this function.



Use the graph to estimate the y -value when x is 3.5. Sea level will rise about 8.75 millimeters in 3.5 years.

4 Look Back

As the number of years increases, sea level also increases, so the graph is reasonable. When x is between 3 and 4, y is between 7.5 and 10. Since 3.5 is between 3 and 4, it is reasonable to estimate y to be 8.75 when x is 3.5.



- The fastest recorded Hawaiian lava flow moved at an average speed of 6 miles per hour. The function $y = 6x$ describes the distance y the lava moved on average in x hours. Graph the function. Use the graph to estimate how many miles the lava moved after 5.5 hours.

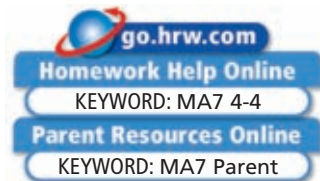
THINK AND DISCUSS

- How do you find the range of a function if the domain is all real numbers?
- Explain how to use a graph to find the value of a function for a given value of x .
- GET ORGANIZED** Copy and complete the graphic organizer. Explain how to graph a function for each situation.



4-4

Exercises



GUIDED PRACTICE

SEE EXAMPLE 1 Graph each function for the given domain.

p. 256

- $3x - y = 1$; D: $\{-3, -1, 0, 4\}$
- $f(x) = -|x|$; D: $\{-5, -3, 0, 3, 5\}$
- $f(x) = x + 4$; D: $\{-5, -3, 0, 4\}$
- $y = x^2 - 1$; D: $\{-3, -1, 0, 1, 3\}$

SEE EXAMPLE 2 Graph each function.

p. 257

- $f(x) = 6x + 4$
- $y = \frac{1}{2}x + 4$
- $x + y = 0$
- $y = |x| - 4$
- $f(x) = 2x^2 - 7$
- $y = -x^2 + 5$

SEE EXAMPLE 3 Use a graph of the function $f(x) = \frac{1}{2}x - 2$ to find the value of y when $x = 2$. Check your answer.

p. 258

SEE EXAMPLE 4 **Oceanography** The floor of the Atlantic Ocean is spreading at an average rate of 1 inch per year. The function $y = x$ describes the number of inches y the ocean floor spreads in x years. Graph the function. Use the graph to estimate the number of inches the ocean floor will spread in $10\frac{1}{2}$ years.

p. 259

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
13–16	1
17–24	2
25–26	3
27	4

Graph each function for the given domain.

- $2x + y = 4$; D: $\{-3, -1, 4, 7\}$
- $y = |x| - 1$; D: $\{-4, -2, 0, 2, 4\}$
- $f(x) = -7x$; D: $\{-2, -1, 0, 1\}$
- $y = (x + 1)^2$; D: $\{-2, -1, 0, 1, 2\}$

Graph each function.

- $y = -3x + 5$
- $f(x) = 3x$
- $x + y = 8$
- $f(x) = 2x + 2$
- $y = -|x| + 10$
- $f(x) = -5 + x^2$
- $y = |x + 1| + 1$
- $y = (x - 2)^2 - 1$

Extra Practice

Skills Practice p. S12
Application Practice p. S31

- Use a graph of the function $f(x) = -2x - 3$ to find the value of y when $x = -4$. Check your answer.
- Use a graph of the function $f(x) = \frac{1}{3}x + 1$ to find the value of y when $x = 6$. Check your answer.

27. **Transportation** An electric motor scooter can travel at 0.25 miles per minute. The function $y = 0.25x$ describes the number of miles y the scooter can travel in x minutes. Graph the function. Use the graph to estimate the number of miles an electric motor scooter travels in 15 minutes.

Graph each function.

28. $f(x) = x - 1$ 29. $12 - x - 2y = 0$ 30. $3x - y = 13$
 31. $y = x^2 - 2$ 32. $x^2 - y = -4$ 33. $2x^2 = f(x)$
 34. $f(x) = |2x| - 2$ 35. $y = |-x|$ 36. $-|2x + 1| = y$

37. Find the value of x so that $(x, 12)$ satisfies $y = 4x + 8$.

38. Find the value of x so that $(x, 6)$ satisfies $y = -x - 4$.

39. Find the value of y so that $(-2, y)$ satisfies $y = -2x^2$.

For each function, determine whether the given points are on the graph.

40. $y = 7x - 2$; $(1, 5)$ and $(2, 10)$ 41. $y = |x| + 2$; $(3, 5)$ and $(-1, 3)$

42. $y = x^2$; $(1, 1)$ and $(-3, -9)$ 43. $y = \frac{1}{4}x - 2$; $(1, -\frac{3}{4})$ and $(4, -1)$

44. **ERROR ANALYSIS** Student A says that $(3, 2)$ is on the graph of $y = 4x - 5$, but student B says that it is not. Who is incorrect? Explain the error.

A

$y = 4x - 5$	
3	$4(2) - 5$
3	$8 - 5$
3	3 ✓

B

$y = 4x - 5$	
2	$4(3) - 5$
2	$12 - 5$
2	7 ✗

Determine whether $(0, -7)$, $(-6, -\frac{5}{3})$, and $(-2, -3)$ lie on the graph of each function.

45. $x + 3y = -11$ 46. $y + |x| = -1$ 47. $x^2 - y = 7$

For each function, find three ordered pairs that lie on the graph of the function.

48. $-6 = 3x + 2y$ 49. $y = 1.1x + 2$

50. $y = \frac{4}{5}x$ 51. $y = 3x - 1$

52. $y = |x| + 6$ 53. $y = x^2 - 5$

54. **Critical Thinking** Graph the functions $y = |x|$ and $y = -|x|$. Describe how they are alike. How are they different?

**MULTI-STEP
TEST PREP**

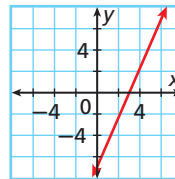


55. This problem will prepare you for the Multi-Step Test Prep on page 264.

A pool containing 10,000 gallons of water is being drained. Every hour, the volume of the water in the pool decreases by 1500 gallons.

- Write an equation to describe the volume v of water in the pool after h hours.
- How much water is in the pool after 1 hour?
- Create a table of values showing the volume of the water in gallons in the pool as a function of the time in hours and graph the function.

56. **Estimation** Use the graph to estimate the value of y when $x = 2.117$.

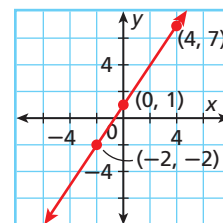


57. **Write About It** Why is a graph a convenient way to show the ordered pairs that satisfy a function?



58. Which function is graphed?

- (A) $2y - 3x = 2$ (C) $y = 2x - 1$
 (B) $5x + y = 1$ (D) $y = 5x + 8$



59. Which ordered pair is NOT on the graph of $y = 4 - |x|$?

- (F) $(0, 4)$ (H) $(-1, 3)$
 (G) $(4, 0)$ (J) $(3, -1)$

60. Which function has $(3, 2)$ on its graph?

- (A) $2x - 3y = 12$ (C) $y = -\frac{2}{3}x + 4$
 (B) $-2x - 3y = 12$ (D) $y = -\frac{3}{2}x + 4$

61. Which statement(s) is true about the function $y = x^2 + 1$?

- I. All points on the graph are above the origin.
 II. All ordered pairs have positive x -values.
 III. All ordered pairs have positive y -values.

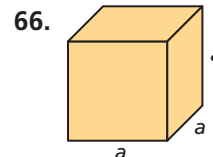
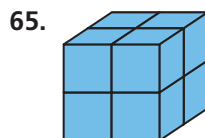
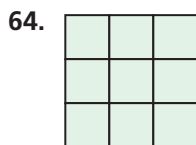
- (F) I Only (G) II Only (H) I and II (J) I and III

CHALLENGE AND EXTEND

62. Graph the function $y = x^3$. Make sure you have enough ordered pairs to see the shape of the graph.
63. The temperature of a liquid that started at 64°F is increasing by 4°F per hour. Write a function that describes the temperature of the liquid over time. Graph the function to show the temperatures over the first 10 hours.

SPIRAL REVIEW

Write the power represented by each geometric model. (Lesson 1-4)



Solve each inequality and graph the solutions. (Lesson 3-3)

67. $5p < -20$

68. $18 > -9k$

69. $\frac{3}{4}b \geq 15$

Evaluate each function for the given input values. (Lesson 4-3)

70. For $f(x) = -2x - 3$, find $f(x)$ when $x = -4$ and when $x = 2$.

71. For $h(t) = \frac{2}{3}t + 1$, find $h(t)$ when $t = -6$ and when $t = 9$.



Connect Function Rules, Tables, and Graphs

You can use a graphing calculator to understand the connections among function rules, tables, and graphs.

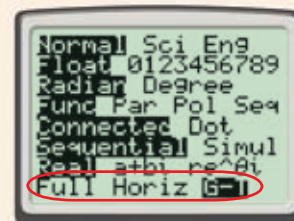
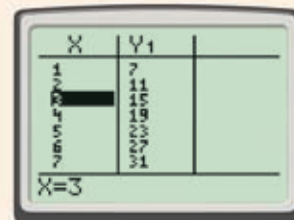
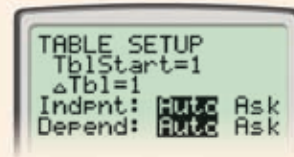
Use with Lesson 4-4



Activity

Make a table of values for the function $f(x) = 4x + 3$. Then graph the function.

- Press **Y=** and enter the function rule $4x + 3$.
- Press **2nd** **TBLSET** **WINDOW**. Make sure **Indpnt: Auto** and **Depend: Auto** are selected.
- To view the table, press **2nd** **TABLE** **GRAPH**. The x -values and the corresponding y -values appear in table form. Use the up and down arrow keys to scroll through the table.
- To view the table with the graph, press **MODE** and select **G-T** view. Press **ENTER**. Be sure to use the standard window.
- Press **TRACE** to see both the graph and a table of values.
- Press the left arrow key several times to move the cursor. Notice that the point on the graph and the values in the table correspond.



Try This

Make a table of values for each function. Then graph the function.

- $f(x) = 2x - 1$
- $f(x) = 1.5x$
- $f(x) = \frac{1}{2}x + 2$
- Explain the relationship between a function, its table of values, and the graph of the function.

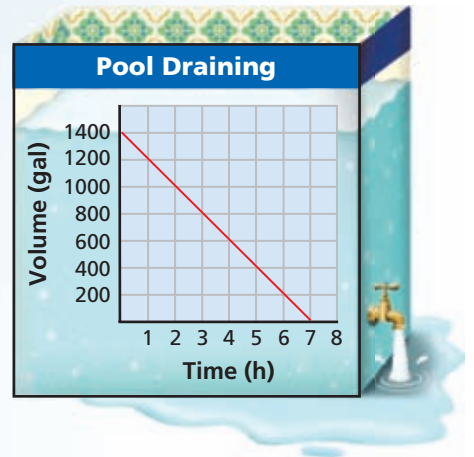
MULTI-STEP TEST PREP



Function Concepts

Down the Drain The graph shows the relationship between the number of hours that have passed since a pool began to drain and the amount of water in the pool.

- Describe in words the relationship between the amount of water in the pool and the number of hours that have passed since the pool began to drain.
- What are the domain and range for the graph?
- Use the graph to determine how much water is in the pool after 3 hours. How much water is in the pool after $4\frac{1}{2}$ hours?
- Copy and complete the table.



Draining Pool	
Time (h)	Volume (gal)
0	1400
1	■
2	■
3	■
4	■
5	■
6	■
7	■

- Write an equation to describe the relationship between the volume v and the time t . Use the equation to find how much water is in the pool after 5.2 hours.



4-5

Scatter Plots and Trend Lines



Objectives

Create and interpret scatter plots.

Use trend lines to make predictions.

Vocabulary

scatter plot
correlation
positive correlation
negative correlation
no correlation
trend line

Who uses this?

Ecologists can use scatter plots to help them analyze data about endangered species, such as ocelots. (See Example 1.)

In this chapter, you have examined relationships between sets of ordered pairs, or data. Displaying data visually can help you see relationships.

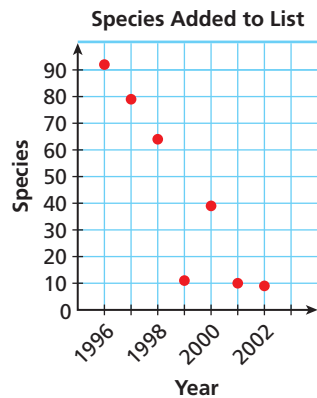
A **scatter plot** is a graph with points plotted to show a possible relationship between two sets of data. A scatter plot is an effective way to display some types of data.

EXAMPLE 1 Graphing a Scatter Plot from Given Data

The table shows the number of species added to the list of endangered and threatened species in the United States during the given years. Graph a scatter plot using the given data.

Increase in List							
Calendar Year	1996	1997	1998	1999	2000	2001	2002
Species	91	79	62	11	39	10	9

Source: U.S. Fish and Wildlife Service



Use the table to make ordered pairs for the scatter plot.

The x-value represents the calendar year and the y-value represents the number of species added.

Plot the ordered pairs.

Helpful Hint

The point (2000, 39) tells you that in the year 2000, the list increased by 39 species.



- The table shows the number of points scored by a high school football team in the first four games of a season. Graph a scatter plot using the given data.

Game	1	2	3	4
Score	6	21	46	34

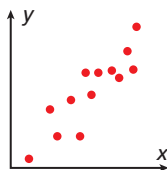
A **correlation** describes a relationship between two data sets. A graph may show the correlation between data. The correlation can help you analyze trends and make predictions. There are three types of correlations between data.



Correlations

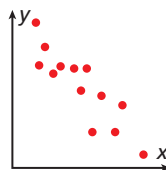
Positive Correlation

Both sets of data values increase.



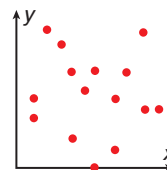
Negative Correlation

One set of data values increases as the other set decreases.



No Correlation

There is no relationship between the data sets.



In the endangered species graph, as time increases, the number of new species added decreases. So the correlation between the data is negative.

EXAMPLE 2 Describing Correlations from Scatter Plots

Describe the correlation illustrated by the scatter plot.

TV Watching and Test Scores

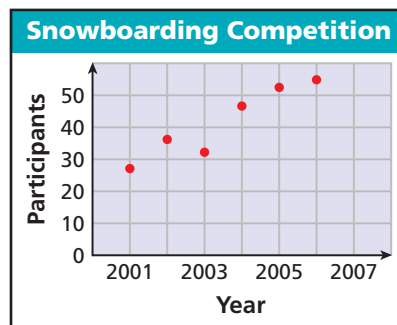


As the number of hours spent watching TV increased, test scores decreased.

There is a negative correlation between the two data sets.



2. Describe the correlation illustrated by the scatter plot.



EXAMPLE 3 Identifying Correlations

Identify the correlation you would expect to see between each pair of data sets. Explain.

- A** the number of empty seats in a classroom and the number of students seated in the class

You would expect to see a negative correlation. As the number of students increases, the number of empty seats decreases.

- B** the number of pets a person owns and the number of books that person read last year

You would expect to see no correlation. The number of pets a person owns has nothing to do with how many books the person has read.

Identify the correlation you would expect to see between each pair of data sets. Explain.

- C** the monthly rainfall and the depth of water in a reservoir
 You would expect to see a positive correlation. As more rain falls, there is more water in the reservoir.

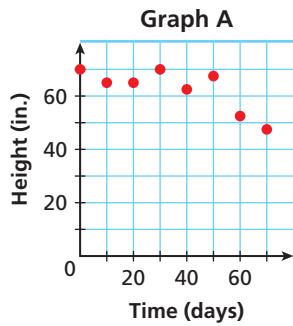


Identify the correlation you would expect to see between each pair of data sets. Explain.

- 3a. the temperature in Houston and the number of cars sold in Boston
 3b. the number of members in a family and the size of the family's grocery bill
 3c. the number of times you sharpen your pencil and the length of your pencil

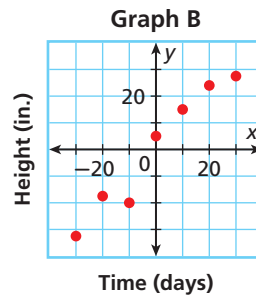
EXAMPLE 4 Matching Scatter Plots to Situations

Choose the scatter plot that best represents the relationship between the number of days since a sunflower seed was planted and the height of the plant. Explain.



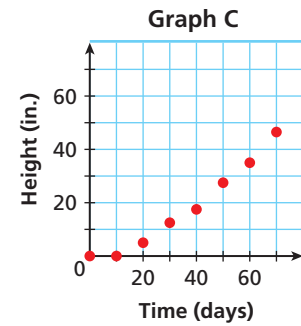
There will be a positive correlation between the number of days and the height because the plant will grow each day.

Graph A has a negative correlation, so it is incorrect.



Neither the number of days nor the plant heights can be negative.

Graph B shows negative values, so it is incorrect.

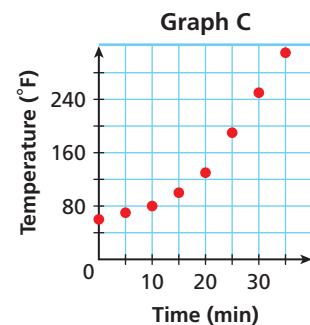
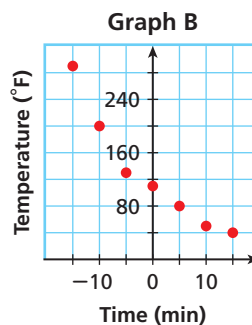
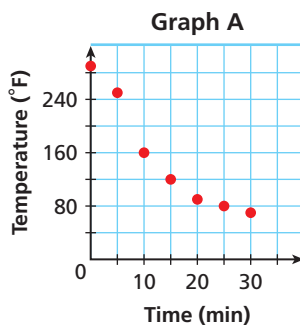


This graph shows all positive coordinates and a positive correlation, so it could represent the data sets.

Graph C is the correct scatter plot.



4. Choose the scatter plot that best represents the relationship between the number of minutes since a pie has been taken out of the oven and the temperature of the oven. Explain.

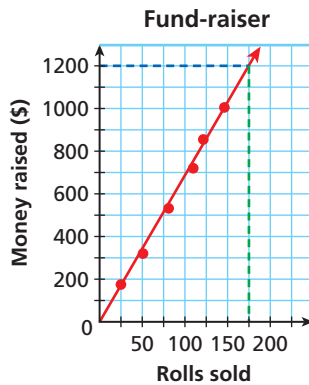
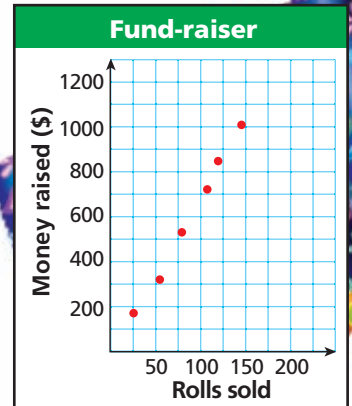


You can graph a line on a scatter plot to help show a relationship in the data. This line, called a **trend line**, helps show the correlation between data sets more clearly. It can also be helpful when making predictions based on the data.

EXAMPLE 5 Fund-raising Application

The scatter plot shows a relationship between the total amount of money collected and the total number of rolls of wrapping paper sold as a school fund-raiser. Based on this relationship, predict how much money will be collected when 175 rolls have been sold.

Draw a trend line and use it to make a prediction.



Draw a line that has about the same number of points above and below it. Your line may or may not go through data points.

Find the point on the line whose x-value is 175. The corresponding y-value is 1200.

Based on the data, \$1200 is a reasonable prediction of how much money will be collected when 175 rolls have been sold.



5. Based on the trend line above, predict how many wrapping paper rolls need to be sold to raise \$500.

THINK AND DISCUSS

- Is it possible to make a prediction based on a scatter plot with no correlation? Explain your answer.
- GET ORGANIZED** Copy and complete the graphic organizer with either a scatter plot, a real-world example, or both.



	Graph	Example
Positive Correlation		
Negative Correlation		The amount of water in a watering can and the number of flowers watered
No Correlation		

GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

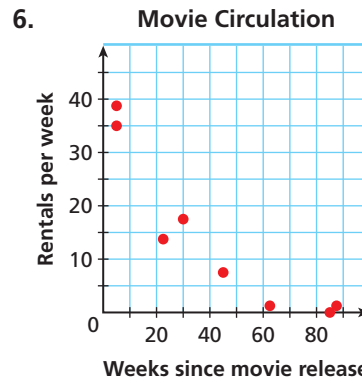
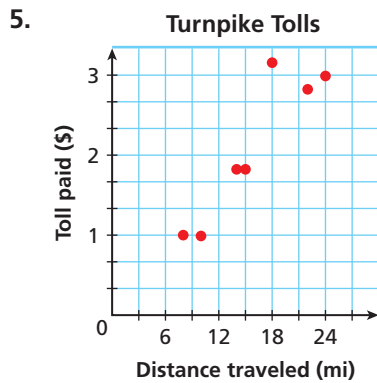
1. Give an example of a graph that is not a *scatter plot*.
2. How is a scatter plot that shows *no correlation* different from a scatter plot that shows a *negative correlation*?
3. Does a *trend line* always pass through every point on a scatter plot? Explain.
4. Graph a scatter plot using the given data.

SEE EXAMPLE 1
 p. 266

Garden Statue	Cupid	Gnome	Lion	Flamingo	Wishing well
Height (in.)	32	18	35	28	40
Price (\$)	50	25	80	15	75

SEE EXAMPLE 2
 p. 267

Describe the correlation illustrated by each scatter plot.



SEE EXAMPLE 3
 p. 267

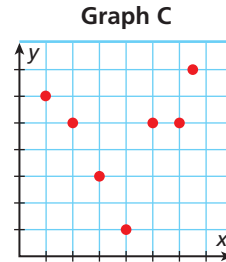
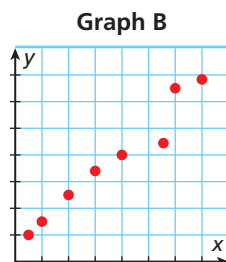
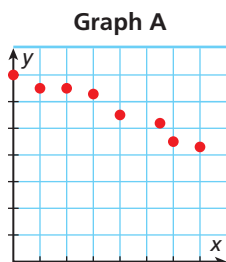
Identify the correlation you would expect to see between each pair of data sets. Explain.

7. the volume of water poured into a container and the amount of empty space left in the container
8. a person's shoe size and the length of the person's hair
9. the outside temperature and the number of people at the beach

SEE EXAMPLE 4
 p. 268

Choose the scatter plot that best represents the described relationship. Explain.

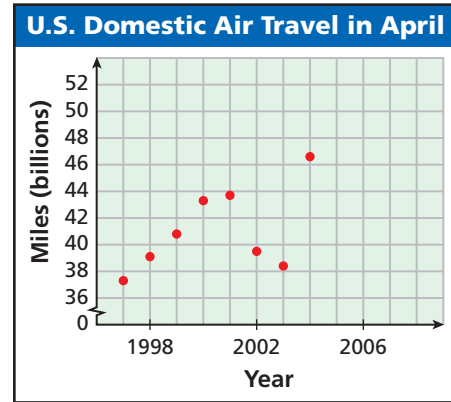
10. age of car and number of miles traveled
11. age of car and sales price of car
12. age of car and number of states traveled to



SEE EXAMPLE 5

p. 270

13. **Transportation** The scatter plot shows the total number of miles passengers flew on U.S. domestic flights in the month of April for the years 1997–2004. Based on this relationship, predict how many miles passengers flew in April 2008.



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14	1
15–16	2
17–18	3
19–20	4
21	5

Extra Practice

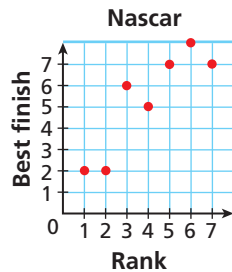
Skills Practice p. S12
Application Practice p. S31

14. Graph a scatter plot using the given data.

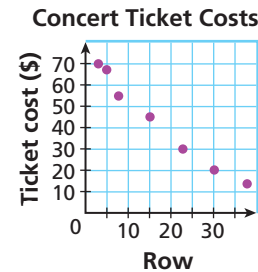
Train Arrival Time	6:45 A.M.	7:30 A.M.	8:15 A.M.	9:45 A.M.	10:30 A.M.
Passengers	160	148	194	152	64

Describe the correlation illustrated by each scatter plot.

- 15.



- 16.

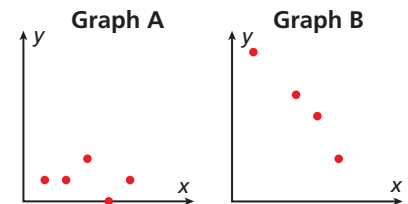


Identify the correlation you would expect to see between each pair of data sets. Explain.

17. the speed of a runner and the distance she can cover in 10 minutes
18. the year a car was made and the total mileage

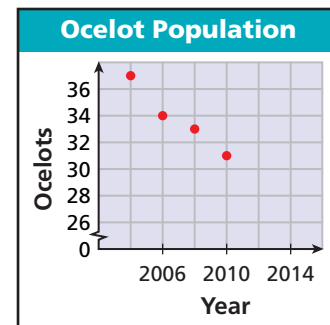
Choose the scatter plot that best represents the described relationship. Explain.

19. the number of college classes taken and the number of roommates
20. the number of college classes taken and the hours of free time.



- 21.

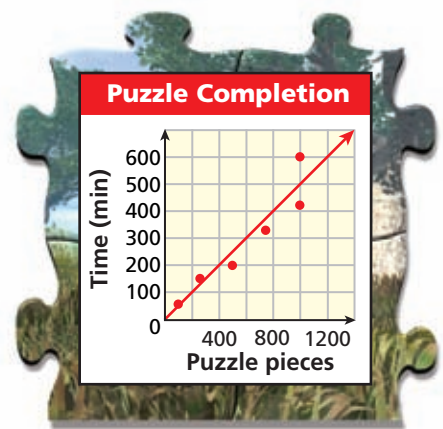
Ecology The scatter plot shows a projection of the average ocelot population living in Laguna Atascosa National Wildlife Refuge near Brownsville, Texas. Based on this relationship, predict the number of ocelots living at the wildlife refuge in 2014 if nothing is done to help manage the ocelot population.



LINK
Ecology

The ocelot population in Texas is dwindling due in part to their habitat being destroyed. The ocelot population at Laguna Atascosa National Wildlife Refuge is monitored by following 5–10 ocelots yearly by radio telemetry.

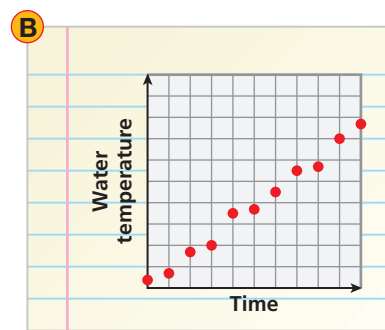
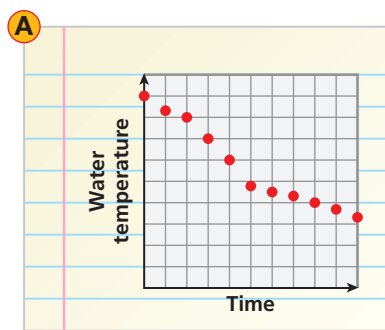
22. **Estimation** Angie enjoys putting jigsaw puzzles together. The scatter plot shows the number of puzzle pieces and the time in minutes it took her to complete each of her last six puzzles. Use the trend line to estimate the time in minutes it will take Angie to complete a 1200-piece puzzle.



23. **Critical Thinking** Describe the correlation between the number of left shoes sold and the number of right shoes sold.
24. Roma had guests for dinner at her house eight times and has recorded the number of guests and the total cost for each meal in the table.

Guests	3	4	4	6	6	7	8	8
Cost (\$)	30	65	88	90	115	160	150	162

- Graph a scatter plot of the data.
 - Describe the correlation.
 - Draw a trend line.
 - Based on the trend line you drew, predict the cost of dinner for 11 guests.
 - What if...?** Suppose that each cost in the table increased by \$5. How will this affect the cost of dinner for 11 guests?
25. **///ERROR ANALYSIS///** Students graphed a scatter plot for the temperature of hot bath water and time if no new water is added. Which graph is incorrect? Explain the error.



26. **Critical Thinking** Will more people or fewer people buy an item if the price goes up? Explain the relationship and describe the correlation.

**MULTI-STEP
TEST PREP**

27. This problem will prepare you for the Multi-Step Test Prep on page 282.

Juan and his parents are visiting a university 205 miles from their home. As they travel, Juan uses the car odometer and his watch to keep track of the distance.

- Make a scatter plot for this data set.
- Describe the correlation. Explain.
- Draw a trend line for the data and predict the distance Juan would have traveled going to a university 4 hours away.

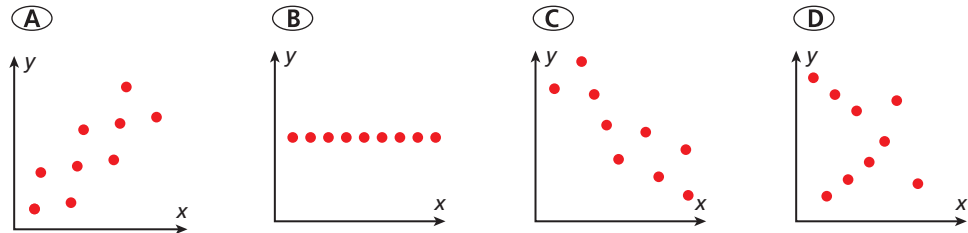
Time (min)	Distance (mi)
0	0
30	28
60	58
90	87
120	117
150	148
180	178
210	205



28. **Write About It** Conduct a survey of your classmates to find the number of siblings they have and the number of pets they have. Predict whether there will be a positive, negative, or no correlation. Then graph the data in a scatter plot. What is the relationship between the two data sets? Was your prediction correct?



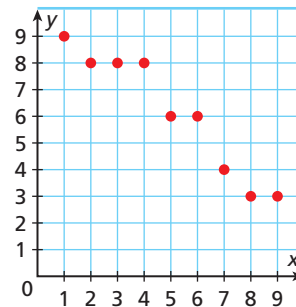
29. Which graph is the best example of a negative correlation?



30. Which situation best describes a positive correlation?

- (F) The amount of rainfall on Fridays
 (G) The height of a candle and the amount of time it stays lit
 (H) The price of a pizza and the number of toppings added
 (J) The temperature of a cup of hot chocolate and the length of time it sits

31. **Short Response** Write a real-world situation for the graph. Explain your answer.



CHALLENGE AND EXTEND

32. Describe a situation that involves a positive correlation. Gather data on the situation. Make a scatter plot showing the correlation. Use the scatter plot to make a prediction. Repeat for a negative correlation and for no correlation.
33. Research an endangered or threatened species in your state. Gather information on its population for several years. Make a scatter plot using the data you gather. Is there a positive or negative correlation? Explain. Draw a trend line and make a prediction about the species population over the next 5 years.

SPIRAL REVIEW

Write an equation to represent each relationship. Then solve the equation. (Lesson 2-4)

34. Five times a number increased by 2 is equal to twice the number decreased by 4.
 35. Five times the sum of a number and 2 is equal to 8 less than twice the number.

Solve each inequality. (Lesson 3-5)

36. $4(6 + x) \geq -2x$ 37. $3(x - 1) > 3x$ 38. $2(3 - x) < 2(1 + x)$

Graph each function. (Lesson 4-4)

39. $y = 2x - 3$ 40. $y = -|x| + 3$ 41. $y = x^2 - 4$



Interpret Scatter Plots and Trend Lines

You can use a graphing calculator to graph a trend line on a scatter plot.

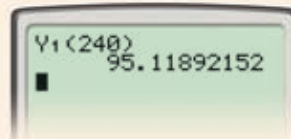
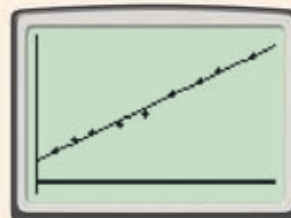
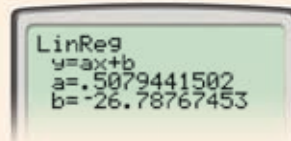
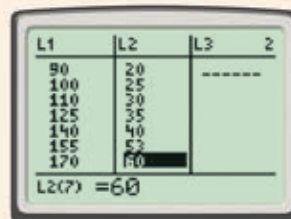
Use with Lesson 4-5

Activity

The table shows the recommended dosage of a particular medicine as related to a person's weight. Graph a scatter plot of the given data. Draw the trend line. Then predict the dosage for a person weighing 240 pounds.

Weight (lb)	90	100	110	125	140	155	170	180	200
Dosage (mg)	20	25	30	35	40	53	60	66	75

- First enter the data. Press **STAT** and select **1: Edit**. In **L1**, enter the first weight. Press **ENTER**. Continue entering all weights. Use **▶** to move to **L2**. Enter the first dosage. Press **ENTER**. Continue entering all dosages.
- To view the scatter plot, press **2nd** **Y=**. Select **Plot 1**. Select **On**, the first plot type, and the plot mark **+**. Press **ZOOM**. Select **9: ZoomStat**. You should see a scatter plot of the data.
- To find the trend line, press **STAT** and select the **CALC** menu. Select **LinReg (ax+b)**. Press **ENTER**. This gives you the values of a and b in the trend line.
- To enter the equation for the trend line, press **Y=**, and then input $.5079441502x - 26.78767453$. Press **GRAPH**.
- Now predict the dosage for a weight of 240 pounds. Press **VAR**. Select **Y-VARS** menu and select **1:Function**. Select **1:Y1**. Enter **(240)**. Press **ENTER**. The dosage is about 95 milligrams.



Try This

- The table shows the price of a stock over an 8-month period. Graph a scatter plot of the given data. Draw the trend line. Then predict what the price of one share of stock will be in the twelfth month.

Month	1	2	3	4	5	6	7	8
Price (\$)	32	35	37	41	46	50	54	59



Median-Fit Line

You have learned about trend lines. Now you will learn about another line of fit called the *median-fit line*.

Example

At a water raft rental shop, a group of up to four people can rent a single raft. The table shows the number of rafts rented to different groups of people one morning. Graph the median-fit line for the data.

People x	1	2	4	5	5	5	7	9	10	11	12	15
Rafts Rented y	1	1	1	3	4	5	4	7	5	3	4	6

- Plot the points on a coordinate plane.
- Divide the data into three sections of equal size. Find the medians of the x -values and the y -values for each section. Plot the three median points with an X.

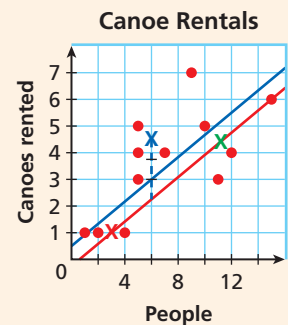
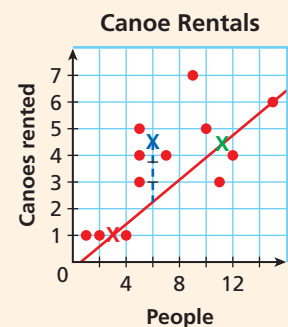
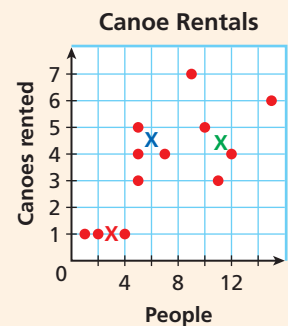
1	2	4	5	5	5	7	9	10	11	12	15
1	1	1	3	4	5	4	7	5	3	4	6

Median point:
(3, 1)

Median point:
(6, 4.5)

Median point:
(11.5, 4.5)

- Connect the outside, or first and third, median points with a line.
- Lightly draw a dashed line straight down from the middle median point to the line just drawn. Mark the dashed line to create three equal segments.
- Keeping your ruler parallel to the first line you drew, move your ruler to the mark closest to the line. Draw the line. This is the median-fit line.



Try This

- A manager at a restaurant kept track one afternoon of the number of people in a party and the time it took to seat them. Graph the median-fit line for the data.
- Use your median-fit line to predict the time it would take to seat a party of 6.

People x	3	7	8	8	10	12
Wait Time y (min)	1	5	3	9	6	6

4-6

Arithmetic Sequences

Objectives

Recognize and extend an arithmetic sequence.

Find a given term of an arithmetic sequence.

Vocabulary

sequence

term

arithmetic sequence

common difference

Why learn this?

The distance between you and a lightning strike can be approximated by using an arithmetic sequence.

During a thunderstorm, you can estimate your distance from a lightning strike by counting the number of seconds from the time you see the lightning until the time you hear the thunder.

Time (s)	Distance (mi)
1	0.2
2	0.4
3	0.6
4	0.8
5	1.0
6	1.2
7	1.4
8	1.6

+ 0.2
+ 0.2
+ 0.2
+ 0.2
+ 0.2
+ 0.2
+ 0.2

When you list the times and distances in order, each list forms a *sequence*. A **sequence** is a list of numbers that may form a pattern. Each number in a sequence is a **term**.

In the distance sequence, each distance is 0.2 mi greater than the previous distance. When the terms of a sequence differ by the same nonzero number d , the sequence is an **arithmetic sequence** and d is the **common difference**. The distances in the table form an arithmetic sequence with $d = 0.2$.

The variable a is often used to represent terms in a sequence. The variable a_9 , read “ a sub 9,” is the ninth term in a sequence. To designate any term, or the n th term, in a sequence, you write a_n , where n can be any number.

To find a term in an arithmetic sequence, add d to the previous term.

Know It!

Note

Finding a Term of an Arithmetic Sequence

The n th term of an arithmetic sequence with **common difference** d is

$$a_n = a_{n-1} + d.$$

EXAMPLE 1 Identifying Arithmetic Sequences

Determine whether each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms in the sequence.

A 12, 8, 4, 0, ...

Step 1 Find the difference between successive terms.

$$12, 8, 4, 0, \dots$$

$-4 \quad -4 \quad -4$

Add -4 to each term to find the next term.
The common difference is -4 .

Step 2 Use the common difference to find the next 3 terms.

$$12, 8, 4, 0, -4, -8, -12$$

$-4 \quad -4 \quad -4$

$$a_n = a_{n-1} + d$$

The sequence appears to be an arithmetic sequence with a common difference of -4 . The next 3 terms are -4 , -8 , -12 .

Reading Math

The three dots at the end of a sequence are called an ellipsis. They mean that the sequence continues and can be read as “and so on.”

B 1, 4, 9, 16, ...

Find the difference between successive terms.



The difference between successive terms is not the same.

This sequence is not an arithmetic sequence.



Determine whether each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms.

1a. $-\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \dots$

1b. $-4, -2, 1, 5, \dots$

To find the n th term of an arithmetic sequence when n is a large number, you need an equation or rule. Look for a pattern to find a rule for the sequence below.

1	2	3	4...	n ← Position
↓	↓	↓	↓	
3,	5,	7,	9...	← Term
a_1	a_2	a_3	a_4	a_n

The sequence starts with 3. The common difference d is 2. You can use the first term and the common difference to write a rule for finding a_n .

Words	Numbers	Algebra
1st term	3	a_1
2nd term = 1st term plus common difference	$3 + (1)2 = 5$	$a_1 + 1d$
3rd term = 1st term plus 2 common differences	$3 + (2)2 = 7$	$a_1 + 2d$
4th term = 1st term plus 3 common differences	$3 + (3)2 = 9$	$a_1 + 3d$
⋮	⋮	⋮
n th term = 1st term plus $(n - 1)$ common differences	$3 + (n - 1)2$	$a_1 + (n - 1)d$

The pattern in the table shows that to find the n th term, add the first term to the product of $(n - 1)$ and the common difference.



Finding the n th Term of an Arithmetic Sequence

The n th term of an arithmetic sequence with common difference d and first term a_1 is

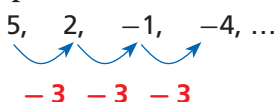
$$a_n = a_1 + (n - 1)d.$$

EXAMPLE 2 Finding the n th Term of an Arithmetic Sequence

Find the indicated term of each arithmetic sequence.

A 22nd term: 5, 2, -1, -4, ...

Step 1 Find the common difference.



The common difference is -3 .

Step 2 Find the 22nd term.

$$a_n = a_1 + (n - 1)d$$

$$a_{22} = 5 + (22 - 1)(-3)$$

$$= 5 + (21)(-3)$$

$$= 5 - 63$$

$$= -58$$

Write the rule to find the n th term.

Substitute 5 for a_1 , 22 for n , and -3 for d .

Simplify the expression in parentheses.

Multiply.

Subtract.

B 15th term: $a_1 = 7$; $d = 3$

$$a_n = a_1 + (n - 1)d$$

$$a_{15} = 7 + (15 - 1)3$$

$$= 7 + (14)3$$

$$= 7 + 42$$

$$= 49$$

Write the rule to find the n th term.

Substitute 7 for a_1 , 15 for n , and 3 for d .

Simplify the expression in parentheses.

Multiply.

Add.



Find the indicated term of each arithmetic sequence.

2a. 60th term: 11, 5, -1 , -7 , ... 2b. 12th term: $a_1 = 4.2$; $d = 1.4$

EXAMPLE 3 Travel Application

The odometer on a car reads 60,473 on day 1. Every day, the car is driven 54 miles. If this pattern continues, what is the odometer reading on day 20?

Notice that the sequence for the situation is arithmetic with $d = 54$ because the odometer reading will increase by 54 miles per day.

Since the odometer reading on day 1 is 60,473 miles, $a_1 = 60,473$.

Since you want to find the odometer reading on day 20, you will need to find the 20th term of the sequence, so $n = 20$.

$$a_n = a_1 + (n - 1)d$$

Write the rule to find the n th term.

$$a_{20} = 60,473 + (20 - 1)54$$

Substitute 60,473 for a_1 , 54 for d , and 21 for n .

$$= 60,473 + (19)54$$

Simplify the expression in parentheses.

$$= 60,473 + 1026$$

Multiply.

$$= 61,499$$

Add.

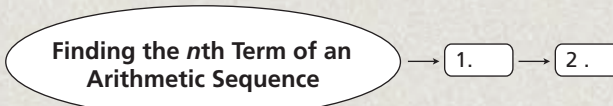
The odometer will read 61,499 miles on day 20.



3. Each time a truck stops, it drops off 250 pounds of cargo. After stop 1, its cargo weighed 2000 pounds. How much does the load weigh after stop 6?

THINK AND DISCUSS

1. Explain how to determine if a sequence appears to be arithmetic.
2. **GET ORGANIZED** Copy and complete the graphic organizer with steps for finding the n th term of an arithmetic sequence.



33. **Recreation** The rates for a go-cart course are shown.
- Explain why the relationship described on the flyer could be represented by an arithmetic sequence.
 - Find the cost for 1, 2, 3, and 4 laps. Write a rule to find the n th term of the sequence.
 - How much would 15 laps cost?
 - What if...?** After 9 laps, you get the 10th one free. Will the sequence still be arithmetic? Explain.

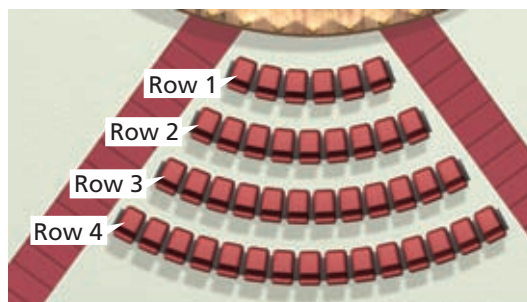


Find the given term of each arithmetic sequence.

34. 2.5, 8.5, 14.5, 20.5, ...; 30th term 35. 189.6, 172.3, 155, 137.7, ...; 18th term
36. $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \dots$; 15th term 37. $\frac{2}{3}, \frac{11}{12}, \frac{7}{6}, \frac{17}{12}, \dots$; 25th term

38. **Number Theory** The sequence 1, 1, 2, 3, 5, 8, 13, ... is a famous sequence called the Fibonacci sequence. After the first two terms, each term is the sum of the previous two terms.
- Write the first 10 terms of the Fibonacci sequence. Is the Fibonacci sequence arithmetic? Explain.
 - Notice that the third term is divisible by 2. Are the 6th and 9th terms also divisible by 2? What conclusion can you draw about every third term? Why is this true?
 - Can you find any other patterns? (*Hint*: Look at every 4th and 5th term.)

39. **Entertainment** Seats in a concert hall are arranged in the pattern shown.
- The numbers of seats in the rows form an arithmetic sequence. Write a rule for the arithmetic sequence.
 - How many seats are in the 15th row?
 - A ticket costs \$40. Suppose every seat in the first 10 rows is filled. What is the total revenue from those seats?



- d. **What if...?** An extra chair is added to each row. Write the new rule for the arithmetic sequence and find the new total revenue from the first 10 rows.
40. **Write About It** Explain how to find the common difference of an arithmetic sequence. How can you determine whether the arithmetic sequence has a positive common difference or a negative common difference?

LINK
Number Theory

Fibonacci numbers occur frequently throughout nature. The number of petals on many flowers are numbers of the Fibonacci sequence. Two petals on a flower are rare but 3, 5, and even 34 petals are common.

41. This problem will prepare you for the Multi-Step Test Prep on page 282.

Juan is traveling to visit universities. He notices mile markers along the road. He records the mile marker every 10 minutes. His father is driving at a constant speed.

- Copy and complete the table.
- Write the rule for the sequence.
- What does the common difference represent?
- If this sequence continues, find the mile marker for time interval 10.

Time Interval	Mile Marker
1	520
2	509
3	498
4	■
5	■
6	■

MULTI-STEP TEST PREP



42. What are the next three terms in the arithmetic sequence $-21, -12, -3, 6, \dots$?
 Ⓐ 9, 12, 15 Ⓑ 15, 24, 33 Ⓒ 12, 21, 27 Ⓓ 13, 20, 27

43. What is the common difference for the data listed in the second column?
 Ⓕ -1.8 Ⓗ 2.8
 Ⓖ 1.8 Ⓙ -3.6

Altitude (ft)	Boiling Point of Water ($^{\circ}\text{F}$)
1000	210.2
2000	208.4
3000	206.6

44. Which of the following sequences is NOT arithmetic?
 Ⓐ $-4, 2, 8, 14, \dots$ Ⓑ $9, 4, -1, -6, \dots$ Ⓒ $2, 4, 8, 16, \dots$ Ⓓ $\frac{1}{3}, 1\frac{1}{3}, 2\frac{1}{3}, 3\frac{1}{3}, \dots$

CHALLENGE AND EXTEND

45. The first term of an arithmetic sequence is 2, and the common difference is 9. Find two consecutive terms of the sequence that have a sum of 355. What positions in the sequence are the terms?
46. The 60th term of an arithmetic sequence is 106.5, and the common difference is 1.5. What is the first term of the sequence?
47. **Athletics** Verona is training for a marathon. The first part of her training schedule is shown below.

Session	1	2	3	4	5	6
Distance Run (mi)	3.5	5	6.5	8	9.5	11

- a. If Verona continues this pattern, during which training session will she run 26 miles? Is her training schedule an arithmetic sequence? Explain.
- b. If Verona's training schedule starts on a Monday and she runs every third day, on which day will she run 26 miles?

SPIRAL REVIEW

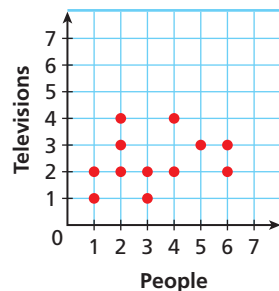
48. Three sides of a triangle are represented by $x, x + 3$ and $x + 5$. The perimeter of the triangle is 35 units. Solve for x . (Lesson 2-3)
49. The length of a rectangle is 2 and the width is represented by $x + 4$. The area of the rectangle is 40 square units. Find the width of the rectangle. (Lesson 2-3)

Solve each compound inequality and graph the solutions. (Lesson 3-6)

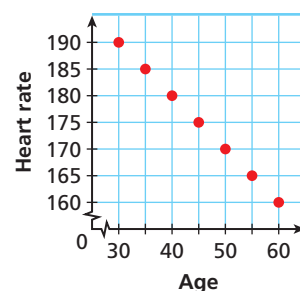
50. $4 < 2n + 6 \leq 20$ 51. $t + 5 > 7$ OR $2t - 8 < -12$

Describe the correlation illustrated by each scatter plot. (Lesson 4-5)

52. Household Televisions



53. Safe Heart Rate



MULTI-STEP TEST PREP

STATE

Applying Functions

College Knowledge Myra is helping her brother plan a college visit 10 hours away from their home. She creates a table listing approximate travel times and distances from their home.

1. Create a scatter plot for the data.
2. Draw a trend line through the data.
3. Based on the trend line, how many miles will they have traveled after 5 hours?
4. If Myra's brother decided to visit a college 13 hours away from their home, approximately how many miles will they travel?
5. To find the average speed for the entire trip, find $\frac{\text{change in distance}}{\text{change in time}}$ between the initial ordered pair and the final ordered pair. Include the units.

Time (h)	Distance (mi)
0	0
2	123
3	190
4	207
6	355
8	472
10	657



Quiz for Lessons 4-5 Through 4-6

4-5 Scatter Plots and Trend Lines

The table shows the time it takes different people to read a given number of pages.

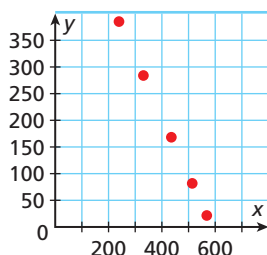
Pages Read	2	6	6	8	8	10	10
Time (min)	10	15	20	15	30	25	30

- Graph a scatter plot using the given data.
- Describe the correlation illustrated by the scatter plot.

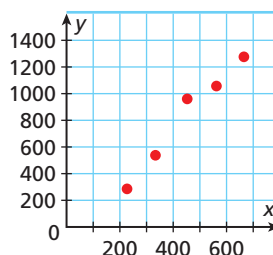
Choose the scatter plot that best represents the described relationship. Explain.

- number of movie tickets sold and number of available seats
- number of movie tickets sold and amount of concession sales
- number of movie tickets sold and length of movie

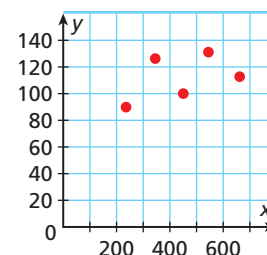
Graph A



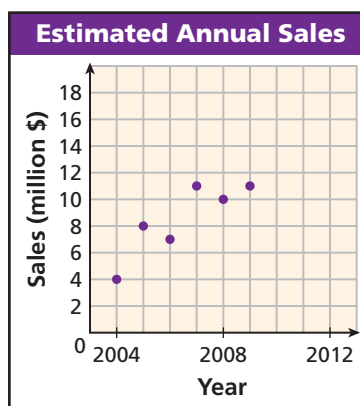
Graph B



Graph C



- The scatter plot shows the estimated annual sales for an electronics and appliance chain of stores for the years 2004–2009. Based on this relationship, predict the annual sales in 2012.



4-6 Arithmetic Sequences

Determine whether each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms.

- 7, 3, -1, -5, ...
- 3, 6, 12, 24, ...
- 3.5, -2, -0.5, 1, ...

Find the indicated term of the arithmetic sequence.

- 31st term: 12, 7, 2, -3, ...
- 22nd term: $a_1 = 6$; $d = 4$
- With no air resistance, an object would fall 16 feet during the first second, 48 feet during the second second, 80 feet during the third second, 112 feet during the fourth second, and so on. How many feet will the object fall during the ninth second?

Vocabulary

arithmetic sequence	276	function	241	range	240
common difference	276	function notation	250	relation	240
continuous graph	235	function rule	250	scatter plot	266
correlation	266	independent variable	250	sequence	276
dependent variable	250	negative correlation	267	term	276
discrete graph	235	no correlation	267	trend line	269
domain	240	positive correlation	267		

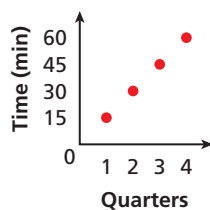
Complete the sentences below with vocabulary words from the list above.

- The set of x -coordinates of the ordered pairs of a relation is called the ____? ____.
- If one set of data values increases as another set of data values decreases, the relationship can be described as having a(n) ____? ____.
- A sequence is an ordered list of numbers where each number is a(n) ____? ____.

4-1 Graphing Relationships (pp. 234–239)**EXAMPLES**

Sketch a graph for each situation. Tell whether the graph is continuous or discrete.

- A parking meter has a limit of 1 hour. The cost is \$0.25 per 15 minutes and the meter accepts quarters only.

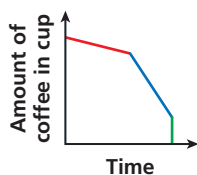


Since only quarters are accepted, the points are not connected.

The graph is discrete.

- Ian bought a cup of coffee. At first, he sipped slowly. As it cooled, he drank more quickly. The last bit was cold, and he dumped it out.

As time passes the coffee was **sipped slowly**, **drank more quickly**, and then **dumped out**.



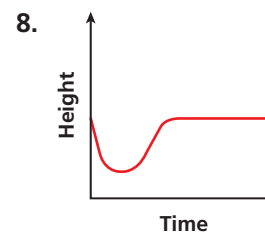
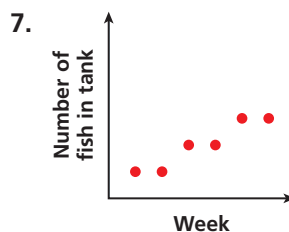
The graph is continuous.

EXERCISES

Sketch a graph for each situation. Tell whether the graph is continuous or discrete.

- A girl was walking home at a steady pace. Then she stopped to talk to a friend. After her friend left, she jogged the rest of the way home.
- A ball is dropped from a second story window and bounces to a stop on the patio below.
- Jason was on the second floor when he got a call to attend a meeting on the sixth floor. He took the stairs. After the meeting, he took the elevator to the first floor.

Write a possible situation for each graph.



4-2 Relations and Functions (pp. 240–246)

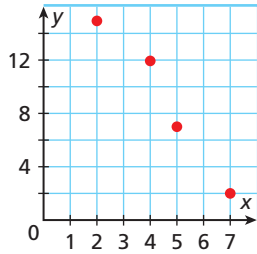
EXAMPLES

- Express the relation $\{(2, 15), (4, 12), (5, 7), (7, 2)\}$ as a table, as a graph, and as a mapping diagram.

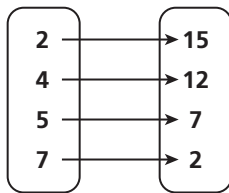
Table

x	y
2	15
4	12
5	7
7	2

Graph



Mapping Diagram



Give the domain and range of each relation. Tell whether the relation is a function. Explain.

- | x | y |
|----|---|
| -3 | 0 |
| -2 | 0 |
| -1 | 1 |

D: $\{-3, -2, -1\}$
R: $\{0, 1\}$

The relation is a function because each domain value is paired with exactly one range value.

- D: $\{1, 2\}$
R: $\{-5, -4, 4\}$

The relation is not a function because one domain value is paired with two range values.

- D: $-4 \leq x \leq 4$
R: $-2 \leq y \leq 6$
The relation is a function because every x -value is paired with exactly one y -value.

EXERCISES

Express each relation as a table, as a graph, and as a mapping diagram.

9. $\{(-1, 0), (0, 1), (2, 1)\}$
10. $\{(-2, -1), (-1, 1), (2, 3), (3, 4)\}$

Give the domain and range of each relation.

11. $\{(-4, 5), (-2, 3), (0, 1), (2, -1)\}$
12. $\{(-2, -1), (-1, 0), (0, -1), (1, 0), (2, -1)\}$

13.

x	0	1	4	1	4
y	0	-1	-2	1	2

- 14.

Give the domain and range of each relation. Tell whether the relation is a function. Explain.

15. $\{(-5, -3), (-3, -2), (-1, -1), (1, 0)\}$

- 16.

17.

x	1	2	3	4	1
y	3	2	1	0	-1

18. A local parking garage charges \$5.00 for the first hour plus \$1.50 for each additional hour or part of an hour. Write a relation as a set of ordered pairs in which the x -value represents the number of hours and the y -value represents the cost for x hours. Use a domain of 1, 2, 3, 4, 5. Is this relation a function? Explain.

19. A baseball coach is taking the team for ice cream. Four students can ride in each car. Create a mapping diagram to show the number of cars needed to transport 8, 10, 14, and 16 students. Is this relation a function? Explain.

4-3 Writing Functions (pp. 249–255)

EXAMPLES

- Determine a relationship between the x - and y -values in the table. Write an equation.

x	1	2	3	4
y	-3	-6	-9	-12

What are possible relationships between the x -values and the y -values?

$$1 - 4 = -3 \quad 1(-3) = -3$$

$$2 - 4 \neq -6 \quad 2(-3) = -6 \quad \checkmark$$

$$3(-3) = -9 \quad \checkmark$$

$$4(-3) = -12 \quad \checkmark$$

$$y = -3x \quad \text{Write an equation.}$$

- Nia earns \$5.25 per hour. Identify the independent and dependent variables. Write an equation in function notation for the situation.

Nia's **pay** depends on the **number of hours** she works.

Dependent: **pay**

Independent: **hours**

Let h represent the number of hours Nia works.

The function for Nia's pay is $f(h) = 5.25h$.

EXERCISES

Determine the relationship between the x - and y -values. Write an equation.

20.

x	1	2	3	4
y	-6	-5	-4	-3

21. $\{(1, 9), (2, 18), (3, 27), (4, 36)\}$

Identify the independent and dependent variables. Write an equation in function notation for the situation.

22. A baker spends \$6 on ingredients for each cake he bakes.

23. Tim will buy twice as many CDs as Raul.

Evaluate each function for the given input values.

24. For $f(x) = -2x + 4$, find $f(x)$ when $x = -5$.

25. For $g(n) = -n^2 - 2$, find $g(n)$ when $n = -3$.

26. For $h(t) = 7 - |t + 3|$, find $h(t)$ when $t = -4$ and when $t = 5$.

4-4 Graphing Functions (pp. 256–262)

EXAMPLE

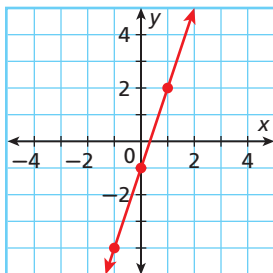
- Graph the function $y = 3x - 1$.

Step 1 Choose several values of x to generate ordered pairs.

x	$y = 3x - 1$	y
-1	$y = 3(-1) - 1 = -4$	-4
0	$y = 3(0) - 1 = -1$	-1
1	$y = 3(1) - 1 = 2$	2
2	$y = 3(2) - 1 = 5$	5

Step 2 Plot enough points to see a pattern.

Step 3 Draw a line through the points to show all the ordered pairs that satisfy this function.



EXERCISES

Graph each function for the domain $\{-2, -1, 1, 2\}$.

27. $4x + y = 2$

28. $y = (1 - x)^2$

Graph each function.

29. $3x - y = 1$

30. $y = 2 - |x|$

31. $y = x^2 - 6$

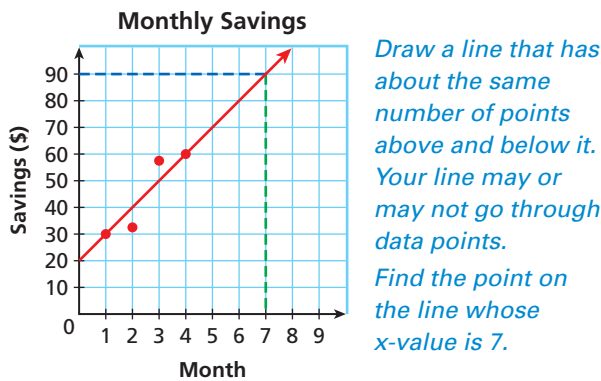
32. $y = |x + 5| + 1$

33. The function $y = 6.25x$ describes the amount of money y Peter gets paid after x hours. Graph the function. Use the graph to estimate how much money Peter gets paid after 7 hours.

4-5 Scatter Plots and Trend Lines (pp. 266–273)

EXAMPLE

- The graph shows the amount of money in a savings account. Based on this relationship, predict how much money will be in the account in month 7.



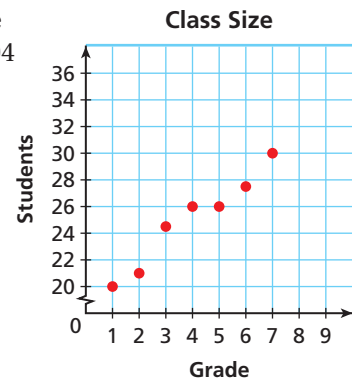
Based on the data, \$90 is a reasonable prediction.

EXERCISES

34. The table shows the value of a car for the given years. Graph a scatter plot using the given data. Describe the correlation illustrated by the scatter plot.

Year	2000	2001	2002	2003
Value (thousand \$)	28	25	23	20

35. The graph shows the results of a 2003–2004 survey on class size at the given grade levels. Based on this relationship, predict the class size for the 9th grade.



4-6 Arithmetic Sequences (pp. 276–281)

EXAMPLES

- Determine whether the sequence appears to be arithmetic. If so, find the common difference and the next three terms.

$-8, -5, -2, 1, \dots$

Step 1 Find the difference between successive terms.

$-8, -5, -2, 1, \dots$ *The common difference is 3.*
 $\quad \quad \quad \nearrow \quad \nearrow \quad \nearrow$
 $\quad \quad \quad +3 \quad +3 \quad +3$

Step 2 Use the common difference to find the next 3 terms.

$-8, -5, -2, 1, \quad 4, \quad 7, \quad 10$
 $\quad \quad \quad \nearrow \quad \nearrow \quad \nearrow$
 $\quad \quad \quad +3 \quad +3 \quad +3$

- Find the 18th term of the arithmetic sequence for which $a_1 = -4$ and $d = 6$.

$a_n = a_1 + (n - 1)d$ *Write the rule.*
 $a_{18} = -4 + (18 - 1)6$ *Substitute.*
 $= -4 + (17)6$ *Simplify.*
 $= -4 + 102$ *Simplify.*
 $= 98$

The 18th term is 98.

EXERCISES

Determine whether each sequence appears to be arithmetic. If so, find the common difference and the next three terms.

36. $20, 14, 8, 2, \dots$

37. $-15, -12, -9, -4, \dots$

38. $5, 4, 2, -1, \dots$

39. $-8, -5.5, -3, -0.5, \dots$

Find the indicated term of each arithmetic sequence.

40. 31st term: $-15, -11, -7, -3, \dots$

41. 24th term: $a_1 = 7; d = -3$

42. 17th term: $a_1 = -20; d = 2.5$

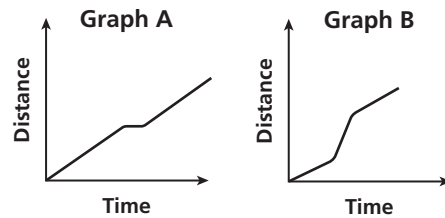
43. Marie has \$180 in a savings account in week 1. She plans to deposit \$12 each following week. Assuming that she does not withdraw any money from her account, what will her balance be in week 20?

44. The table shows the temperature at the given heights above sea level. Use an arithmetic sequence to find the temperature at 8000 feet above sea level.

Height Above Sea Level (thousand feet)	1	2	3	4
Temperature ($^{\circ}\text{C}$)	30	23.5	17	10.5

Choose the graph that best represents each situation.

1. A person walks leisurely, stops, and then continues walking.
2. A person jogs, then runs, and then jogs again.

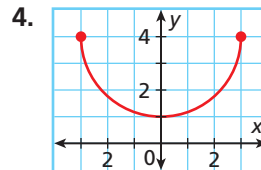


Give the domain and range for each relation.

Tell whether the relation is a function. Explain.

3.

x	-2	1	0	1	3
y	3	2	1	0	-1



5. Bowling costs \$3 per game plus \$2.50 for shoe rental. Identify the independent and dependent variables. Write an equation in function notation for the situation.

Evaluate each function for the given input values.

6. For $f(x) = -3x + 4$, find $f(x)$ when $x = -2$.
7. For $f(x) = 2x^2$, find $f(x)$ when $x = -3$.
8. An engraver charges a \$10 fee plus \$6 for each line of engraving. Write a function to describe the situation. Find a reasonable domain and range for the function for up to 8 lines.

Graph each function for the given domain.

9. $3x + y = 4$; D: $\{-2, -1, 0, 1, 2\}$
10. $y = |x - 1|$; D: $\{-3, 0, 1, 3, 5\}$
11. $y = x^2 - 1$; D: $\{-2, -1, 0, 1, 2\}$

Graph each function.

12. $y = x - 5$
13. $y = x^2 - 5$
14. $y = |x| + 3$
15. The function $y = 30x$ describes the amount of interest y earned in a savings account in x years. Graph the function. Use the graph to estimate the total amount of interest earned in 7 years.

The table shows possible recommendations for the amount of sleep that children should get every day.

16. Graph a scatter plot of the given data.
17. Describe the correlation illustrated by the scatter plot.
18. Predict how many hours of sleep a 16-year-old needs.

Age (yr)	1	2	3	4	5	14
Sleep Needed (h)	14	13	12	12	11	9

Determine whether each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms.

19. 11, 6, 1, -4, ...
20. -4, -3, -1, 2, ...
21. 7, 21, 30, 45, ...

Find the indicated term of the arithmetic sequence.

22. 32nd term: 18, 11, 4, -3, ...
23. 24th term: $a_1 = 4$; $d = 6$
24. Mandy's new job has a starting salary of \$16,000 and annual increases of \$800. How much will she earn during her fifth year?

COLLEGE ENTRANCE EXAM PRACTICE



FOCUS ON ACT

Questions on the ACT Mathematics Test do not require the use of a calculator, but you may bring one to use with the test. Make sure that it is a calculator that is on the approved list for the ACT.



When taking the test, you will be more comfortable using a calculator that you are used to. If you already have a calculator, make sure it is one of the permitted calculators. If you plan to use a new one, make sure to practice using it before the test.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

1. The soccer team is ordering new uniforms. There is a one-time setup charge of \$50.00, and each uniform costs \$23.50. Which of the following best describes the total cost C for ordering uniforms for p players?

- (A) $C = 23.50p$
- (B) $C = 50p$
- (C) $C = 73.50p$
- (D) $C = 23.50p + 50$
- (E) $C = 50p + 23.50$

2. In the given relation, what domain value corresponds to the range value -2 ?

$$\{(-1, 2), (-2, 4), (2, 5), (0, -2), (2, 0)\}$$

- (F) -2
- (G) 0
- (H) 2
- (J) 4
- (K) 5

3. Evaluate $h(x) = \frac{1}{2}(5 - 6x) + 9x$ when $x = \frac{2}{3}$.

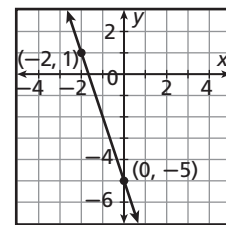
- (A) $\frac{9}{2}$
- (B) $\frac{13}{2}$
- (C) 7
- (D) $\frac{19}{2}$
- (E) $\frac{23}{2}$

4. What is the seventh term of the arithmetic sequence $-4, -1, 2 \dots$?

- (F) 5
- (G) 10
- (H) 11
- (J) 14
- (K) 17

5. The graph of which function is shown below?

- (A) $y = -3x - 5$
- (B) $y = -\frac{1}{3}x - \frac{5}{3}$
- (C) $y = -5x - 3$
- (D) $y = 3x - 5$
- (E) $y = 5x + 3$



6. Which of the following relations is NOT a function?

- (F) $\{(0, 1), (1, 2), (2, 3), (3, 4)\}$
- (G) $\{(1, 2), (2, 2), (3, 3), (4, 3)\}$
- (H) $\{(0, 2), (2, 4), (4, 1), (1, 3)\}$
- (J) $\{(1, 3), (4, 2), (2, 0), (3, 4)\}$
- (K) $\{(0, 2), (1, 3), (4, 3), (1, 2)\}$



Extended Response: Understand the Scores

Extended response test items are typically multipart questions that require a high level of thinking. The responses are scored using a 4-point rubric. To receive full credit, you must correctly answer all parts of the question and provide a clear explanation. A partial answer is worth 2 or 3 points, an incorrect solution is worth 1 point, and no response is worth 0 points.

EXAMPLE 1

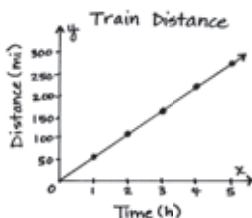
Extended Response A train traveling from Boston, Massachusetts, to Richmond, Virginia, averages about 55 miles per hour. Define variables, write an equation, make a table, and draw a graph to show the distance the train travels in 5 hours.

Here are examples of four different responses and their scores using the rubric shown.

4-point response:

Let d = the distance the train travels.
Let t = the time the train travels.
 $d = 55t$

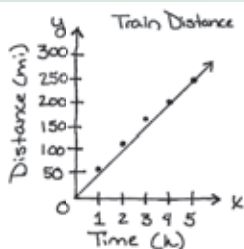
Time (h)	0	1	2	3	4	5
Distance (mi)	0	55	110	165	220	275



3-point response:

Let d = the distance the train travels
Let t = the time the train travels
 $d = 55t$

Time (h)	0	1	2	3	4	5
Distance (mi)	0	55	110	165	200	250

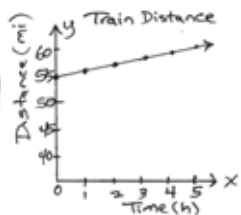


The student shows all of the work, but there are two minor computation errors when $t = 4$ and $t = 5$.

2-point response:

$$d = 55 + t$$

Time (h)	0	1	2	3	4	5
Distance (mi)	55	56	57	58	59	60



The student writes an incorrect equation and uses it to create an incorrect table and graph.

1-point response:

$$d = 55t$$

The student does not answer two parts of the question.



Never leave an extended-response test item blank. At least try to define variables or write equations where appropriate. You will get some points just for trying.

Read each test item and answer the questions that follow using the rubric below.

Scoring Rubric:

4 points: The student shows all of the work, correctly answers all parts of the question, and provides a clear explanation.

3 points: The student shows most of the work and provides a clear explanation but has a minor computation error, or the student shows all of the work and arrives at a correct solution but does not provide a clear explanation.

2 points: The student makes major errors resulting in an incorrect solution, or the student gives a correct solution but does not show any work nor provide an explanation.

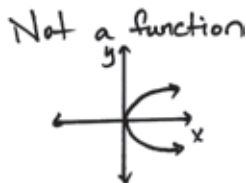
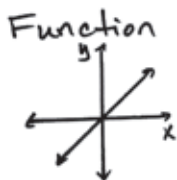
1 point: The student shows no work and gives an incorrect solution.

0 points: The student gives no response.

Item A

Extended Response Draw a graph that is a function. Explain why it is a function. Then draw a graph that is NOT a function. Explain why it is not a function.

1. What should be included in a 4-point response?
2. Explain how would you score the response below.



The first graph is a function because each x -value has exactly one y -value. When $x=1$, $y=1$. The second graph is not a function because there is more than one y -value for each x -value. When $x=1$, $y=1$, and $y=-1$. Therefore, the second graph is not a function.

Item B

Extended Response A car travels at a steady rate of 60 miles per hour. Identify the independent and dependent variables. Describe the domain and range. Write an equation to describe the situation.

3. Ana wrote the response below.

The equation is $y = 60x$. The independent variable is time and the dependent variable is distance. The domain and range are all real numbers.

Explain how would you score Ana's response.

4. If you did not give Ana full credit, what should be added to Ana's response, if anything, so that it receives full credit?

Item C

Extended Response Lara bought 8 notebooks and 4 binders. She spent \$14 total without tax. How much did each notebook cost if each binder cost \$2.50? Write an equation and find the solution.

5. Explain how would you score the response below.

Let s = the cost of each notebook.

Let b = the cost of each binder.

$$8s + 4b = 14$$

$$8s + 4(2.50) = 14$$

$$8s + 10 = 14$$

$$8s = 4$$

$$s = 2 \text{ The notebooks cost } \$2 \text{ each}$$

6. If you did not give the response full credit, what should be added to the response, if anything, so that it receives full credit?



STANDARDIZED TEST PREP

CUMULATIVE ASSESSMENT, CHAPTERS 1–4

Multiple Choice

1. Find the value of $|a| - b^2$ when $a = -3$ and $b = -5$.

(A) -28 (C) -7
 (B) -22 (D) 4

2. Benito has x apples. He cuts each apple in half and gives each half to a different horse. Which expression represents the number of horses Benito feeds?

(F) $x \cdot \frac{1}{2}$ (H) $x \cdot 1\frac{1}{2}$
 (G) $x \div \frac{1}{2}$ (J) $x \div 1\frac{1}{2}$

3. If the value of a^5 is positive, then which is true?

(A) a is positive.
 (B) a is negative.
 (C) a^5 is odd.
 (D) a^5 is even.

4. Find the value of $\frac{2a}{a^3}$ if $4 - a = -6$.

(F) $\frac{1}{50}$ (H) 8
 (G) $\frac{1}{2}$ (J) 10

5. There are 36 flowers in a bouquet. Two-thirds of the flowers are roses. One-fourth of the roses are red. What percent of the bouquet is made up of red roses?

(A) 9% (C) 25%
 (B) $16\frac{2}{3}\%$ (D) $66\frac{2}{3}\%$

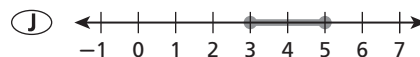
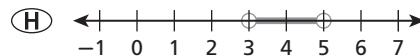
6. A large tree should be planted at least 70 feet away from a power line. Which inequality shows the acceptable number of feet x between a large tree and a power line?

(F) $x < 70$ (H) $x > 70$
 (G) $x \leq 70$ (J) $x \geq 70$

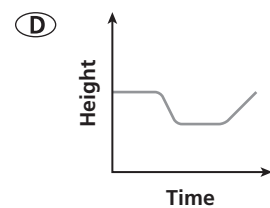
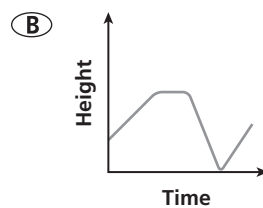
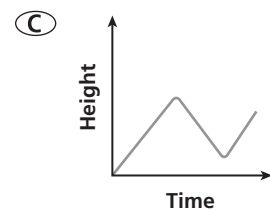
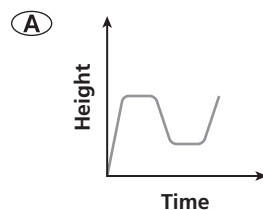
7. Which statement is modeled by $3f + 2 > -16$?

(A) Two added to 3 times f is at least -16 .
 (B) Three times the sum of f and 2 is at most -16 .
 (C) The sum of 2 and 3 times f is more than -16 .
 (D) The product of $3f$ and 2 is no more than -16 .

8. Jo Ann needs at least 3 pounds of peaches for a recipe. At the market, she calculates that she has enough money to buy 5 pounds at most. Which graph shows all possible numbers of pounds of peaches Jo Ann can buy so that she has enough for the recipe?



9. A bird flies from the ground to the top of a tree, sits there and sings for a while, flies down to the top of a picnic table to eat crumbs, and then flies back to the top of the tree to sing some more. Which graph best represents this situation?



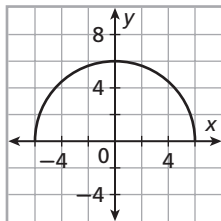
10. Which relation is NOT a function?

(F) $\{(1, -5), (3, 1), (-5, 4), (4, -2)\}$
 (G) $\{(2, 7), (3, 7), (4, 7), (5, 8)\}$
 (H) $\{(1, -5), (-1, 6), (1, 5), (6, -3)\}$
 (J) $\{(3, -2), (5, -6), (7, 7), (8, 8)\}$



If possible, use the same calculator you usually use in math class. A timed test is not the right place to figure out where buttons are and how they work. Also, replace your batteries the night before the test. If your batteries run out, you may be given a replacement calculator you are not familiar with.

11. The graph below shows a function.



What is the domain of the function?

- Ⓐ $x \geq 0$
 Ⓑ $x \geq -6$
 Ⓒ $0 \leq x \leq 6$
 Ⓓ $-6 \leq x \leq 6$
12. Which situation best describes a negative correlation?
- Ⓕ The speed of a runner and the time it takes to run a race
 Ⓖ The number of apples in a bag and the weight of the bag of apples
 Ⓗ The time it takes to repair a car and the amount of the bill
 Ⓙ The number of people in a household and the amount of mail in their mailbox

13. Which of the following is a solution of $x + 1 \leq \frac{3}{2}$ AND $x - 1 \geq -\frac{5}{4}$?

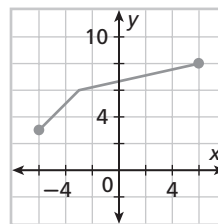
- Ⓐ $\frac{3}{2}$ Ⓒ $-\frac{1}{3}$
 Ⓑ $\frac{1}{3}$ Ⓓ $-\frac{3}{2}$

Gridded Response

14. What is the value of x when $3(x + 7) - 6x = 4 - (x + 1)$?
15. For $h(x) = x^3 + 2x$, find $h(4)$.
16. WalkieTalkie phone company charges \$18.00 for basic phone service per month and \$0.15 per minute for long distance calls. Arena Calls charges \$80.00 per month with no fee for long distance calls. What is the minimum number of minutes of long distance calls for which the cost of WalkieTalkie is more than the cost of Arena Calls?

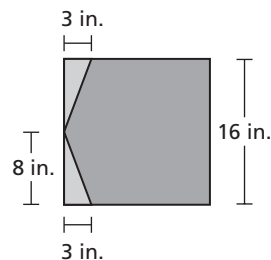
Short Response

17. A function is graphed below.



What is the domain and range of the function?

18. Rory made a pentagon by cutting two triangles from a square piece of cardboard as shown.



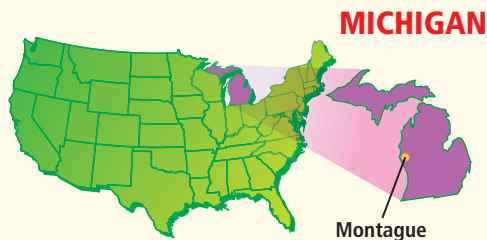
What is the area of the pentagon? Show your work or explain how you got your answer.

19. The manager of a new restaurant needs at most 12 servers. He has already hired 7 servers.
- a. Write and solve an inequality to determine how many more servers the manager could hire.
 b. Graph the solutions to the inequality you solved in part a.
20. Study the sequence below.
 18, 24.5, 31, 37.5, 44, ...
- a. Could this sequence be arithmetic? Explain.
 b. Find the 100th term of the sequence. Show your work.

Extended Response

21. A relation is shown in the table.
- a. Express the relation as a mapping diagram.
 b. Is the relation a function? Explain why or why not.
 c. Write a possible real-life situation for the relation.

x	y
2	12
3	15
3	18
5	40
6	64



★ The World's Largest Weather Vane

The world's largest working weather vane weighs approximately 4300 pounds and is located in Montague, Michigan. Weather vanes are used to indicate the direction of the wind. When the wind blows, the vane points in the direction from which the wind is coming.

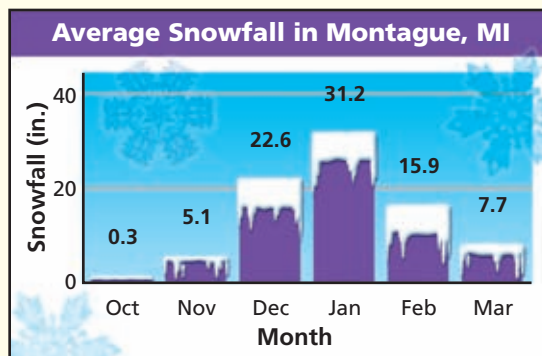
Choose one or more strategies to solve each problem.

1. The arrow on the world's largest weather vane has a length of 26 feet. The height of the weather vane is 4 feet shorter than twice its length. What is the height of the world's largest weather vane?

At the base of the weather vane is a working weather station. Weather stations include instruments such as thermometers, rain gauges, and wind gauges that measure different characteristics of the weather. The data gathered from weather stations are used to make predictions about future weather.

For 2 and 3, use the graph.

2. The data in the graph represent the average snowfall for each month, measured over a 30-year time period. The average amount of snowfall received in April is less than $\frac{1}{4}$ of the average snowfall in March. What is the greatest possible average snowfall for April? Round to the nearest tenth of an inch.
3. Which month do you predict will get the most snowfall next year? Explain your reasoning.



★ Maple Syrup

Michigan produces about 90,000 gallons of maple syrup each year. This places the state among the top ten states in U.S. production of maple syrup. Maple syrup is made from the sap of maple trees, but only about 1% of Michigan's maple trees are used in maple syrup production.



Choose one or more strategies to solve each problem.

1. The standard sugar concentration level of maple syrup is 66%. At certain levels above 66%, the product develops into maple cream, soft maple sugar, or hard maple sugar. The sugar concentration never reaches 100%, even in hard maple sugar. What is the range of sugar concentration levels in the various maple products? Show your answer on a number line.

For 2 and 3, use the table.

2. How many Calories are in 1 cup of maple syrup?
(Hint: 4 tbsp = $\frac{1}{4}$ c)
3. Approximately how many tablespoons of maple syrup would you need to have the same number of Calories that are in 7 tablespoons of honey? Round to the nearest tablespoon.
4. It takes 40 gallons of maple sap to make 1 gallon of maple syrup. Each tap hole in a maple tree will produce about 10 gallons of sap in an average year. How many gallons of maple syrup could be made with the sap from 20 tap holes?
5. It is recommended that maple trees be at least 10 inches in diameter before they are tapped. Only one tap should be placed in trees that are 10 to 18 inches in diameter, while 2 taps can be placed in trees greater than 18 inches in diameter. An orchard has 130 trees that are less than 10 inches in diameter, 104 trees that are 10–18 inches in diameter, and 48 trees that are greater than 18 inches in diameter. What is the maximum number of tap holes this orchard should have?

Sweetener	Calories (per tbsp)
Blackstrap molasses	43
Granulated sugar	46
Maple syrup	50
Corn syrup	57
Honey	64

