CHAPTER 5

Linear Functions

Chapter Focus

- Translate among different representations of linear functions.
- Find and interpret slopes and intercepts of linear equations that model real-world problems
- Solve real-world problems involving linear equations.

Take *Flight*

You can use linear functions to describe patterns and relationships in flight times.

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KEYWORD: MA7 ChProj

ARE YOU READY?

✔ Vocabulary

Match each term on the left with a definition on the right.

1. coefficient
2. coordinate plane
3. transformation
4. perpendicular

A. a change in the size or position of a figure

B. forming right angles

C. a two-dimensional system formed by the intersection of a horizontal number line and a vertical number line

D. an ordered pair of numbers that gives the location of a point

E. a number that is multiplied by a variable

✔ Ordered Pairs

Graph each point on the same coordinate plane.

5. $A(2, 5)$
6. $B(-1, -3)$
7. $C(-5, 2)$
8. $D(4, -4)$
9. $E(-2, 0)$
10. $F(0, 3)$
11. $G(8, 7)$
12. $H(-8, -7)$

✔ Solve for a Variable

Solve each equation for the indicated variable.

13. $2x + y = 8; y$
14. $5y = 5x - 10; y$
15. $2y = 6x - 8; y$
16. $10x + 25 = 5y; y$

✔ Evaluate Expressions

Evaluate each expression for the given value of the variable.

17. $4g - 3; g = -2$
18. $8p - 12; p = 4$
19. $4x + 8; x = -2$
20. $-5t - 15; t = 1$

✔ Connect Words and Algebra

21. The value of a stock begins at \$0.05 and increases by \$0.01 each month. Write an equation representing the value of the stock v in any month m.

22. Write a situation that could be modeled by the equation $b = 100 - s$.

✔ Rates and Unit Rates

Find each unit rate.

23. 322 miles on 14 gallons of gas
24. \$14.25 for 3 pounds of deli meat
25. 32 grams of fat in 4 servings
26. 120 pictures on 5 rolls of film

Study Guide: Preview

Where You've Been

Previously, you
- wrote equations in function notation.
- graphed functions.
- identified the domain and range of functions.
- identified independent and dependent variables.

In This Chapter

You will study
- writing and graphing linear functions.
- identifying and interpreting the components of linear graphs, including the *x*-intercept, *y*-intercept, and slope.
- graphing and analyzing families of functions.

Where You're Going

You can use the skills in this chapter
- to solve systems of linear equations in Chapter 6.
- to identify rates of change in linear data in biology and economics.
- to make calculations and comparisons in your personal finances.

Key Vocabulary/Vocabulario

constant of variation	constante de variación
direct variation	variación directa
family of functions	familia de funciones
linear function	función lineal
parallel lines	líneas paralelas
perpendicular lines	líneas perpendiculares
slope	pendiente
transformation	transformación
x-intercept	intersección con el eje *x*
y-intercept	intersección con el eje *y*

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. What shape do you think is formed when a **linear function** is graphed on a coordinate plane?

2. The meaning of *intercept* is similar to the meaning of *intersection*. What do you think an **x-intercept** might be?

3. **Slope** is a word used in everyday life, as well as in mathematics. What is your understanding of the word *slope*?

4. A family is a group of related people. Use this concept to define **family of functions** .

Reading *and* Writing Math

Study Strategy: Use Multiple Representations

Representing a math concept in more than one way can help you understand it more clearly. As you read the explanations and example problems in your text, note the use of tables, lists, graphs, diagrams, and symbols, as well as words to explain a concept.

From Lesson 4-4:

In this example from Chapter 4, the given function is described using an equation, a table, ordered pairs, and a graph.
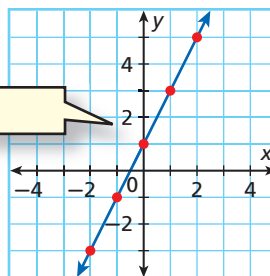

Graphing Functions

Graph each function.

A $2x + 1 = y$ — **Equation**

Step 1 Choose several values of x and generate ordered pairs.

Step 2 Plot enough points to see a pattern.

Table

x	$2x + 1 = y$	(x, y)
-3	$2(-3) + 1 = -5$	$(-3, -5)$
-2	$2(-2) + 1 = -3$	$(-2, -3)$
-1	$2(-1) + 1 = -1$	$(-1, -1)$
0	$2(0) + 1 = 1$	$(0, 1)$
1	$2(1) + 1 = 3$	$(1, 3)$
2	$2(2) + 1 = 5$	$(2, 5)$
3	$2(3) + 1 = 7$	$(3, 7)$

Graph

Ordered Pairs

Step 3 The ordered pairs appear to form a line. Draw a line through all the points to show all the ordered pairs that satisfy the function. Draw arrowheads on both "ends" of the line.

Try This

1. If an employee earns $8.00 an hour, $y = 8x$ gives the total pay y the employee will earn for working x hours. For this equation, make a table of ordered pairs and a graph. Explain the relationships between the equation, the table, and the graph. How does each one describe the situation?

2. What situations might make one representation more useful than another?

Identifying Linear Functions

Objectives
Identify linear functions and linear equations.

Graph linear functions that represent real-world situations and give their domain and range.

Vocabulary
linear function
linear equation

Why learn this?
Linear functions can describe many real-world situations, such as distances traveled at a constant speed.

Most people believe that there is no speed limit on the German autobahn. However, many stretches have a speed limit of 120 km/h. If a car travels continuously at this speed, $y = 120x$ gives the number of kilometers y that the car would travel in x hours. Solutions are shown in the graph.

The graph represents a function because each domain value (x-value) is paired with exactly one range value (y-value). Notice that the graph is a straight line. A function whose graph forms a straight line is called a **linear function**.
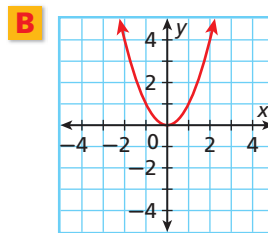
Distance Traveled
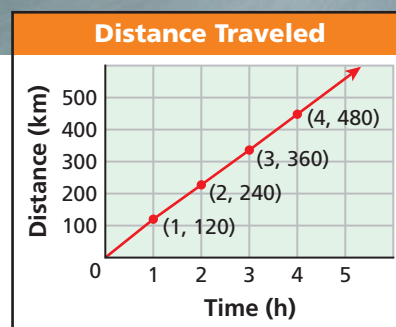
EXAMPLE **1** **Identifying a Linear Function by Its Graph**

Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?

A
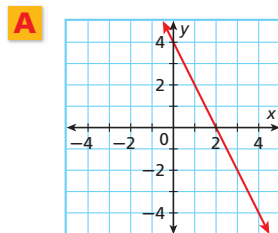
Each domain value is paired with exactly one range value. The graph forms a line.

linear function

B

Each domain value is paired with exactly one range value. The graph is not a line.

not a linear function

C

The only domain value, 3, is paired with many different range values.

not a function

Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?
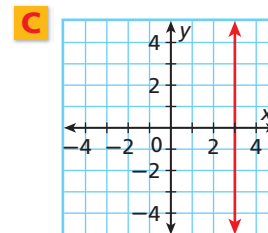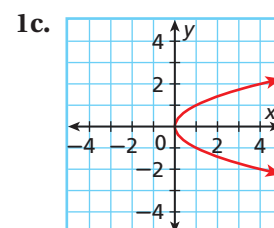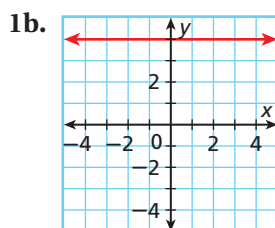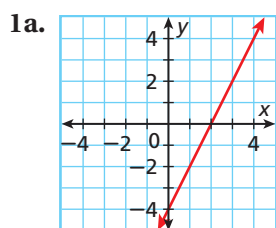
1a.

1b.

1c.

You can sometimes identify a linear function by looking at a table or a list of ordered pairs. In a linear function, a constant change in x corresponds to a constant change in y.

x	y
−2	7
−1	4
0	1
1	−2
2	−5

+1 → −3
+1 → −3
+1 → −3
+1 → −3

x	y
−2	6
−1	3
0	2
1	3
2	6

+1 → −3
+1 → −1
+1 → +1
+1 → +3

Caution!

If you find a constant change in the y-values, check for a constant change in the x-values. Both need to be constant for the function to be linear.

In this table, a constant change of +1 in x corresponds to a constant change of −3 in y. These points satisfy a linear function.

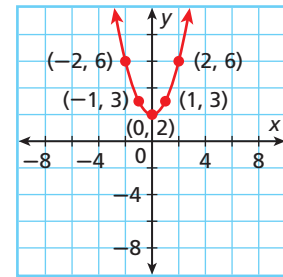*The points from this table lie on a line.*
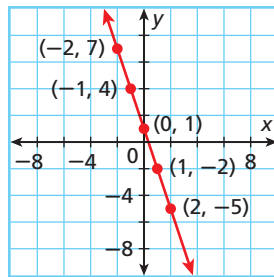
In this table, a constant change of +1 in x does *not* correspond to a constant change in y. These points do *not* satisfy a linear function.

The points from this table do not lie on a line.

EXAMPLE 2 Identifying a Linear Function by Using Ordered Pairs

Tell whether each set of ordered pairs satisfies a linear function. Explain.

A $\{(2, 4), (5, 3), (8, 2), (11, 1)\}$

x	y
2	4
5	3
8	2
11	1

+3 → −1
+3 → −1
+3 → −1

Write the ordered pairs in a table. Look for a pattern.

A constant change of +3 in x corresponds to a constant change of −1 in y.

These points satisfy a linear function.

B $\{(-10, 10), (-5, 4), (0, 2), (5, 0)\}$

x	y
−10	10
−5	4
0	2
5	0

+5 → −6
+5 → −2
+5 → −2

Write the ordered pairs in a table. Look for a pattern.

A constant change of +5 in x corresponds to different changes in y.

These points do not satisfy a linear function.

CHECK IT OUT!

2. Tell whether the set of ordered pairs $\{(3, 5), (5, 4), (7, 3), (9, 2), (11, 1)\}$ satisfies a linear function. Explain.

Another way to determine whether a function is linear is to look at its equation. A function is linear if it is described by a *linear equation*. A **linear equation** is any equation that can be written in the *standard form* shown below.

> **Know it!**
> .Note

> **Standard Form of a Linear Equation**
>
> $Ax + By = C$ where A, B, and C are real numbers and A and B are not both 0

Notice that when a linear equation is written in standard form
- x and y both have exponents of 1.
- x and y are not multiplied together.
- x and y do not appear in denominators, exponents, or radical signs.

Linear		Not Linear	
$3x + 2y = 10$	*Standard form*	$3xy + x = 1$	*x and y are multiplied.*
$y - 2 = 3x$	*Can be written as* $3x - y = -2$	$x^3 + y = -1$	*x has an exponent other than 1.*
$-y = 5x$	*Can be written as* $5x + y = 0$	$x + \dfrac{6}{y} = 12$	*y is in a denominator.*

For any two points, there is exactly one line that contains them both. This means you need only two ordered pairs to graph a line.

EXAMPLE 3 **Graphing Linear Functions**

Tell whether each function is linear. If so, graph the function.

A $y = x + 3$

$$y = x + 3 \qquad \textit{Write the equation in standard form.}$$
$$\underline{-x \quad -x} \qquad \textit{Subtraction Property of Equality}$$
$$y - x = 3$$
$$-x + y = 3 \qquad \textit{The equation is in standard form } (A = -1, B = 1, C = 3).$$

> **Remember!**
> - $y - x = y + (-x)$
> - $y + (-x) = -x + y$
> - $-x = -1x$
> - $y = 1y$

The equation can be written in standard form, so the function is linear.

To graph, choose three values of x, and use them to generate ordered pairs. (You only need two, but graphing three points is a good check.)

Plot the points and connect them with a straight line.

x	$y = x + 3$	(x, y)
0	$y = 0 + 3 = 3$	$(0, 3)$
1	$y = 1 + 3 = 4$	$(1, 4)$
2	$y = 2 + 3 = 5$	$(2, 5)$
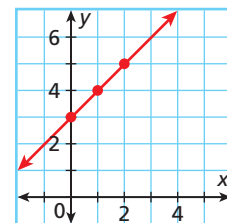
B $y = x^2$

This is not linear, because x has an exponent other than 1.

> **CHECK IT OUT!** Tell whether each function is linear. If so, graph the function.
> **3a.** $y = 5x - 9$ **3b.** $y = 12$ **3c.** $y = 2^x$

For linear functions whose graphs are not horizontal, the domain and range are all real numbers. However, in many real-world situations, the domain and range must be restricted. For example, some quantities cannot be negative, such as time.

Sometimes domain and range are restricted even further to a set of points. For example, a quantity such as number of people can only be whole numbers. When this happens, the graph is not actually connected because every point on the line is not a solution. However, you may see these graphs shown connected to indicate that the linear pattern, or trend, continues.
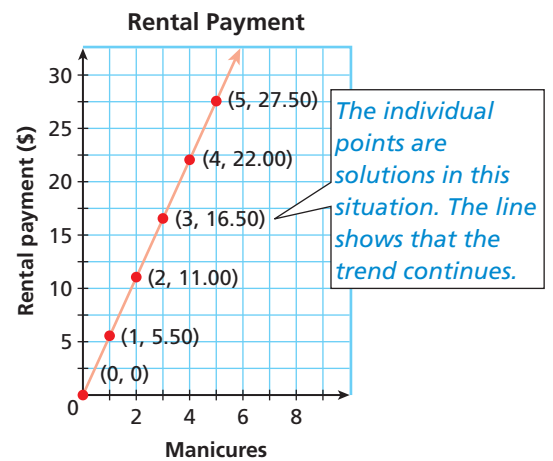
E X A M P L E **4** *Career Application*

Sue rents a manicure station in a salon and pays the salon owner $5.50 for each manicure she gives. The amount Sue pays each day is given by $f(x) = 5.50x$, where x is the number of manicures. Graph this function and give its domain and range.

Choose several values of x and make a table of ordered pairs.

Graph the ordered pairs.

x	$f(x) = 5.50x$
0	$f(0) = 5.50(0) = 0$
1	$f(1) = 5.50(1) = 5.50$
2	$f(2) = 5.50(2) = 11.00$
3	$f(3) = 5.50(3) = 16.50$
4	$f(4) = 5.50(4) = 22.00$
5	$f(5) = 5.50(5) = 27.50$

> **Remember!**
>
> $f(x) = y$, so in Example 4, graph the function values (dependent variable) on the y-axis.

Rental Payment

The individual points are solutions in this situation. The line shows that the trend continues.

The number of manicures must be a whole number, so the domain is $\{0, 1, 2, 3, ...\}$. The range is $\{0, 5.50, 11.00, 16.50, ...\}$.

CHECK IT OUT!

4. **What if...?** At another salon, Sue can rent a station for $10.00 per day plus $3.00 per manicure. The amount she would pay each day is given by $f(x) = 3x + 10$, where x is the number of manicures. Graph this function and give its domain and range.

THINK AND DISCUSS

1. Suppose you are given five ordered pairs that satisfy a function. When you graph them, four lie on a straight line, but the fifth does not. Is the function linear? Why or why not?

2. In Example 4, why is every point on the line not a solution?

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe how to use the information to identify a linear function. Include an example.

Determining Whether a Function Is Linear

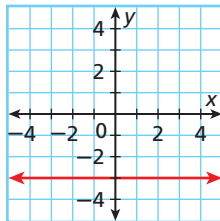| From its graph | From its equation | From a list of ordered pairs |

GUIDED PRACTICE

1. **Vocabulary** Is the *linear equation* $3x - 2 = y$ in standard form? Explain.
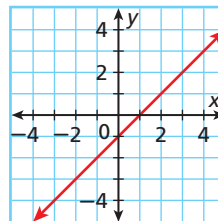
SEE EXAMPLE 1
p. 300

Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?
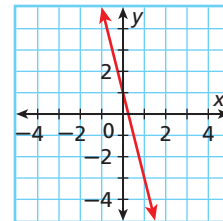
2.

3.

4.

SEE EXAMPLE 2
p. 301

Tell whether the given ordered pairs satisfy a linear function. Explain.

5.

x	5	4	3	2	1
y	0	2	4	6	8

6.

x	1	4	9	16	25
y	1	2	3	4	5

7. $\{(0, 5), (-2, 3), (-4, 1), (-6, -1), (-8, -3)\}$

8. $\{(2, -2), (-1, 0), (-4, 1), (-7, 3), (-10, 6)\}$

SEE EXAMPLE 3
p. 302

Tell whether each function is linear. If so, graph the function.

9. $2x + 3y = 5$

10. $2y = 8$

11. $\dfrac{x^2 + 3}{5} = y$

12. $\dfrac{x}{5} = \dfrac{y}{3}$

SEE EXAMPLE 4
p. 303

13. **Transportation** A train travels at a constant speed of 75 mi/h. The function $f(x) = 75x$ gives the distance that the train travels in x hours. Graph this function and give its domain and range.

14. **Entertainment** A movie rental store charges a $6.00 membership fee plus $2.50 for each movie rented. The function $f(x) = 2.50x + 6$ gives the cost of renting x movies. Graph this function and give its domain and range.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
15–17	1
18–20	2
21–24	3
25	4

Extra Practice

Skills Practice p. S12
Application Practice p. S32

Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?

15.

16.

17.
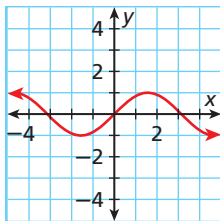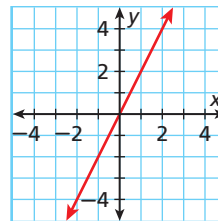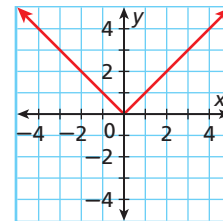
Tell whether the given ordered pairs satisfy a linear function. Explain.

18.

x	−3	0	3	6	9
y	−2	−1	0	2	4

19.

x	−1	0	1	2	3
y	−3	−2	−1	0	1

20. $\{(3, 4), (0, 2), (-3, 0), (-6, -2), (-9, -4)\}$

Tell whether each function is linear. If so, graph the function.

21. $y = 5$
22. $4y - 2x = 0$
23. $\frac{3}{x} + 4y = 10$
24. $5 + 3y = 8$

25. Transportation The gas tank in Tony's car holds 15 gallons, and the car can travel 25 miles for each gallon of gas. When Tony begins with a full tank of gas, the function $f(x) = -\frac{1}{25}x + 15$ gives the amount of gas $f(x)$ that will be left in the tank after traveling x miles (if he does not buy more gas). Graph this function and give its domain and range.

Tell whether the given ordered pairs satisfy a function. If so, is it a linear function?

26. $\{(2, 5), (2, 4), (2, 3), (2, 2), (2, 1)\}$
27. $\{(-8, 2), (-6, 0), (-4, -2), (-2, -4), (0, -6)\}$

28.

x	-10	-6	-2	2	4
y	0	0.25	0.50	0.75	1

29.

x	-5	-1	3	7	11
y	1	1	1	1	1

Tell whether each equation is linear. If so, write the equation in standard form and give the values of A, B, and C.

30. $2x - 8y = 16$
31. $y = 4x + 2$
32. $2x = \frac{y}{3} - 4$
33. $\frac{4}{x} = y$

34. $\frac{x+4}{2} = \frac{y-4}{3}$
35. $x = 7$
36. $xy = 6$
37. $3x - 5 + y = 2y - 4$

38. $y = -x + 2$
39. $5x = 2y - 3$
40. $2y = -6$
41. $y = \sqrt{x}$

Graph each linear function.

42. $y = 3x + 7$
43. $y = x + 25$
44. $y = 8 - x$
45. $y = 2x$

46. $-2y = -3x + 6$
47. $y - x = 4$
48. $y - 2x = -3$
49. $x = 5 + y$

50. Measurement One inch is equal to approximately 2.5 centimeters. Let x represent inches and y represent centimeters. Write an equation in standard form relating x and y. Give the values of A, B, and C.

51. Wages Molly earns \$8.00 an hour at her job.
   a. Let x represent the number of hours that Molly works. Write a function using x and $f(x)$ that describes Molly's pay for working x hours.
   b. Graph this function and give its domain and range.

52. Write About It For $y = 2x - 1$, make a table of ordered pairs and a graph. Describe the relationships between the equation, the table, and the graph.

53. Critical Thinking Describe a real-world situation that can be represented by a linear function whose domain and range must be limited. Give your function and its domain and range.

MULTI-STEP TEST PREP

54. This problem will prepare you for the Multi-Step Test Prep on page 342.
   a. Juan is running on a treadmill. The table shows the number of Calories Juan burns as a function of time. Explain how you can tell that this relationship is linear by using the table.
   b. Create a graph of the data.
   c. How can you tell from the graph that the relationship is linear?

Time (min)	Calories
3	27
6	54
9	81
12	108
15	135
18	162
21	189

55. Physical Science A ball was dropped from a height of 100 meters. Its height above the ground in meters at different times after its release is given in the table. Do these ordered pairs satisfy a linear function? Explain.

Time (s)	0	1	2	3
Height (m)	100	90.2	60.8	11.8

56. Critical Thinking Is the equation $x = 9$ a linear equation? Does it describe a linear function? Explain.

57. Which is NOT a linear function?

Ⓐ $y = 8x$ Ⓑ $y = x + 8$ Ⓒ $y = \dfrac{8}{x}$ Ⓓ $y = 8 - x$

58. The speed of sound in 0 °C air is about 331 feet per second. Which function could be used to describe the distance in feet d that sound will travel in air in s seconds?

Ⓕ $d = s + 331$ Ⓖ $d = 331s$ Ⓗ $s = 331d$ Ⓙ $s = 331 - d$

59. Extended Response Write your own linear function. Show that it is a linear function in at least three different ways. Explain any connections you see between your three methods.

CHALLENGE AND EXTEND

60. What equation describes the x-axis? the y-axis? Do these equations represent linear functions?

Geometry Copy and complete each table below. Then tell whether the table shows a linear relationship.

61.

Perimeter of a Square	
Side Length	Perimeter
1	■
2	■
3	■
4	■

62.

Area of a Square	
Side Length	Area
1	■
2	■
3	■
4	■

63.

Volume of a Cube	
Side Length	Volume
1	■
2	■
3	■
4	■

SPIRAL REVIEW

Simplify each expression. *(Lesson 1-4)*

64. 8^2 **65.** $(-1)^3$ **66.** $(-4)^4$ **67.** $\left(\dfrac{1}{3}\right)^2$

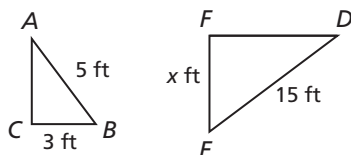**Solve each equation. Check your answer.** *(Lesson 2-4)*

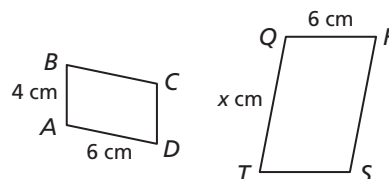**68.** $6m + 5 = 3m - 4$ **69.** $2(t - 4) = 3 - (3t + 1)$ **70.** $9y + 5 - 2y = 2y + 5 - y + 3$

Find the value of x in each diagram. *(Lesson 2-8)*

71. $\triangle ABC \sim \triangle DEF$

72. $ABCD \sim QRST$

5-2 Using Intercepts

The x-intercept is 4. It represents the time that the diver reaches the surface, or when depth = 0.

Objectives
Find x- and y-intercepts and interpret their meanings in real-world situations.

Use x- and y-intercepts to graph lines.

Vocabulary
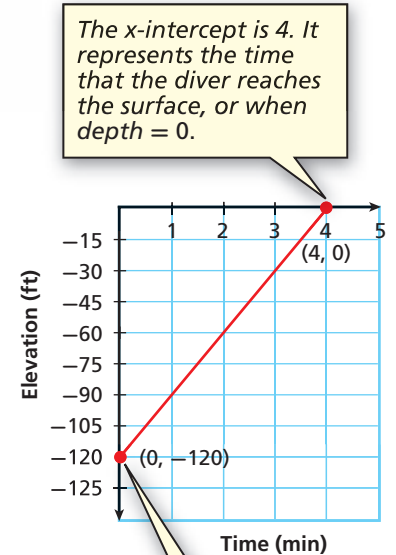y-intercept
x-intercept

Who uses this?
Divers can use intercepts to determine the time a safe ascent will take.

A diver explored the ocean floor 120 feet below the surface and then ascended at a rate of 30 feet per minute. The graph shows the diver's elevation below sea level during the ascent.

The **y-intercept** is the y-coordinate of the point where the graph intersects the y-axis. The x-coordinate of this point is always 0.

The **x-intercept** is the x-coordinate of the point where the graph intersects the x-axis. The y-coordinate of this point is always 0.

The y-intercept is −120. It represents the diver's elevation at the start of the ascent, when time = 0.

EXAMPLE 1 **Finding Intercepts**

Find the x- and y-intercepts.

A
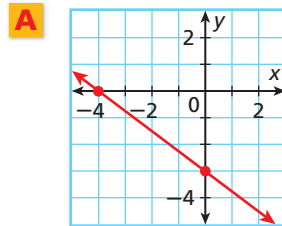
The graph intersects the x-axis at (−4, 0).
The x-intercept is −4.

The graph intersects the y-axis at (0, −3).
The y-intercept is −3.

B $3x - 2y = 12$

To find the x-intercept, replace y with 0 and solve for x.

$$3x - 2y = 12$$
$$3x - 2(0) = 12$$
$$3x - 0 = 12$$
$$3x = 12$$
$$\frac{3x}{3} = \frac{12}{3}$$
$$x = 4$$

The x-intercept is 4.

To find the y-intercept, replace x with 0 and solve for y.

$$3x - 2y = 12$$
$$3(0) - 2y = 12$$
$$0 - 2y = 12$$
$$-2y = 12$$
$$\frac{-2y}{-2} = \frac{12}{-2}$$
$$y = -6$$

The y-intercept is −6.

CHECK IT OUT!

Find the x- and y-intercepts.

1a.

1b. $-3x + 5y = 30$

1c. $4x + 2y = 16$

Finding Intercepts

I use the "cover-up" method to find intercepts. To use this method, make sure the equation is in standard form first.

If I have $4x - 3y = 12$:

First, I cover $4x$ with my finger and solve the equation I can still see.

$$\text{✋} - 3y = 12$$
$$y = -4$$

The y-intercept is -4.

Then I cover $-3y$ with my finger and do the same thing.

$$4x \, \text{✋} = 12$$
$$x = 3$$

The x-intercept is 3.
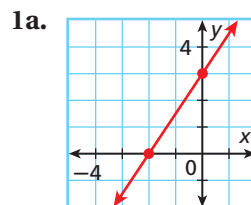
Madison Stewart
Jefferson High School

**EXAMPLE 2**

Travel Application

The Sandia Peak Tramway in Albuquerque, New Mexico, travels a distance of about 4500 meters to the top of Sandia Peak. Its speed is 300 meters per minute. The function $f(x) = 4500 - 300x$ gives the tram's distance in meters from the top of the peak after x minutes. Graph this function and find the intercepts. What does each intercept represent?

Neither time nor distance can be negative, so choose several nonnegative values for x. Use the function to generate ordered pairs.

x	0	2	5	10	15
$f(x) = 4500 - 300x$	4500	3900	3000	1500	0

Graph the ordered pairs. Connect the points with a line.

Sandia Peak Tramway
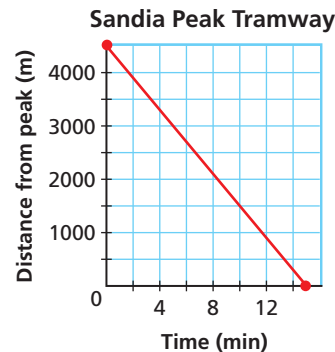
- y-intercept: 4500. This is the starting distance from the top (time = 0).
- x-intercept: 15. This the time when the tram reaches the peak (distance = 0).

Caution!

The graph is not the path of the tram. Even though the line is descending, the graph describes the distance from the peak as the tram goes *up* the mountain.

CHECK IT OUT!

2. The school store sells pens for $2.00 and notebooks for $3.00. The equation $2x + 3y = 60$ describes the number of pens x and notebooks y that you can buy for $60.

   a. Graph the function and find its intercepts.

   b. What does each intercept represent?

Remember, to graph a linear function, you need to plot only two ordered pairs. It is often simplest to find the ordered pairs that contain the intercepts.

EXAMPLE **3** **Graphing Linear Equations by Using Intercepts**

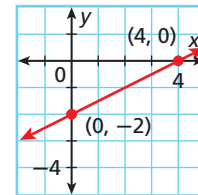Use intercepts to graph the line described by each equation.

A $2x - 4y = 8$

Step 1 Find the intercepts.

x-intercept:	y-intercept:
$2x - 4y = 8$	$2x - 4y = 8$
$2x - 4(0) = 8$	$2(0) - 4y = 8$
$2x = 8$	$-4y = 8$
$\dfrac{2x}{2} = \dfrac{8}{2}$	$\dfrac{-4y}{-4} = \dfrac{8}{-4}$
$x = 4$	$y = -2$

Step 2 Graph the line.
Plot (4, 0) and (0, −2).
Connect with a straight line.

Helpful Hint

You can use a third point to check your line. Either choose a point from your graph and check it in the equation, or use the equation to generate a point and check that it is on your graph.

B $\dfrac{2}{3}y = 4 - \dfrac{1}{2}x$

Step 1 Write the equation in standard form.

$$6\left(\dfrac{2}{3}y\right) = 6\left(4 - \dfrac{1}{2}x\right) \qquad \text{Multiply both sides by 6, the LCD of the}$$
$$4y = 24 - 3x \qquad\qquad \text{fractions, to clear the fractions.}$$
$$3x + 4y = 24 \qquad\qquad \text{Write the equation in standard form.}$$

Step 2 Find the intercepts.

x-intercept:	y-intercept:
$3x + 4y = 24$	$3x + 4y = 24$
$3x + 4(0) = 24$	$3(0) + 4y = 24$
$3x = 24$	$4y = 24$
$\dfrac{3x}{3} = \dfrac{24}{3}$	$\dfrac{4y}{4} = \dfrac{24}{4}$
$x = 8$	$y = 6$

Step 3 Graph the line.
Plot (8, 0) and (0, 6).
Connect with a straight line.
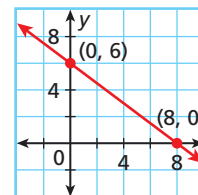
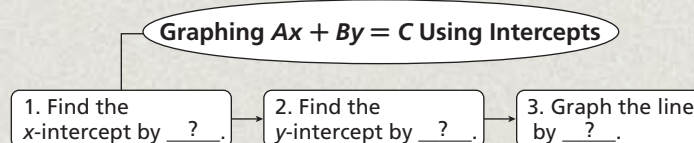**CHECK IT OUT!** Use intercepts to graph the line described by each equation.

3a. $-3x + 4y = -12$ **3b.** $y = \dfrac{1}{3}x - 2$

THINK AND DISCUSS

1. A function has x-intercept 4 and y-intercept 2. Name two points on the graph of this function.

2. What is the y-intercept of $2.304x + y = 4.318$? What is the x-intercept of $x - 92.4920y = -21.5489$?

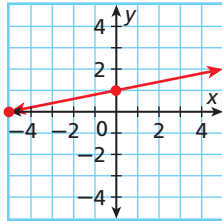**3. GET ORGANIZED** Copy and complete the graphic organizer.

Graphing $Ax + By = C$ Using Intercepts

1. Find the x-intercept by __?__.	2. Find the y-intercept by __?__.	3. Graph the line by __?__.

GUIDED PRACTICE

1. **Vocabulary** The _____?_____ is the y-coordinate of the point where a graph crosses the y-axis. (*x-intercept* or *y-intercept*)
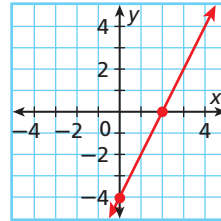
SEE EXAMPLE 1
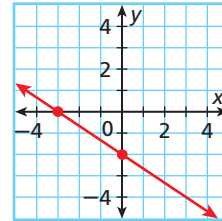p. 307

Find the x- and y-intercepts.

2.

3.

4.

5. $2x - 4y = 4$

6. $-2y = 3x - 6$

7. $4y + 5x = 2y - 3x + 16$

SEE EXAMPLE 2
p. 308

8. **Biology** To thaw a specimen stored at -25 °C, the temperature of a refrigeration tank is raised 5 °C every hour. The temperature in the tank after x hours can be described by the function $f(x) = -25 + 5x$.

  a. Graph the function and find its intercepts.

  b. What does each intercept represent?

SEE EXAMPLE 3
p. 309

Use intercepts to graph the line described by each equation.
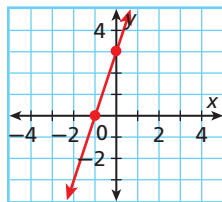
9. $4x - 5y = 20$
10. $y = 2x + 4$
11. $\frac{1}{3}x - \frac{1}{4}y = 2$
12. $-5y + 2x = -10$

PRACTICE AND PROBLEM SOLVING

Find the x- and y-intercepts.

13.

14.
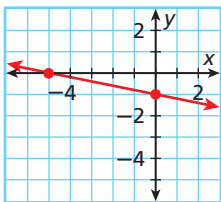
15.
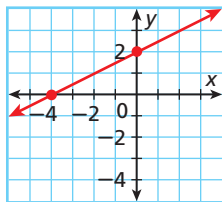
16. $6x + 3y = 12$
17. $4y - 8 = 2x$
18. $-2y + x = 2y - 8$

19. $4x + y = 8$
20. $y - 3x = -15$
21. $2x + y = 10x - 1$

22. **Environmental Science** A fishing lake was stocked with 300 bass. Each year, the population decreases by 25. The population of bass in the lake after x years is represented by the function $f(x) = 300 - 25x$.

  a. Graph the function and find its intercepts.

  b. What does each intercept represent?

23. **Sports** Julie is running a 5-kilometer race. She runs 1 kilometer every 5 minutes. Julie's distance from the finish line after x minutes is represented by the function $f(x) = 5 - \frac{1}{5}x$.

  a. Graph the function and find its intercepts.

  b. What does each intercept represent?

Use intercepts to graph the line described by each equation.

24. $4x - 6y = 12$

25. $2x + 3y = 18$

26. $\frac{1}{2}x - 4y = 4$

27. $y - x = -1$

28. $5x + 3y = 15$

29. $x - 3y = -1$

Bamboo is the world's fastest-growing woody plant. Some varieties can grow more than 30 centimeters a day and up to 40 meters tall.

30. Biology A bamboo plant is growing 1 foot per day. When you first measure it, it is 4 feet tall.

  a. Write an equation to describe the height y, in feet, of the bamboo plant x days after you measure it.

  b. What is the y-intercept?

  c. What is the meaning of the y-intercept in this problem?

31. Estimation Look at the scatter plot and trend line.

  a. Estimate the x- and y-intercepts.

  b. What is the real-world meaning of each intercept?

Tropical Forests

32. Personal Finance A bank employee notices an abandoned checking account with a balance of $412. If the bank charges a $4 monthly fee for the account, the function $b = 412 - 4m$ shows the balance b in the account after m months.

  a. Graph the function and give its domain and range. (*Hint:* The bank will keep charging the monthly fee even after the account is empty.)

  b. Find the intercepts. What does each intercept represent?

  c. When will the bank account balance be 0?

33. Critical Thinking Complete the following to learn about intercepts and horizontal and vertical lines.

  a. Graph $x = -6$, $x = 1$, and $x = 5$. Find the intercepts.

  b. Graph $y = -3$, $y = 2$, and $y = 7$. Find the intercepts.

  c. Write a rule describing the intercepts of linear equations whose graphs are horizontal and vertical lines.
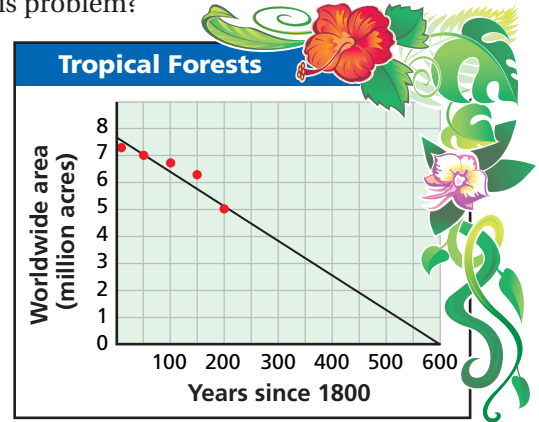
Match each equation with a graph.

34. $-2x - y = 4$ **35.** $y = 4 - 2x$ **36.** $2y + 4x = 8$ **37.** $4x - 2y = 8$

A.

B.

C.

D.

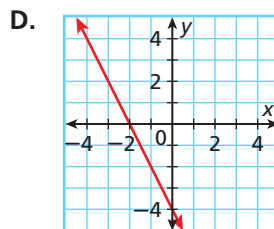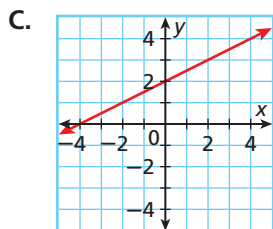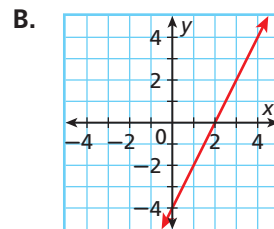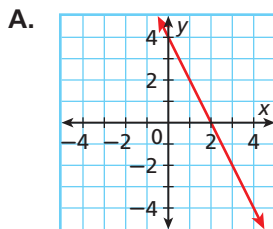**38.** This problem will prepare you for the Multi-Step Test Prep on page 342.

Kristyn rode a stationary bike at the gym. She programmed the timer for 20 minutes. The display counted backward to show how much time remained in her workout. It also showed her mileage.
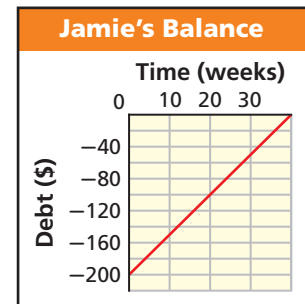
a. What are the intercepts?

b. What do the intercepts represent?

Time Remaining (min)	Distance Covered (mi)
20	0
16	0.35
12	0.70
8	1.05
4	1.40
0	1.75

39. **Write About It** Write a real-world problem that could be modeled by a linear function whose x-intercept is 5 and whose y-intercept is 60.

40. Which is the x-intercept of $-2x = 9y - 18$?

(A) −9   (B) −2   (C) 2   (D) 9

41. Which of the following situations could be represented by the graph?

(F) Jamie owed her uncle $200. Each week for 40 weeks she paid him $5.

(G) Jamie owed her uncle $200. Each week for 5 weeks she paid him $40.

(H) Jamie owed her uncle $40. Each week for 5 weeks she paid him $200.

(J) Jamie owed her uncle $40. Each week for 200 weeks she paid him $5.

Jamie's Balance

42. **Gridded Response** What is the y-intercept of $60x + 55y = 660$?

CHALLENGE AND EXTEND

Use intercepts to graph the line described by each equation.

43. $\frac{1}{2}x + \frac{1}{5}y = 1$ **44.** $0.5x - 0.2y = 0.75$ **45.** $y = \frac{3}{8}x + 6$

46. For any linear equation $Ax + By = C$, what are the intercepts?

47. Find the intercepts of $22x - 380y = 20,900$. Explain how to use the intercepts to determine appropriate scales for the graph.

SPIRAL REVIEW

48. Marlon's fish tank is 80% filled with water. Based on the measurements shown, what volume of the tank is NOT filled with water? *(Lesson 2-9)*
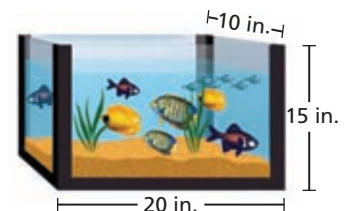
Solve each inequality and graph the solutions. *(Lesson 3-3)*

49. $3c > 12$ **50.** $-4 \geq \frac{t}{2}$ **51.** $\frac{1}{2}m \geq -3$ **52.** $-2w > 14$

Tell whether the given ordered pairs satisfy a linear function. Explain. *(Lesson 5-1)*

53. $\{(-2, 0), (0, 3), (2, 6), (4, 9), (6, 12)\}$ **54.** $\{(0, 0), (1, 1), (4, 2), (9, 3), (16, 4)\}$
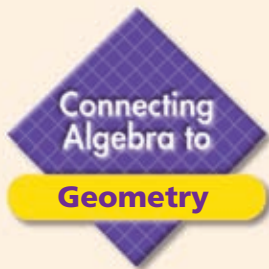
Area in the Coordinate Plane

Lines in the coordinate plane can form the sides of polygons. You can use points on these lines to help you find the areas of these polygons.

Example

Find the area of the triangle formed by the x-axis, the y-axis, and the line described by $3x + 2y = 18$.

Step 1 Find the intercepts of $3x + 2y = 18$.

x-intercept:

$3x + 2y = 18$
$3x + 2(0) = 18$
$3x = 18$
$x = 6$

y-intercept:

$3x + 2y = 18$
$3(0) + 2y = 18$
$2y = 18$
$y = 9$

Step 2 Use the intercepts to graph the line. The x-intercept is 6, so plot $(6, 0)$. The y-intercept is 9, so plot $(0, 9)$. Connect with a straight line. Then shade the triangle formed by the line and the axes, as described.

Step 3 Recall that the area of a triangle is given by $A = \frac{1}{2}bh$.

• The length of the base is **6**.

• The height is **9**.

Step 4 Substitute these values into the formula.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(6)(9)$ *Substitute into the area formula.*

$= \frac{1}{2}(54)$ *Simplify.*

$= 27$

The area of the triangle is 27 square units.
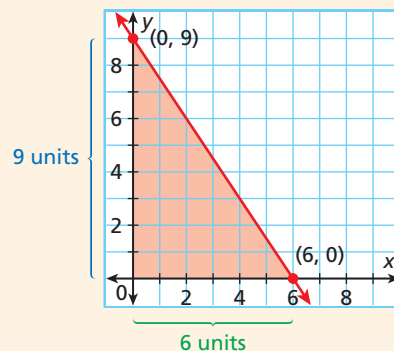
Try This

1. Find the area of the triangle formed by the x-axis, the y-axis, and the line described by $3x + 2y = 12$.

2. Find the area of the triangle formed by the x-axis, the y-axis, and the line described by $y = 6 - x$.

3. Find the area of the polygon formed by the x-axis, the y-axis, the line described by $y = 6$, and the line described by $x = 4$.

Rate of Change and Slope

Objectives
Find rates of change and slopes.

Relate a constant rate of change to the slope of a line.

Vocabulary
rate of change
rise
run
slope

Why learn this?
Rates of change can be used to find how quickly costs have increased.

In 1985, the cost of sending a 1-ounce letter was 22 cents. In 1988, the cost was 25 cents. How fast did the cost change from 1985 to 1988? In other words, at what *rate* did the cost change?

A **rate of change** is a ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable.

$$\text{rate of change} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

EXAMPLE 1   *Consumer Application*

The table shows the cost of mailing a 1-ounce letter in different years. Find the rate of change in cost for each time interval. During which time interval did the cost increase at the greatest rate?

Year	1988	1990	1991	2004	2008
Cost (¢)	25	25	29	37	42

Step 1 Identify the dependent and independent variables.

dependent: cost   **independent: year**

Step 2 Find the rates of change.

1988 to 1990 $\dfrac{\text{change in cost}}{\text{change in years}} = \dfrac{25 - 25}{1990 - 1988} = \dfrac{0}{2} = 0$ *0 cents / year*

1990 to 1991 $\dfrac{\text{change in cost}}{\text{change in years}} = \dfrac{29 - 25}{1991 - 1990} = \dfrac{4}{1} = 4$ *4 cents / year*

1991 to 2004 $\dfrac{\text{change in cost}}{\text{change in years}} = \dfrac{37 - 29}{2004 - 1991} = \dfrac{8}{13} \approx 0.62 \approx$ *0.62 cents / year*

2004 to 2008 $\dfrac{\text{change in cost}}{\text{change in years}} = \dfrac{42 - 37}{2008 - 2004} = \dfrac{5}{4} = 1.25$ *1.25 cents / year*

The cost increased at the greatest rate from 1990 to 1991.

Caution!
A rate of change of 1.25 cents per year for a 4-year period means that the *average* change was 1.25 cents per year. The *actual* change in each year may have been different.

CHECK IT OUT!

1. The table shows the balance of a bank account on different days of the month. Find the rate of change for each time interval. During which time interval did the balance decrease at the greatest rate?

Day	1	6	16	22	30
Balance ($)	550	285	210	210	175

EXAMPLE 2 Finding Rates of Change from a Graph

Graph the data from Example 1 and show the rates of change.

Postage Costs
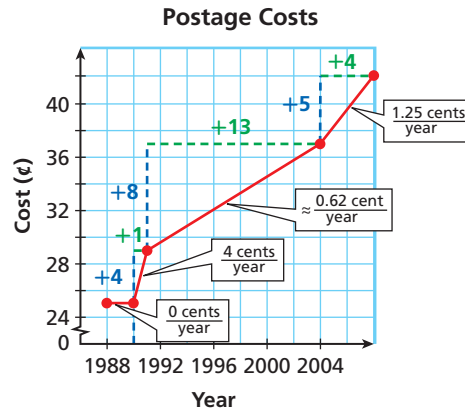
Graph the ordered pairs. The vertical blue segments show the changes in the dependent variable, and the horizontal green segments show the changes in the independent variable.

Notice that the greatest rate of change is represented by the steepest of the red line segments.

Also notice that between 1988 and 1990, when the cost did not change, the red line segment is horizontal.

CHECK IT OUT!
2. Graph the data from Check It Out Problem 1 and show the rates of change.

If all of the connected segments have the same rate of change, then they all have the same steepness and together form a straight line. The constant rate of change of a nonvertical line is called the *slope* of the line.

Know it! Note

Slope of a Line

The **rise** is the difference in the **y-values** of two points on a line.

The **run** is the difference in the **x-values** of two points on a line.

The **slope** of a line is the ratio of rise to run for any two points on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

(Remember that *y* is the **dependent variable** and *x* is the **independent variable**.)

Run = 2 Rise = 3 Run = 2 Rise = 3 Run = 2 Rise = 3 **Slope** $= \frac{3}{2}$
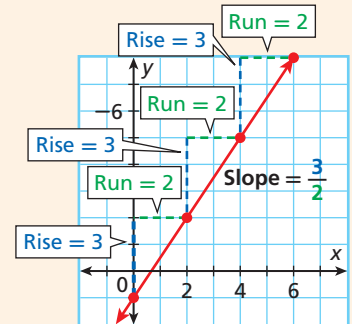
EXAMPLE 3 Finding Slope

Find the slope of the line.

Caution!

Pay attention to the scales on the axes. One square on the grid may not represent 1 unit. In Example 3, each square represents $\frac{1}{2}$ unit.

Run = 1 (2, 3) Rise = 2 Rise = −2 (1, 1) Run = −1
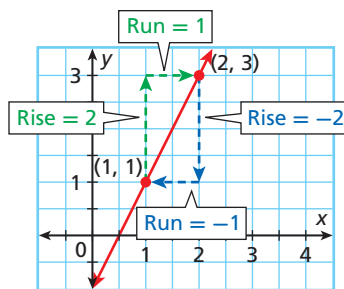
Begin at one point and count vertically to find the rise.

Then count horizontally to the second point to find the run.

It does not matter which point you start with. The slope is the same.

$$\text{slope} = \frac{2}{1} = 2$$

$$\text{slope} = \frac{-2}{-1} = 2$$

CHECK IT OUT!
3. Find the slope of the line that contains $(0, -3)$ and $(5, -5)$.

Finding Slopes of Horizontal and Vertical Lines

Find the slope of each line.

A
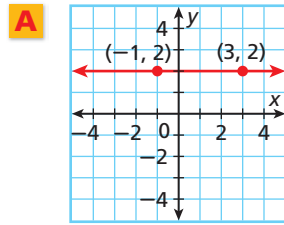
$$\frac{\text{rise}}{\text{run}} = \frac{0}{4} = 0$$

The slope is 0.

B
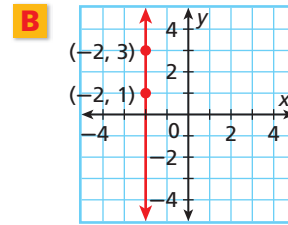
$$\frac{\text{rise}}{\text{run}} = \frac{2}{0}$$ *You cannot divide by 0.*

The slope is undefined.

CHECK IT OUT! Find the slope of each line.

4a.

4b.
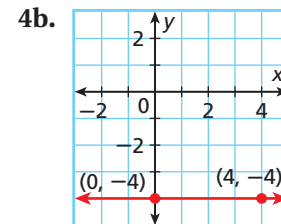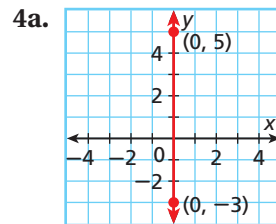
As shown in the previous examples, slope can be positive, negative, zero, or undefined. You can tell which of these is the case by looking at the graph of a line—you do not need to calculate the slope.

Know it! Note

Positive Slope	Negative Slope	Zero Slope	Undefined Slope
Line rises from left to right.	Line falls from left to right.	Horizontal line	Vertical line

E X A M P L E **5** **Describing Slope**

Tell whether the slope of each line is positive, negative, zero, or undefined.

A
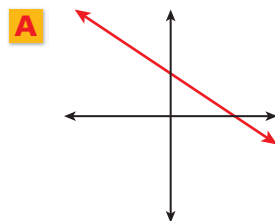
The line falls from left to right.

The slope is negative.

B
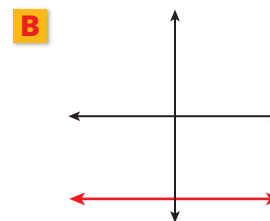
The line is horizontal.

The slope is 0.

CHECK IT OUT! Tell whether the slope of each line is positive, negative, zero, or undefined.

5a.
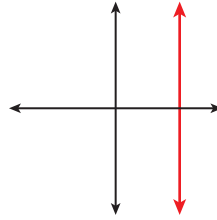
5b.
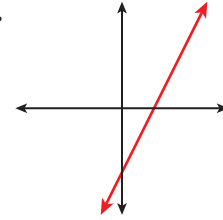
A line's slope is a measure of its steepness. Some lines are steeper than others. As the absolute value of the slope increases, the line becomes steeper.
As the absolute value of the slope decreases, the line becomes less steep.

Comparing Slopes		
Slope $= \frac{1}{2}$ Slope $= 4$	Slope $= -1$ Slope $= -2$	Slope $= -3$ Slope $= \frac{3}{4}$
The line with slope **4** is steeper than the line with slope $\frac{1}{2}$.	The line with slope **−2** is steeper than the line with slope **−1**.	The line with slope **−3** is steeper than the line with slope $\frac{3}{4}$.
$\|4\| > \left\|\frac{1}{2}\right\|$	$\|-2\| > \|-1\|$	$\|-3\| > \left\|\frac{3}{4}\right\|$

THINK AND DISCUSS

1. What is the rise shown in the graph? What is the run? What is the slope?

2. The rate of change of the profits of a company over one year is negative. How have the profits of the company changed over that year?

3. Would you rather climb a hill with a slope of 4 or a hill with a slope of $\frac{5}{2}$? Explain your answer.
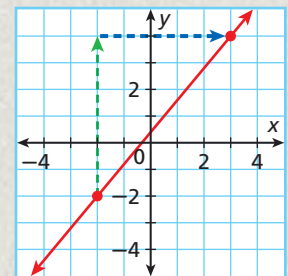
4. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, sketch a line whose slope matches the given description.

Know it! Note

Slope

Positive   Negative   Zero   Undefined
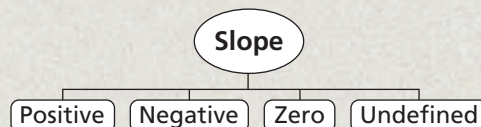
GUIDED PRACTICE

1. **Vocabulary** The *slope* of any nonvertical line is ___?___. (*positive* or *constant*)

SEE EXAMPLE 1
p. 314

2. The table shows the volume of gasoline in a gas tank at different times. Find the rate of change for each time interval. During which time interval did the volume decrease at the greatest rate?

Time (h)	0	1	3	6	7
Volume (gal)	12	9	5	1	1

SEE EXAMPLE 2
p. 315

3. The table shows a person's heart rate over time. Graph the data and show the rates of change.

Time (min)	0	2	5	7	10
Heart Rate (beats/min)	64	92	146	84	64

Find the slope of each line.

SEE EXAMPLE 3
p. 315

4.

5.

SEE EXAMPLE 4
p. 316

6.

7.
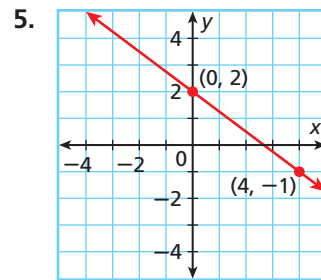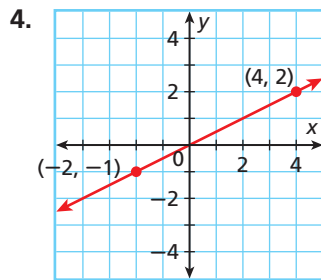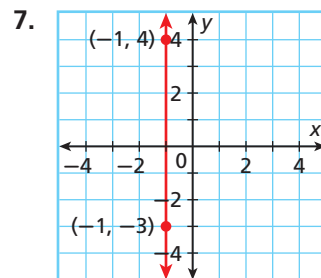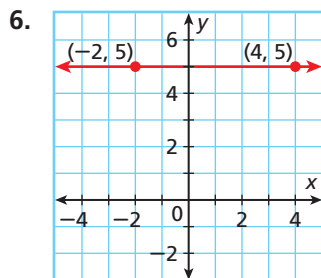
SEE EXAMPLE 5
p. 316

Tell whether the slope of each line is positive, negative, zero, or undefined.

8.

9.

10.

11.
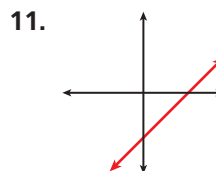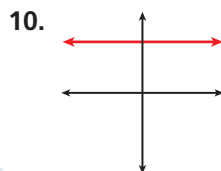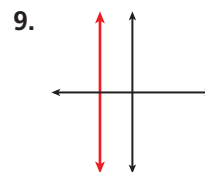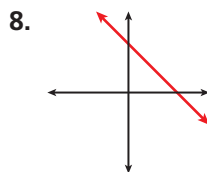
PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
12	1
13	2
14–15	3
16–17	4
18–19	5

Extra Practice

Skills Practice p. S12

Application Practice p. S32

12. The table shows the length of a baby at different ages. Find the rate of change for each time interval. Round your answers to the nearest tenth. During which time interval did the baby have the greatest growth rate?

Age (mo)	3	9	18	26	33
Length (in.)	23.5	27.5	31.6	34.5	36.7

13. The table shows the distance of an elevator from the ground floor at different times. Graph the data and show the rates of change.

Time (s)	0	15	23	30	35
Distance (m)	30	70	0	45	60

Find the slope of each line.
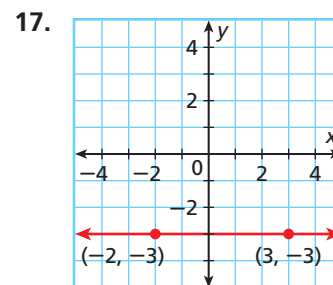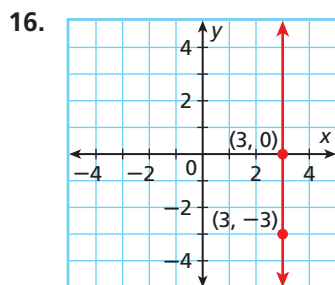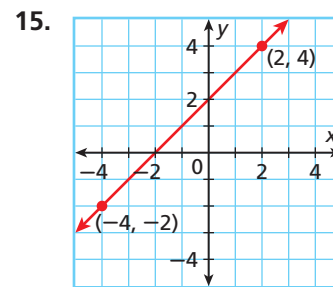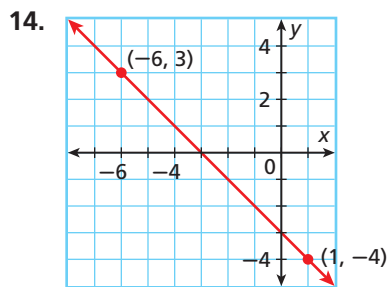
14.

15.

16.

17.

Tell whether the slope of each line is positive, negative, zero, or undefined.

18.

19.
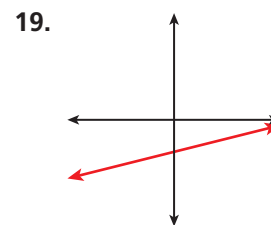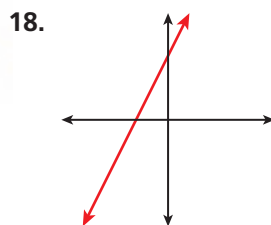
Travel

The Incline Railway's climb up Lookout Mountain has been called "America's Most Amazing Mile." A round-trip on the railway lasts about 1.5 hours.

20. Travel The Lookout Mountain Incline Railway in Chattanooga, Tennessee, is the steepest passenger railway in the world. A section of the railway has a slope of about 0.73. In this section, a vertical change of 1 unit corresponds to a horizontal change of what length? Round your answer to the nearest hundredth.

21. Critical Thinking In Lesson 5-1, you learned that in a linear function, a constant change in x corresponds to a constant change in y. How is this related to slope?

5-3 Rate of Change and Slope **319**

MULTI-STEP TEST PREP

22. This problem will prepare you for the Multi-Step Test Prep on page 342.

a. The graph shows a relationship between a person's age and his or her estimated maximum heart rate in beats per minute. Find the slope.

b. Describe the rate of change in this situation.

Estimated Maximum Heart Rate

Maximum heart rate (beats/min) vs. Age (yr). Points shown: (20, 200) and (60, 160).
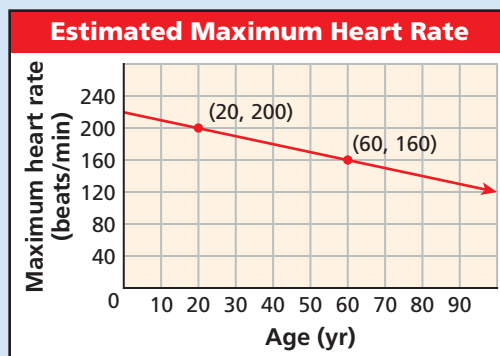
23. Construction Most staircases in use today have 9-inch treads and $8\frac{1}{2}$-inch risers. What is the slope of a staircase with these measurements?

24. A ladder is leaned against a building. The bottom of the ladder is 9 feet from the building. The top of the ladder is 16 feet above the ground.

a. Draw a diagram to represent this situation.

b. What is the slope of the ladder?

Tread

Riser
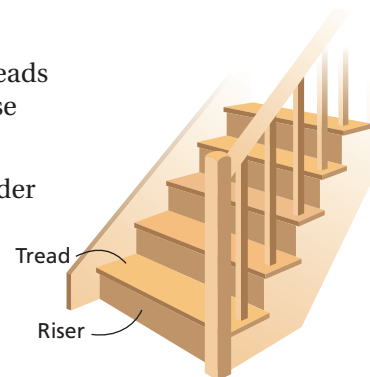
25. Write About It Why will the slope of any horizontal line be 0? Why will the slope of any vertical line be undefined?

26. The table shows the distance traveled by a car during a five-hour road trip.

Time (h)	0	1	2	3	4	5
Distance (mi)	0	40	80	80	110	160

a. Graph the data and show the rates of change.

b. The rate of change represents the average speed. During which hour was the car's average speed the greatest?

27. Estimation The graph shows the number of files scanned by a computer virus detection program over time.

a. Estimate the coordinates of point A.

b. Estimate the coordinates of point B.

c. Use your answers from parts **a** and **b** to estimate the rate of change (in files per second) between points A and B.

Virus Scan

Files scanned vs. Time (s). Points A and B shown.
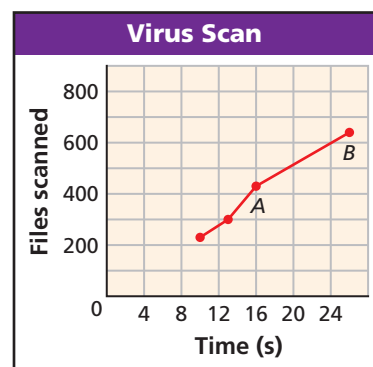
28. Data Collection Use a graphing calculator and a motion detector for the following. Set the equipment so that the graph shows distance on the y-axis and time on the x-axis.

a. Experiment with walking in front of the motion detector. How must you walk to graph a straight line? Explain.

b. Describe what you must do differently to graph a line with a positive slope vs. a line with a negative slope.

c. How can you graph a line with slope 0? Explain.

29. The slope of which line has the greatest absolute value?

(A) line A   (C) line C

(B) line B   (D) line D
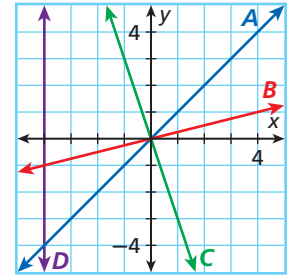
30. For which line is the run equal to 0?

(A) line A   (C) line C

(B) line B   (D) line D

31. Which line has a slope of 4?

(F)   (H)

(G)   (J)
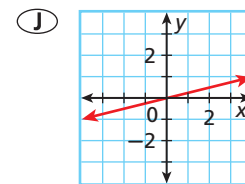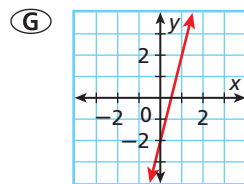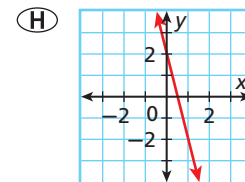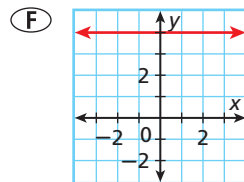
CHALLENGE AND EXTEND

32. Recreation Tara and Jade are hiking up a hill. Each has a different stride. The run for Tara's stride is 32 inches, and the rise is 8 inches. The run for Jade's stride is 36 inches. What is the rise of Jade's stride?

33. Economics The table shows cost in dollars charged by an electric company for various amounts of energy in kilowatt-hours.

Energy (kWh)	0	200	400	600	1000	2000
Cost ($)	3	3	31	59	115	150

   a. Graph the data and show the rates of change.

   b. Compare the rates of change for each interval. Are they all the same? Explain.

   c. What do the rates of change represent?

   d. Describe in words the electric company's billing plan.

SPIRAL REVIEW

Add or subtract. *(Lesson 1-2)*

34. $-5 + 15$ **35.** $9 - 11$ **36.** $-5 - (-25)$

Find the domain and range of each relation, and tell whether the relation is a function. *(Lesson 4-2)*

37. $\{(3, 4), (3, 2), (3, 0), (3, -2)\}$ **38.**

x	0	2	4	-2	-4
y	0	2	4	2	4

Find the x- and y-intercepts. *(Lesson 5-2)*

39. $2x + y = 6$ **40.** $y = -3x - 9$ **41.** $2y = -4x + 1$

Algebra LAB

Use with Lesson 5-3

Explore Constant Changes

There are many real-life situations in which the amount of change is constant. In these activities, you will explore what happens when

- a quantity increases by a constant amount.

- a quantity decreases by a constant amount.

Activity 1

Janice has read 7 books for her summer reading club. She plans to read 2 books each week for the rest of the summer. The table shows the total number of books that Janice will have read after different numbers of weeks have passed.

1 What number is added to the number of books in each row to get the number of books in the next row?

2 What does your answer to Problem 1 represent in Janice's situation? Describe the meaning of the constant change.

3 Graph the ordered pairs from the table. Describe how the points are related.

4 Look again at your answer to Problem 1. Explain how this number affects your graph.

Janice's Summer Reading	
Week	Total Books Read
0	7
1	9
2	11
3	13
4	15
5	17

Try This

At a particular college, a full-time student must take at least 12 credit hours per semester and may take up to 18 credit hours per semester. Tuition costs $200 per credit hour.

1. Copy and complete the table by using the information above.

2. What number is added to the cost in each row to get the cost in the next row?

3. What does your answer to Problem 2 above represent in the situation? Describe the meaning of the constant change.

4. Graph the ordered pairs from the table. Describe how the points are related.

5. Look again at your answer to Problem 2. Explain how this number affects your graph.

6. Compare your graphs from Activity 1 and Problem 4. How are they alike? How are they different?

7. **Make a Conjecture** Describe the graph of any situation that involves repeated addition of a positive number. Why do you think your description is correct?

Tuition Costs	
Credit Hours	Cost ($)
12	▪
13	▪
14	▪
15	▪
16	▪
17	▪
18	▪

Activity 2

An airplane is 3000 miles from its destination. The plane is traveling at a rate of 540 miles per hour. The table shows how far the plane is from its destination after various amounts of time have passed.

1 What number is subtracted from the distance in each row to get the distance in the next row?

2 What does your answer to Problem 1 represent in the situation? Describe the meaning of the constant change.

3 Graph the ordered pairs from the table. Describe how the points are related.

4 Look again at your answer to Problem 1. Explain how this number affects your graph.

Airplane's Distance	
Time (h)	Distance to Destination (mi)
0	3000
1	2460
2	1920
3	1380
4	840

Try This

A television game show begins with 20 contestants. Each week, the players vote 2 contestants off the show.

8. Copy and complete the table by using the information above.

9. What number is subtracted from the number of contestants in each row to get the number of contestants in the next row?

10. What does your answer to Problem 9 represent in the situation? Describe the meaning of the constant change.

11. Graph the ordered pairs from the table. Describe how the points are related.

12. Look again at your answer to Problem 9. Explain how this number affects your graph.

Game Show	
Week	Contestants Remaining
0	20
1	▨
2	▨
3	▨
4	▨
5	▨
6	▨

13. Compare your graphs from Activity 2 and Problem 11. How are they alike? How are they different?

14. **Make a Conjecture** Describe the graph of any situation that involves repeated subtraction of a positive number. Why do you think your description is correct?

15. Compare your two graphs from Activity 1 with your two graphs from Activity 2. How are they alike? How are they different?

16. **Make a Conjecture** How are graphs of situations involving repeated subtraction different from graphs of situations involving repeated addition? Explain your answer.

5-4 The Slope Formula

Objective
Find slope by using the slope formula.

Why learn this?
You can use the slope formula to find how quickly a quantity, such as the amount of water in a reservoir, is changing. (See Example 3.)

In Lesson 5-3, slope was described as the constant rate of change of a line. You saw how to find the slope of a line by using its graph.

There is also a formula you can use to find the slope of a line, which is usually represented by the letter m. To use this formula, you need the coordinates of two different points on the line.

Know it! Note

Slope Formula

WORDS	FORMULA	EXAMPLE
The slope of a line is the ratio of the difference in y-values to the difference in x-values between any two different points on the line.	If (x_1, y_1) and (x_2, y_2) are any two different points on a line, the slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1}$.	If $(2, -3)$ and $(1, 4)$ are two points on a line, the slope of the line is $m = \frac{4 - (-3)}{1 - 2} = \frac{7}{-1} = -7$.

EXAMPLE 1 Finding Slope by Using the Slope Formula

Find the slope of the line that contains $(4, -2)$ and $(-1, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{Use the slope formula.}$$

$$= \frac{2 - (-2)}{-1 - 4} \qquad \text{Substitute } (4, -2) \text{ for } (x_1, y_1) \text{ and } (-1, 2) \text{ for } (x_2, y_2).$$

$$= \frac{4}{-5} \qquad \text{Simplify.}$$

$$= -\frac{4}{5}$$

The slope of the line that contains $(4, -2)$ and $(-1, 2)$ is $-\frac{4}{5}$.

Reading Math
The small numbers to the bottom right of the variables are called subscripts. Read x_1 as "x sub one" and y_2 as "y sub two."

CHECK IT OUT!

1a. Find the slope of the line that contains $(-2, -2)$ and $(7, -2)$.

1b. Find the slope of the line that contains $(5, -7)$ and $(6, -4)$.

1c. Find the slope of the line that contains $\left(\frac{3}{4}, \frac{7}{5}\right)$ and $\left(\frac{1}{4}, \frac{2}{5}\right)$.

Sometimes you are not given two points to use in the formula. You might have to choose two points from a graph or a table.

EXAMPLE 2 **Finding Slope from Graphs and Tables**

Each graph or table shows a linear relationship. Find the slope.

A
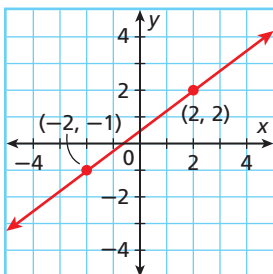
Let $(2, 2)$ be (x_1, y_1) and $(-2, -1)$ be (x_2, y_2).

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ *Use the slope formula.*

$= \dfrac{-1 - 2}{-2 - 2}$ *Substitute $(2, 2)$ for (x_1, y_1) and $(-2, -1)$ for (x_2, y_2).*

$= \dfrac{-3}{-4}$ *Simplify.*

$= \dfrac{3}{4}$

B

x	2	2	2	2
y	0	1	3	5

Step 1 Choose any two points from the table. Let $(2, 0)$ be (x_1, y_1) and $(2, 3)$ be (x_2, y_2).

Step 2 Use the slope formula.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ *Use the slope formula.*

$= \dfrac{3 - 0}{2 - 2}$ *Substitute $(2, 0)$ for (x_1, y_1) and $(2, 3)$ for (x_2, y_2).*

$= \dfrac{3}{0}$ *Simplify.*

The slope is undefined.

CHECK IT OUT! Each graph or table shows a linear relationship. Find the slope.

2a.

2b.

2c.
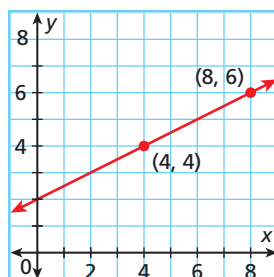
x	0	2	5	6
y	1	5	11	13
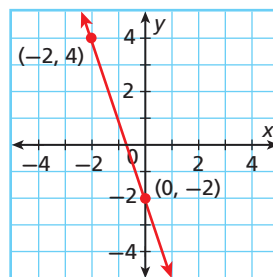
2d.

x	−2	0	2	4
y	3	0	−3	−6

Remember that slope is a rate of change. In real-world problems, finding the slope can give you information about how a quantity is changing.

EXAMPLE 3 *Environmental Science Application*

The graph shows how much water is in a reservoir at different times. Find the slope of the line. Then tell what the slope represents.

Step 1 Use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2000 - 3000}{60 - 20}$$

$$= \frac{-1000}{40} = -25$$

Water in Reservoir
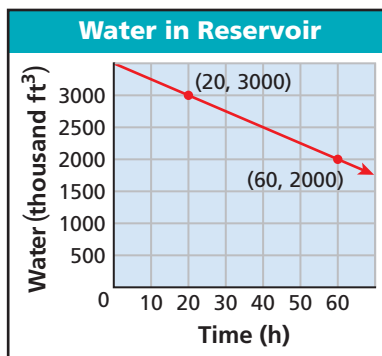
Step 2 Tell what the slope represents.

In this situation, y represents **volume of water** and x represents **time**.

So slope represents $\frac{\text{change in volume}}{\text{change in time}}$ in units of $\frac{\text{thousands of cubic feet}}{\text{hours}}$.

A slope of -25 means the amount of water in the reservoir is decreasing (negative change) at a rate of 25 thousand cubic feet each hour.

CHECK IT OUT!

3. The graph shows the height of a plant over a period of days. Find the slope of the line. Then tell what the slope represents.

Plant Growth
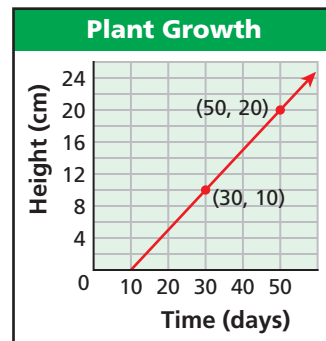
If you know the equation that describes a line, you can find its slope by using any two ordered-pair solutions. It is often easiest to use the ordered pairs that contain the intercepts.

EXAMPLE 4 **Finding Slope from an Equation**

Find the slope of the line described by $6x - 5y = 30$.

Step 1 Find the x-intercept.

$6x - 5y = 30$

$6x - 5(0) = 30$ *Let y = 0.*

$6x = 30$

$\frac{6x}{6} = \frac{30}{6}$

$x = 5$

Step 2 Find the y-intercept.

$6x - 5y = 30$

$6(0) - 5y = 30$ *Let x = 0.*

$-5y = 30$

$\frac{-5y}{-5} = \frac{30}{-5}$

$y = -6$

Step 3 The line contains $(5, 0)$ and $(0, -6)$. Use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 0}{0 - 5} = \frac{-6}{-5} = \frac{6}{5}$$

CHECK IT OUT!

4. Find the slope of the line described by $2x + 3y = 12$.

THINK AND DISCUSS

1. The slope of a line is the difference of the ____?____ divided by the difference of the ____?____ for any two points on the line.

2. Two points lie on a line. When you substitute their coordinates into the slope formula, the value of the denominator is 0. Describe this line.

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe how to find slope using the given method.

Finding Slope

| From a graph | From a table | From an equation |

Know it! Note
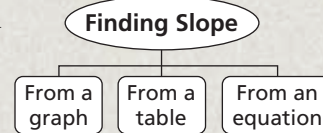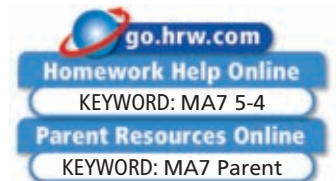
GUIDED PRACTICE

SEE EXAMPLE 1
p. 324

Find the slope of the line that contains each pair of points.

1. $(3, 6)$ and $(6, 9)$ **2.** $(2, 7)$ and $(4, 4)$ **3.** $(-1, -5)$ and $(-9, -1)$

SEE EXAMPLE 2
p. 325

Each graph or table shows a linear relationship. Find the slope.

4.
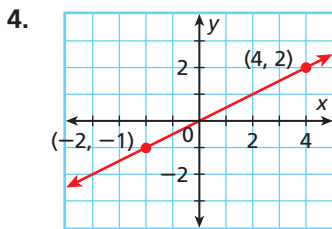
5.

x	y
0	25
2	45
4	65
6	85

SEE EXAMPLE 3
p. 326

Find the slope of each line. Then tell what the slope represents.

6.

Total Pay

7.

Peanut Butter
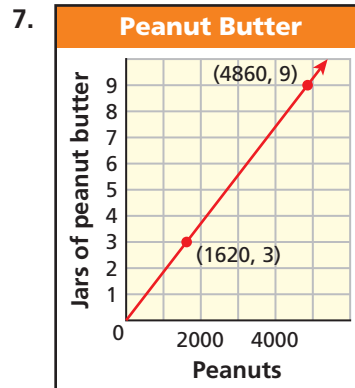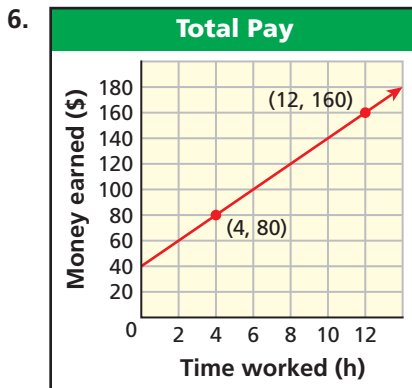
SEE EXAMPLE 4
p. 326

Find the slope of the line described by each equation.

8. $8x + 2y = 96$ **9.** $5x = 90 - 9y$ **10.** $5y = 160 + 9x$

PRACTICE AND PROBLEM SOLVING

Find the slope of the line that contains each pair of points.

11. $(2, 5)$ and $(3, 1)$ **12.** $(-9, -5)$ and $(6, -5)$ **13.** $(3, 4)$ and $(3, -1)$

Each graph or table shows a linear relationship. Find the slope.

14.

x	y
1	18.5
2	22
3	25.5
4	29
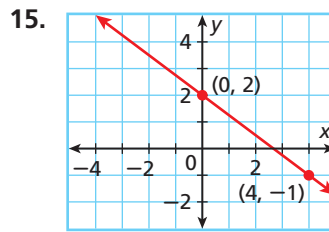
15.

Find the slope of each line. Then tell what the slope represents.

16.

Temperature Conversion

17.

Boiling Point of Water
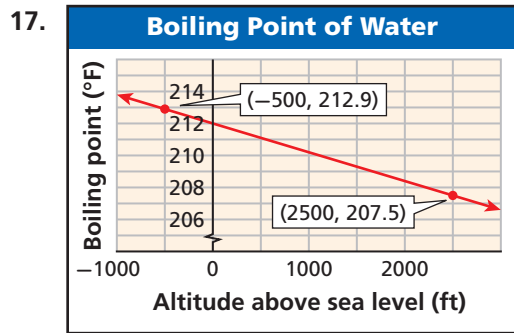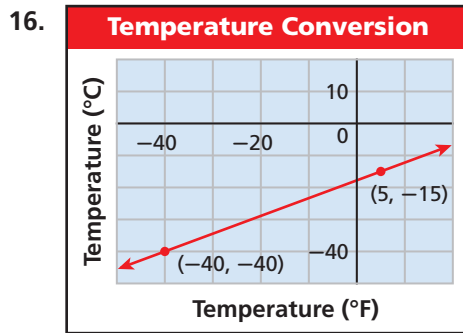
Find the slope of the line described by each equation.

18. $7x + 13y = 91$ **19.** $5y = 130 - 13x$ **20.** $7 - 3y = 9x$

21. ///**ERROR ANALYSIS**/// Two students found the slope of the line that contains $(-6, 3)$ and $(2, -1)$. Who is incorrect? Explain the error.

A
$$m = \frac{-1 - 3}{2 - (-6)} = \frac{-4}{8} = -\frac{1}{2}$$

B
$$m = \frac{-1 - 3}{-6 - 2} = \frac{-4}{-8} = \frac{1}{2}$$

22. Environmental Science The table shows how the number of cricket chirps per minute changes with the air temperature.

Temperature (°F)	40	50	60	70	80	90
Chirps per minute	0	40	80	120	160	200

   a. Find the rates of change.

   b. Is the graph of the data a line? If so, what is the slope? If not, explain why not.

23. Critical Thinking The graph shows the distance traveled by two cars.

   a. Which car is going faster? How much faster?

   b. How are the speeds related to slope?

   c. At what rate is the distance between the cars changing?

Distance Traveled
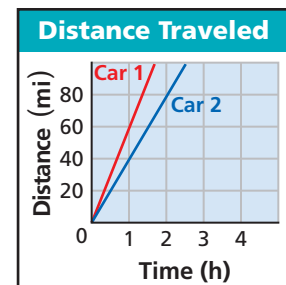
24. Write About It You are given the coordinates of two points on a line. Describe two different ways to find the slope of that line.

25. This problem will prepare you for the Multi-Step Test Prep on page 342.

    a. One way to estimate your maximum heart rate is to subtract your age from 220. Write a function to describe the relationship between maximum heart rate y and age x.

    b. The graph of this function is a line. Find its slope. Then tell what the slope represents.

TEST PREP

26. The equation $2y + 3x = -6$ describes a line with what slope?

   (A) $\dfrac{3}{2}$ (B) 0 (C) $\dfrac{1}{2}$ (D) $-\dfrac{3}{2}$

27. A line with slope $-\dfrac{1}{3}$ could pass through which of the following pairs of points?

   (F) $\left(0, -\dfrac{1}{3}\right)$ and $(1, 1)$ (H) $(0, 0)$ and $\left(-\dfrac{1}{3}, -\dfrac{1}{3}\right)$

   (G) $(-6, 5)$ and $(-3, 4)$ (J) $(5, -6)$ and $(4, 3)$

28. Gridded Response Find the slope of the line that contains $(-1, 2)$ and $(5, 5)$.

CHALLENGE AND EXTEND

Find the slope of the line that contains each pair of points.

29. $(a, 0)$ and $(0, b)$ **30.** $(2x, y)$ and $(x, 3y)$ **31.** (x, y) and $(x + 2, 3 - y)$

Find the value of x so that the points lie on a line with the given slope.

32. $(x, 2)$ and $(-5, 8)$, $m = -1$ **33.** $(4, x)$ and $(6, 3x)$, $m = \dfrac{1}{2}$

34. $(1, -3)$ and $(3, x)$, $m = -1$ **35.** $(-10, -4)$ and (x, x), $m = \dfrac{1}{7}$

36. A line contains the point $(1, 2)$ and has a slope of $\dfrac{1}{2}$. Use the slope formula to find another point on this line.

37. The points $(-2, 4)$, $(0, 2)$, and $(3, x - 1)$ all lie on the same line. What is the value of x? (*Hint:* Remember that the slope of a line is constant for any two points on the line.)

SPIRAL REVIEW

Solve each inequality and graph the solutions. *(Lesson 3-7)*

38. $|x| + 5 < 16$ **39.** $|x + 8| < 3$ **40.** $3|x| \le 12$

41. $|x| - 11 \ge -4$ **42.** $|x - 6| > 10$ **43.** $|x + 1| \ge 7$

Tell whether the given ordered pairs satisfy a linear function. *(Lesson 5-1)*

44. $\{(1, 1), (2, 4), (3, 9), (4, 16)\}$ **45.** $\{(9, 0), (8, -5), (5, -20), (3, -30)\}$

Use the intercepts to graph the line described by each equation. *(Lesson 5-2)*

46. $x - y = 5$ **47.** $3x + y = 9$ **48.** $y = 5x + 10$

5-5 The Midpoint and Distance Formulas

Objectives
Apply the formula for midpoint.

Use the Distance Formula to find the distance between two points.

Vocabulary
midpoint

Why learn this?
You can use the coordinate plane to model and solve problems involving distances, such as the distance across a lake. (See Example 4.)

In Lesson 5-4, you used the coordinates of points to determine the slope of lines. You can also use coordinates to determine the *midpoint* of a line segment on the coordinate plane.

The **midpoint** of a line segment is the point that divides the segment into two congruent segments. *Congruent segments* are segments that have the same length.

You can find the midpoint of a segment by using the coordinates of its endpoints. Calculate the average of the x-coordinates and the average of the y-coordinates of the endpoints.

Know it! Note

Midpoint Formula

The midpoint M of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$
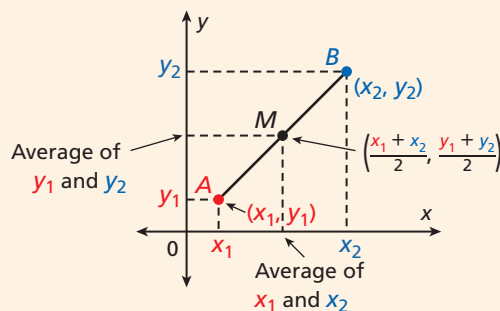
EXAMPLE 1 Finding the Coordinates of a Midpoint

Find the coordinates of the midpoint of $\overline{CD}$ with endpoints $C(-2, -1)$ and $D(4, 2)$.

Remember!

A segment is named by its endpoints. The notation $\overline{CD}$ is read "segment CD."

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \qquad \textit{Write the formula.}$$

$$M\left(\frac{-2 + 4}{2}, \frac{-1 + 2}{2}\right) \qquad \textit{Substitute.}$$

$$M\left(\frac{2}{2}, \frac{1}{2}\right) = M\left(1, \frac{1}{2}\right) \qquad \textit{Simplify.}$$

CHECK IT OUT!

1. Find the coordinates of the midpoint of $\overline{EF}$ with endpoints $E(-2, 3)$ and $F(5, -3)$.

Finding the Coordinates of an Endpoint

M is the midpoint of $\overline{AB}$. *A* has coordinates $(2, 2)$, and *M* has coordinates $(4, -3)$. Find the coordinates of *B*.

Step 1 Let the coordinates of *B* equal (x, y).

Step 2 Use the Midpoint Formula.

$$(4, -3) = \left(\frac{2 + x}{2}, \frac{2 + y}{2} \right)$$

Step 3 Find the *x*-coordinate.   Find the *y*-coordinate.

$4 = \dfrac{2 + x}{2}$	*Set the coordinates equal.*	$-3 = \dfrac{2 + y}{2}$
$2(4) = 2\left(\dfrac{2 + x}{2}\right)$	*Multiply both sides by 2.*	$2(-3) = 2\left(\dfrac{2 + y}{2}\right)$
$8 = \quad 2 + x$	*Simplify.*	$-6 = \quad 2 + y$
$\underline{-2 \quad -2}$	*Subtract 2 from both sides.*	$\underline{-2 \quad -2}$
$6 = \quad\quad x$	*Simplify.*	$-8 = \quad\quad y$

The coordinates of *B* are $(6, -8)$.

Check Graph points *A* and *B* and midpoint *M*.
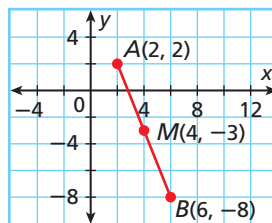
Point M appears to be the midpoint of $\overline{AB}$.

CHECK IT OUT! **2.** *S* is the midpoint of $\overline{RT}$. *R* has coordinates $(-6, -1)$, and *S* has coordinates $(-1, 1)$. Find the coordinates of *T*.

You can also use coordinates to find the distance between two points or the length of a line segment. To find the length of segment *PQ*, draw a horizontal segment from *P* and a vertical segment from *Q* to form a right triangle.

Remember!

The *Pythagorean Theorem* states that if a right triangle has legs of lengths *a* and *b* and a hypotenuse of length *c*, then $a^2 + b^2 = c^2$.
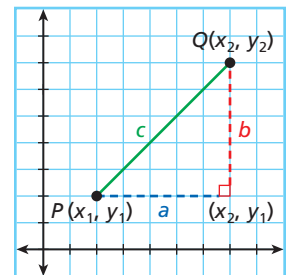
$c^2 = a^2 + b^2$ *Pythagorean Theorem*

$c = \sqrt{a^2 + b^2}$ *Solve for c. Use the positive square root to represent distance.*

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Length of horizontal segment   Length of vertical segment

This equation represents the Distance Formula.

Know it! Note

Distance Formula

In a coordinate plane, the distance *d* between two points (x_1, y_1) and (x_2, y_2) is
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

EXAMPLE 3 **Finding Distance in the Coordinate Plane**

Use the Distance Formula to find the distance,
to the nearest hundredth, from $A(-2, 3)$ to $B(2, -2)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ *Distance Formula*

$d = \sqrt{[2 - (-2)]^2 + (-2 - 3)^2}$ *Substitute (−2, 3) for (x₁, y₁) and (2, −2) for (x₂, y₂).*

$d = \sqrt{4^2 + (-5)^2}$ *Subtract.*

$d = \sqrt{16 + 25}$ *Simplify powers.*

$d = \sqrt{41}$ *Add.*

$d \approx 6.40$ *Find the square root to the nearest hundredth.*
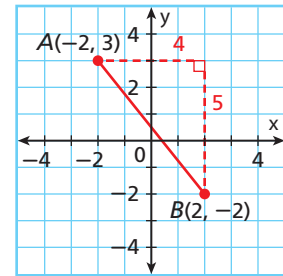
CHECK IT OUT! **3.** Use the Distance Formula to find the distance, to the nearest hundredth, from $R(3, 2)$ to $S(-3, -1)$.

EXAMPLE 4 *Geography Application*

Each unit on the map of Lake Okeechobee represents 1 mile. Delia and her father plan to travel from point A near the town of Okeechobee to point B at Pahokee. To the nearest tenth of a mile, how far do Delia and her father plan to travel?

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(33 - 22)^2 + (13 - 39)^2}$ *Substitute.*

$d = \sqrt{11^2 + (-26)^2}$ *Subtract.*

$d = \sqrt{121 + 676}$ *Simplify powers.*

$d = \sqrt{797}$ *Add.*

$d \approx 28.2$ *Find the square root to the nearest tenth.*

Delia and her father plan to travel about 28.2 miles.
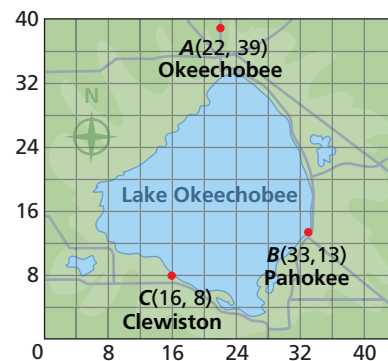
CHECK IT OUT! **4.** Jacob takes a boat from Pahokee to Clewiston. To the nearest tenth of a mile, how far does he travel?

THINK AND DISCUSS

1. If you use the formula $M\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$ to find the midpoint of a segment, will you get the correct coordinates for the midpoint? Explain.

Know it! Note

2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write a formula. Then make a sketch that will illustrate the formula.

Formulas
Midpoint Formula · Distance Formula

Exercises
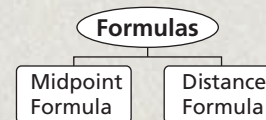
GUIDED PRACTICE

1. Vocabulary In your own words, describe the *midpoint* of a line segment.

SEE EXAMPLE 1
p. 330

Find the coordinates of the midpoint of each segment.

2. $\overline{AB}$ with endpoints $A(4, -6)$ and $B(-4, 2)$

3. $\overline{CD}$ with endpoints $C(0, -8)$ and $D(3, 0)$

4. $\overline{EF}$ with endpoints $E(-8, 17)$ and $F(-12, -16)$

SEE EXAMPLE 2
p. 331

5. M is the midpoint of $\overline{LN}$. L has coordinates $(-3, -1)$, and M has coordinates $(0, 1)$. Find the coordinates of N.

6. B is the midpoint of $\overline{AC}$. A has coordinates $(-3, 4)$, and B has coordinates $\left(-1\frac{1}{2}, 1\right)$. Find the coordinates of C.

SEE EXAMPLE 3
p. 332

Use the Distance Formula to find the distance, to the nearest hundredth, between each pair of points.

7. $A(1, -2)$ and $B(-4, -4)$

8. $X(-2, 7)$ and $Y(-2, -8)$

9. $V(2, -1)$ and $W(-4, 8)$

SEE EXAMPLE 4
p. 332

10. Recreation Each unit on the map of a public park represents 1 kilometer. To the nearest tenth of a kilometer, what is the distance from the campground to the waterfall?
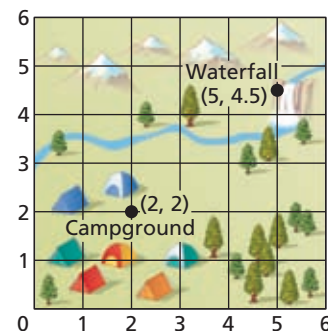
PRACTICE AND PROBLEM SOLVING

Find the coordinates of the midpoint of each segment.

11. $\overline{XY}$ with endpoints $X(-3, -7)$ and $Y(-1, 1)$

12. $\overline{MN}$ with endpoints $M(12, -7)$ and $N(-5, -2)$

13. M is the midpoint of $\overline{QR}$. Q has coordinates $(-3, 5)$, and M has coordinates $(7, -9)$. Find the coordinates of R.

14. D is the midpoint of $\overline{CE}$. E has coordinates $(-3, -2)$, and D has coordinates $\left(2\frac{1}{2}, 1\right)$. Find the coordinates of C.

15. Y is the midpoint of $\overline{XZ}$. X has coordinates $(0, -1)$, and Y has coordinates $\left(1, -4\frac{1}{2}\right)$. Find the coordinates of Z.

Use the Distance Formula to find the distance, to the nearest hundredth, between each pair of points.

16. $U(0, 1)$ and $V(-3, -9)$

17. $M(10, -1)$ and $N(2, -5)$

18. $P(-10, 1)$ and $Q(5, 5)$

19. $F(6, 15)$ and $G(4, 24)$

20. Astronomy Each unit on the map of a section of a moon represents 1 kilometer. To the nearest tenth of a kilometer, what is the distance between the two craters?
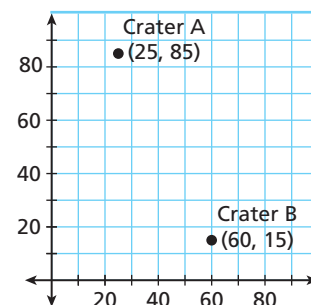
For Exercises 21 and 22, use the map, and round your answers to the nearest tenth of a mile. Each unit on the map represents 1 mile.

21. How far is it from Cedar City to Milltown along Highway 201?

22. A car breaks down on Route 1 halfway between Jefferson and Milltown. A tow truck is sent out from Jefferson. How far does the truck travel to reach the car?

23. Estimation Estimate the distance between the points $(-5.21, 1.84)$ and $(16.62, -23.19)$. Explain how you determined your answer.
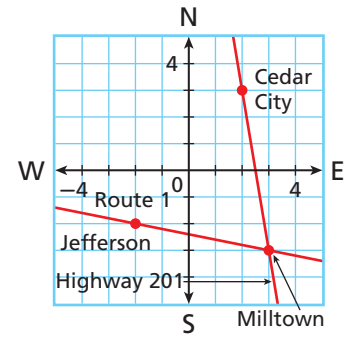
24. Geometry The coordinates of the vertices of $\triangle ABC$ are $A(1, 4)$, $B(-2, -1)$, and $C(3, -2)$. Find the perimeter of $\triangle ABC$ to the nearest whole number.

Find the distance, to the nearest hundredth, between each pair of points.

25. $J(0, 3)$ and $K(-6, -9)$ **26.** $L(-5, 2)$ and $M(-8, 10)$ **27.** $N(4, -6)$ and $P(-2, 7)$

28. Aviation A Coast Guard helicopter receives a distress signal from a boat. The units on the map represent miles. To the nearest minute, how long will it take the helicopter to reach the boat if the helicopter travels at an average speed of 75 miles per hour?
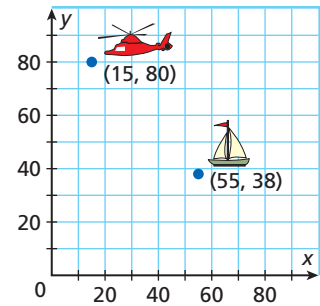
29. Geometry A diameter of a circle has endpoints $(-2, -5)$ and $(2, 1)$.

   a. Find the length of the diameter to the nearest tenth.

   b. Find the coordinates of the center of the circle.

   c. Find the circumference of the circle to the nearest whole number.

30. Travel A group of tourists is traveling by camel in the desert. They are following a straight path from point $(2, 4)$ to point $(8, 12)$ on a map. Each unit on the map represents 1 mile. An oasis lies at the midpoint of the path. The group has already traveled 3.2 miles. How much farther do they need to go to reach the oasis?

31. Multi-Step Use the Distance Formula to order $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ from shortest to longest.

32. Critical Thinking Rebecca found the x-coordinate of the midpoint of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ by dividing the difference between x_2 and x_1 by 2 and then adding the quotient to x_1. Did this method give the correct x-coordinate for the midpoint? Explain.
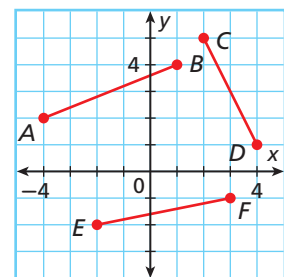
33. Write About It Explain why the Distance Formula is not needed to find the distance between two points that lie on a horizontal or vertical line.

MULTI-STEP TEST PREP

34. This problem will help you prepare for the Multi-Step Test Prep on page 342.

On a map of a city park, the ordered pairs $(3, 5)$ and $(8, 17)$ mark the starting and ending points of a straight jogging trail. Each unit on the map represents 0.1 mile.

   a. What is the length of the trail in miles?

   b. How many back-and-forth trips would Marisol need to make on the trail in order to run at least 7 miles?

35. Which segment has a length closest to 4 units?

(A) $\overline{EF}$ (C) $\overline{JK}$

(B) $\overline{GH}$ (D) $\overline{LM}$
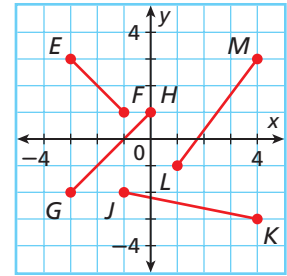
36. What is the distance between the points (7, –3) and (–5, 6)?

(F) 4   (H) 15

(G) 9   (J) 21

37. A coordinate plane is placed over the map of a town. A library is located at (−5, 1), and a museum is located at (3, 5). What is the distance, to the nearest tenth, from the library to the museum?

(A) 4.5 units   (B) 5.7 units   (C) 6.3 units   (D) 8.9 units

38. **Short Response** Brian is driving along a straight highway. His truck can travel 22 miles per gallon of gasoline, and it has 2 gallons of gas remaining. On a map, the truck's current location is (7, 12), and the nearest gas station on the highway is located at (16, 52). Each unit on the map represents 1 mile. Will the truck reach the gas station before running out of gas? Support your answer.

CHALLENGE AND EXTEND

39. **Geometry** Find the area of a trapezoid with vertices $A(−4, 2)$, $B(0, 4)$, $C(3, 3)$, and $D(−3, 0)$. (*Hint*: The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$.)

40. X has coordinates $(a, 3a)$, and Y has coordinates $(−5a, 0)$. Find the coordinates of the midpoint of $\overline{XY}$.

41. The coordinates of P are $(a − 5, 0)$. The coordinates of Q are $(a + 1, a)$. The distance between P and Q is 10 units. Find the value of a.

42. Find two points on the y-axis that are a distance of 5 units from $(4, 2)$.

43. The coordinates of S are $(4, 5)$. The coordinates of the midpoint of $\overline{ST}$ are $(−1, −7)$. Find the length of $\overline{ST}$.

SPIRAL REVIEW

44. In the 2007–2008 season, the Orlando Magic won 52 games and lost 30 games. What percent of its games did the team win? Round your answer to the nearest percent. *(Lesson 2-9)*

Solve each inequality and graph the solutions. *(Lesson 3-5)*

45. $4x < 7x + 6$ **46.** $4y + 3 \geq 5y − 8$ **47.** $6(z + 2) > 3z$

Find the slope of each line. *(Lesson 5-3)*
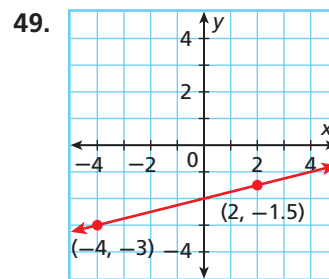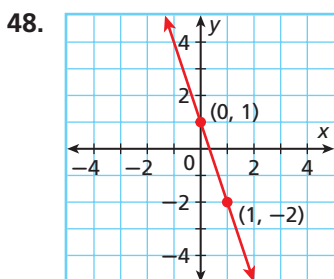
48.

49.

5-6 Direct Variation

Who uses this?
Chefs can use direct variation to determine ingredients needed for a certain number of servings.

A recipe for paella calls for 1 cup of rice to make 5 servings. In other words, a chef needs 1 cup of rice for every 5 servings.

Rice (c) x	1	2	3	4
Servings y	5	10	15	20

Paella is a rice dish that originated in Valencia, Spain.

The equation $y = 5x$ describes this relationship. In this relationship, the number of servings *varies directly* with the number of cups of rice.

A **direct variation** is a special type of linear relationship that can be written in the form $y = kx$, where k is a nonzero constant called the **constant of variation**.

E X A M P L E 1 Identifying Direct Variations from Equations

Tell whether each equation represents a direct variation. If so, identify the constant of variation.

A $y = 4x$

This equation represents a direct variation because it is in the form $y = kx$. The constant of variation is 4.

B $-3x + 5y = 0$

$$-3x + 5y = 0 \qquad \text{Solve the equation for } y.$$
$$\underline{+3x \qquad\qquad +3x} \qquad \text{Since } -3x \text{ is added to } 5y, \text{ add } 3x \text{ to both sides.}$$
$$5y = 3x$$
$$\frac{5y}{5} = \frac{3x}{5} \qquad \text{Since } y \text{ is multiplied by 5, divide both sides by 5.}$$
$$y = \frac{3}{5}x$$

This equation represents a direct variation because it can be written in the form $y = kx$. The constant of variation is $\frac{3}{5}$.

C $2x + y = 10$

$$2x + y = 10 \qquad \text{Solve the equation for } y.$$
$$\underline{-2x \qquad\quad -2x} \qquad \text{Since } 2x \text{ is added to } y, \text{ subtract } 2x \text{ from both sides.}$$
$$y = -2x + 10$$

This equation does not represent a direct variation because it cannot be written in the form $y = kx$.

CHECK IT OUT! Tell whether each equation represents a direct variation. If so, identify the constant of variation.

1a. $3y = 4x + 1$ **1b.** $3x = -4y$ **1c.** $y + 3x = 0$

What happens if you solve $y = kx$ for k?

$$y = kx$$

$$\frac{y}{x} = \frac{kx}{x} \qquad \textit{Divide both sides by x (x $\neq$ 0).}$$

$$\frac{y}{x} = k$$

So, in a direct variation, the ratio $\frac{y}{x}$ is equal to the constant of variation. Another way to identify a direct variation is to check whether $\frac{y}{x}$ is the same for each ordered pair (except where $x = 0$).

EXAMPLE 2 **Identifying Direct Variations from Ordered Pairs**

Tell whether each relationship is a direct variation. Explain.

A

x	1	3	5
y	6	18	30

Method 1 Write an equation.

$y = 6x \qquad$ *Each y-value is 6 times the corresponding x-value.*

This is a direct variation because it can be written as $y = kx$, where $k = 6$.

Method 2 Find $\frac{y}{x}$ for each ordered pair.

$$\frac{6}{1} = \mathbf{6} \qquad\qquad \frac{18}{3} = \mathbf{6} \qquad\qquad \frac{30}{5} = \mathbf{6}$$

This is a direct variation because $\frac{y}{x}$ is the same for each ordered pair.

B

x	2	4	8
y	−2	0	4

Method 1 Write an equation.

$y = x - 4 \qquad$ *Each y-value is 4 less than the corresponding x-value.*

This is not a direct variation because it cannot be written as $y = kx$.

Method 2 Find $\frac{y}{x}$ for each ordered pair.

$$\frac{-2}{2} = \mathbf{-1} \qquad\qquad \frac{0}{4} = \mathbf{0} \qquad\qquad \frac{4}{8} = \mathbf{\frac{1}{2}}$$

This is not a direct variation because $\frac{y}{x}$ is not the same for all ordered pairs.

CHECK IT OUT! Tell whether each relationship is a direct variation. Explain.

2a.
x	y
−3	0
1	3
3	6

2b.
x	y
2.5	−10
5	−20
7.5	−30

2c.
x	y
−2	5
1	3
4	1

If you know one ordered pair that satisfies a direct variation, you can write the equation. You can also find other ordered pairs that satisfy the direct variation.

EXAMPLE 3

Writing and Solving Direct Variation Equations

The value of y varies directly with x, and $y = 6$ when $x = 12$. Find y when $x = 27$.

Method 1 Find the value of k and then write the equation.

$y = kx$	*Write the equation for a direct variation.*
$6 = k(12)$	*Substitute 6 for y and 12 for x. Solve for k.*
$\frac{1}{2} = k$	*Since k is multiplied by 12, divide both sides by 12.*

The equation is $y = \frac{1}{2}x$. When $x = 27$, $y = \frac{1}{2}(27) = 13.5$.

Method 2 Use a proportion.

$\dfrac{6}{12} \diagup \dfrac{y}{27}$	*In a direct variation, $\frac{y}{x}$ is the same for all values of x and y.*
$12y = 162$	*Use cross products.*
$y = 13.5$	*Since y is multiplied by 12, divide both sides by 12.*

CHECK IT OUT!

3. The value of y varies directly with x, and $y = 4.5$ when $x = 0.5$. Find y when $x = 10$.

EXAMPLE 4

Graphing Direct Variations

The three-toed sloth is an extremely slow animal. On the ground, it travels at a speed of about 6 feet per minute. Write a direct variation equation for the distance y a sloth will travel in x minutes. Then graph.

Step 1 Write a direct variation equation.

distance	=	6 feet per minute	times	number of minutes
y	=	6	•	x

Step 2 Choose values of x and generate ordered pairs.

x	$y = 6x$	(x, y)
0	$y = 6(0) = 0$	$(0, 0)$
1	$y = 6(1) = 6$	$(1, 6)$
2	$y = 6(2) = 12$	$(2, 12)$

Step 3 Graph the points and connect.

Speed of a Sloth

CHECK IT OUT!
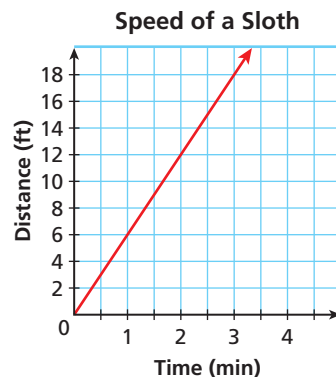
4. The perimeter y of a square varies directly with its side length x. Write a direct variation equation for this relationship. Then graph.

Look at the graph in Example 4. It passes through $(0, 0)$ and has a slope of 6. The graph of any direct variation $y = kx$
- is a line through $(0, 0)$.
- has a slope of k.

THINK AND DISCUSS

1. How do you know that a direct variation is linear?

2. How does the graph of a direct variation differ from the graphs of other types of linear relationships?

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, describe how you can use the given information to identify a direct variation.

Recognizing a Direct Variation		
From an Equation	From Ordered Pairs	From a Graph

5-6 Exercises
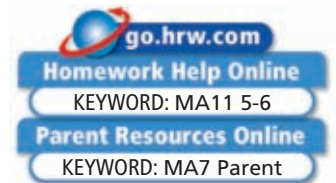
GUIDED PRACTICE

1. Vocabulary If x varies directly with y, then the relationship between the two variables is said to be a ___?___. (*direct variation* or *constant of variation*)

SEE EXAMPLE **1**
p. 336

Tell whether each equation represents a direct variation. If so, identify the constant of variation.

2. $y = 4x + 9$ **3.** $2y = -8x$ **4.** $x + y = 0$

SEE EXAMPLE **2**
p. 337

Tell whether each relationship is a direct variation. Explain.

5.

x	10	5	2
y	12	7	4

6.

x	3	−1	−4
y	−6	2	8

SEE EXAMPLE **3**
p. 338

7. The value of y varies directly with x, and $y = -3$ when $x = 1$. Find y when $x = -6$.

8. The value of y varies directly with x, and $y = 6$ when $x = 18$. Find y when $x = 12$.

SEE EXAMPLE **4**
p. 338

9. Wages Cameron earns $7 per hour at her after-school job. The total amount of her paycheck varies directly with the amount of time she works. Write a direct variation equation for the amount of money y that she earns for working x hours. Then graph.

PRACTICE AND PROBLEM SOLVING

Tell whether each equation represents a direct variation. If so, identify the constant of variation.

10. $y = \frac{1}{6}x$ **11.** $4y = x$ **12.** $x = 2y - 12$

Tell whether each relationship is a direct variation. Explain.

13.

x	6	9	17
y	13.2	19.8	37.4

14.

x	−6	3	12
y	4	−2	−8

15. The value of y varies directly with x, and $y = 8$ when $x = -32$. Find y when $x = 64$.

16. The value of y varies directly with x, and $y = \frac{1}{2}$ when $x = 3$. Find y when $x = 1$.

17. While on his way to school, Norman saw that the cost of gasoline was $2.50 per gallon. Write a direct variation equation to describe the cost y of x gallons of gas. Then graph.

Tell whether each relationship is a direct variation. Explain your answer.

18. The equation $-15x + 4y = 0$ relates the length of a videotape in inches x to its approximate playing time in seconds y.

19. The equation $y - 2.00x = 2.50$ relates the cost y of a taxicab ride to distance x of the cab ride in miles.

Each ordered pair is a solution of a direct variation. Write the equation of direct variation. Then graph your equation and show that the slope of the line is equal to the constant of variation.

20. $(2, 10)$ **21.** $(-3, 9)$ **22.** $(8, 2)$ **23.** $(1.5, 6)$

24. $(7, 21)$ **25.** $(1, 2)$ **26.** $(2, -16)$ **27.** $\left(\frac{1}{7}, 1\right)$

28. $(-2, 9)$ **29.** $(9, -2)$ **30.** $(4, 6)$ **31.** $(3, 4)$

32. $(5, 1)$ **33.** $(1, -6)$ **34.** $\left(-1, \frac{1}{2}\right)$ **35.** $(7, 2)$

Astronomy

36. **Astronomy** Weight varies directly with gravity. A Mars lander weighed 767 pounds on Earth but only 291 pounds on Mars. Its accompanying Mars rover weighed 155 pounds on Mars. How much did it weigh on Earth? Round your answer to the nearest pound.

37. **Environment** Mischa bought an energy-efficient washing machine. She will save about 15 gallons of water per wash load.

   a. Write an equation of direct variation to describe how many gallons of water y Mischa saves for x loads of laundry she washes.

   b. Graph your direct variation from part **a.** Is every point on the graph a solution in this situation? Why or why not?

   c. If Mischa does 2 loads of laundry per week, how many gallons of water will she have saved at the end of a year?

The Mars rover *Spirit* landed on Mars in January 2004 and immediately began sending photos of the planet's surface back to Earth.

38. **Critical Thinking** If you double an x-value in a direct variation, will the corresponding y-value double? Explain.

39. **Write About It** In a direct variation $y = kx$, k is sometimes called the "constant of proportionality." How are proportions related to direct variations?

MULTI-STEP TEST PREP

40. This problem will prepare you for the Multi-Step Test Prep on page 342.

Rhea exercised on a treadmill at the gym. When she was finished, the display showed that she had walked at an average speed of 3 miles per hour.

   a. Write an equation that gives the number of miles y that Rhea would cover in x hours if she walked at this speed.

   b. Explain why this is a direct variation and find the value of k. What does this value represent in Rhea's situation?

41. Which equation does NOT represent a direct variation?

Ⓐ $y = \frac{1}{3}x$ Ⓑ $y = -2x$ Ⓒ $y = 4x + 1$ Ⓓ $6x - y = 0$

42. Identify which set of data represents a direct variation.

Ⓕ

x	1	2	3
y	1	2	3

Ⓗ

x	1	2	3
y	3	5	7

Ⓖ

x	1	2	3
y	0	1	2

Ⓙ

x	1	2	3
y	3	4	5

43. Two yards of fabric cost $13, and 5 yards of fabric cost $32.50. Which equation relates the cost of the fabric c to its length ℓ?

Ⓐ $c = 2.6\ell$ Ⓑ $c = 6.5\ell$ Ⓒ $c = 13\ell$ Ⓓ $c = 32.5\ell$

44. Gridded Response A car is traveling at a constant speed. After 3 hours, the car has traveled 180 miles. If the car continues to travel at the same constant speed, how many hours will it take to travel a total of 270 miles?

CHALLENGE AND EXTEND

45. Transportation The function $y = 20x$ gives the number of miles y that a sport-utility vehicle (SUV) can travel on x gallons of gas. The function $y = 60x$ gives the number of miles y that a hybrid car can travel on x gallons of gas.

   a. If you drive 120 miles, how much gas will you save by driving the hybrid instead of the SUV?

   b. Graph both functions on the same coordinate plane. Will the lines ever meet other than at the origin? Explain.

   c. What if…? Shannon drives 15,000 miles in one year. How many gallons of gas will she use if she drives the SUV? the hybrid?

46. Suppose the equation $ax + by = c$, where a, b, and c are real numbers, describes a direct variation. What do you know about the value of c?

SPIRAL REVIEW

Solve for the indicated variable. *(Lesson 2-5)*

47. $p + 4q = 7$; p **48.** $\frac{s - 5}{t} = 2$; s **49.** $xy + 2y = 4$; x

Determine a relationship between the x- and y-values and write an equation. *(Lesson 4-3)*

50.

x	y
1	−5
2	−4
3	−3
4	−2

51.

x	y
1	−2
2	−4
3	−6
4	−8

52.

x	y
−3	9
−2	6
−1	3
0	0

Find the slope of the line described by each equation. *(Lesson 5-4)*

53. $4x + y = -9$ **54.** $6x - 3y = -9$ **55.** $5x = 10y - 5$

MULTI-STEP TEST PREP

Characteristics of Linear Functions

Heart Health People who exercise need to be aware of their maximum heart rate.

1. One way to estimate your maximum heart rate m is to subtract 85% of your age in years from 217. Create a table of values that shows the maximum heart rates for people ages 13 to 18. Then write an equation to describe the data in the table.

2. Use your table from Problem 1 to graph the relationship between age and maximum heart rate. What are the intercepts? What is the slope?

3. What do the intercepts represent in this situation?

4. What does the slope represent? Explain why the slope is negative.

5. Another formula for estimating maximum heart rate is $m = 206.3 - 0.711a$, where a represents age in years. Describe how this equation is different from your equation in Problem 1. Include slope and intercepts in your description.

6. Which equation gives a higher maximum heart rate for people ages 75 and younger?

7. To be exercising in your *aerobic training zone* means that your heart rate is 70% to 80% of your maximum heart rate. Write two equations that someone could use to estimate the range of heart rates that are within his or her aerobic training zone. Use your equation for maximum heart rate from Problem 1.

READY TO GO ON?

Quiz for Lessons 5-1 Through 5-6

5-1 Identifying Linear Functions

Tell whether the given ordered pairs satisfy a linear function. Explain.

1.

x	−2	−1	0	1	2
y	1	0	1	4	9

2. $\{(-3, 8), (-2, 6), (-1, 4), (0, 2), (1, 0)\}$

5-2 Using Intercepts

Use intercepts to graph the line described by each equation.

3. $2x - 4y = 16$ **4.** $-3y + 6x = -18$ **5.** $y = -3x + 3$

5-3 Rate of Change and Slope

6. The chart gives the amount of water in a rain gauge in inches at various times. Graph the data and show the rates of change.

Time (h)	1	2	3	4	5
Rain (in.)	0.2	0.4	0.7	0.8	1.0

5-4 The Slope Formula

Find the slope of each line. Then tell what the slope represents.

7.

Cost of Peppers

8.

Toy Race Car

9.

Temperatures at Various Altitudes
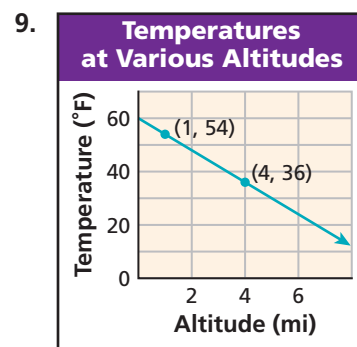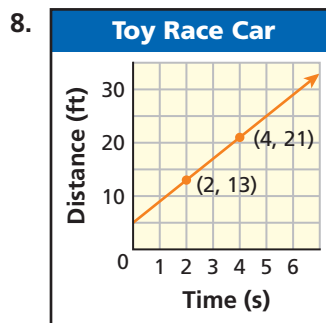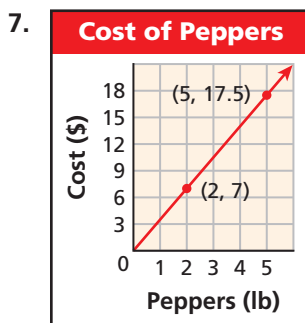
5-5 The Midpoint and Distance Formulas

10. Find the coordinates of the midpoint of $\overline{XY}$ with endpoints $X(-4, 6)$ and $Y(3, 8)$.

11. On a treasure map, the coordinates of a crooked palm tree are (3, 6), and the coordinates of a buried treasure chest are (12, 18). Each unit on the map represents 10 feet. What is the distance in feet between the palm tree and the treasure chest?

5-6 Direct Variation

Tell whether each relationship is a direct variation. If so, identify the constant of variation.

12.

x	1	4	8	12
y	3	6	10	14

13.

x	−6	−2	0	3
y	−3	−1	0	1.5

Slope-Intercept Form

Objectives
Write a linear equation in slope-intercept form.

Graph a line using slope-intercept form.

Who uses this?
Consumers can use slope-intercept form to model and calculate costs, such as the cost of renting a moving van. (See Example 4.)

You have seen that you can graph a line if you know two points on the line. Another way is to use the slope of the line and the point that contains the *y*-intercept.

EXAMPLE **1**

Graphing by Using Slope and *y*-intercept

Graph the line with slope -2 and *y*-intercept 4.

Writing Math

Any integer can be written as a fraction with 1 in the denominator.
$$-2 = \frac{-2}{1}$$

Step 1 The *y*-intercept is **4**, so the line contains (0, **4**). Plot (0, 4).

Step 2 Slope $= \dfrac{\text{change in } y}{\text{change in } x} = \dfrac{-2}{1}$

Count **2 units down** and **1 unit right** from (0, 4) and plot another point.

Step 3 Draw the line through the two points.

Rise $= -2$
Run $= 1$

Graph each line given the slope and *y*-intercept.
1a. slope $= 2$, *y*-intercept $= -3$ **1b.** slope $= -\dfrac{2}{3}$, *y*-intercept $= 1$

If you know the slope of a line and the *y*-intercept, you can write an equation that describes the line.

Step 1 If a line has slope **2** and the *y*-intercept is **3**, then $m = 2$ and $(0, 3)$ is on the line. Substitute these values into the slope formula.

Slope formula $\rightarrow m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $2 = \dfrac{y - 3}{x - 0}$ ←Since you don't know (x_2, y_2), use (x, y).

Step 2 Solve for *y*: $2 = \dfrac{y - 3}{x - 0}$

$2 = \dfrac{y - 3}{x}$ *Simplify the denominator.*

$2 \cdot x = \left(\dfrac{y - 3}{x}\right) \cdot x$ *Multiply both sides by x.*

$2x = y - 3$

$\underline{+ 3 \quad\quad + 3}$ *Add 3 to both sides.*

$2x + 3 = y$, or $y = 2x + 3$

Slope-Intercept Form of a Linear Equation

If a line has **slope m** and the **y-intercept** is **b**, then the line is described by the equation $y = mx + b$.

Any linear equation can be written in slope-intercept form by solving for y and simplifying. In this form, you can immediately see the slope and y-intercept. Also, you can quickly graph a line when the equation is written in slope-intercept form.

EXAMPLE 2   **Writing Linear Equations in Slope-Intercept Form**

Write the equation that describes each line in slope-intercept form.

A   slope $= \dfrac{1}{3}$, y-intercept $= 6$

$y = mx + b$ *Substitute the given values for m and b.*

$y = \dfrac{1}{3}x + 6$ *Simplify if necessary.*

B   slope $= 0$, y-intercept $= -5$

$y = mx + b$

$y = 0x + (-5)$

$y = -5$

C
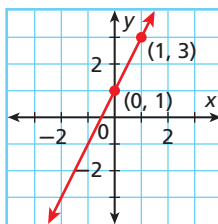
Step 1 Find the y-intercept. The graph crosses the y-axis at $(0, 1)$, so $b = 1$.

Step 2 Find the slope. The line contains the points $(0, 1)$ and $(1, 3)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ *Use the slope formula.*

$m = \dfrac{3 - 1}{1 - 0} = \dfrac{2}{1} = 2$ *Substitute (0, 1) for (x₁, y₁) and (1, 3) for (x₂, y₂).*

Step 3 Write the equation.

$y = mx + b$ *Write the slope-intercept form.*

$y = 2x + 1$ *Substitute 2 for m and 1 for b.*

D   slope $= 4$, $(2, 5)$ is on the line

Step 1 Find the y-intercept.

$y = mx + b$ *Write the slope-intercept form.*

$5 = 4(2) + b$ *Substitute 4 for m, 2 for x, and 5 for y.*

$5 = 8 + b$ *Solve for b. Since 8 is added to b, subtract 8 from both sides to undo the addition.*

$\underline{-8 \quad -8}$

$-3 = b$

Step 2 Write the equation.

$y = mx + b$ *Write the slope-intercept form.*

$y = 4x + (-3)$ *Substitute 4 for m and −3 for b.*

$y = 4x - 3$

CHECK IT OUT!   Write the equation that describes each line in slope-intercept form.

2a. slope $= -12$, y-intercept $= -\dfrac{1}{2}$

2b. slope $= 1$, y-intercept $= 0$

2c. slope $= 8$, $(-3, 1)$ is on the line.

Using Slope-Intercept Form to Graph

Write each equation in slope-intercept form. Then graph the line described by the equation.

A $y = 4x - 3$

$y = 4x - 3$ is in the form $y = mx + b$.

slope: $m = 4 = \dfrac{4}{1}$

y-intercept: $b = -3$

Step 1 Plot $(0, -3)$.

Step 2 Count **4 units up** and **1 unit right** and plot another point.

Step 3 Draw the line connecting the two points.
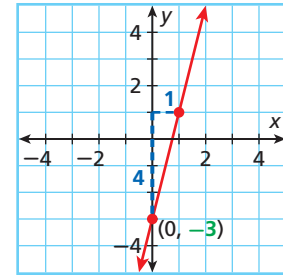
B $y = -\dfrac{2}{3}x + 2$

$y = -\dfrac{2}{3}x + 2$ is in the form $y = mx + b$.

slope: $m = -\dfrac{2}{3} = \dfrac{-2}{3}$

y-intercept: $b = 2$

Step 1 Plot $(0, 2)$

Step 2 Count **2 units down** and **3 units right** and plot another point.

Step 3 Draw the line connecting the two points.
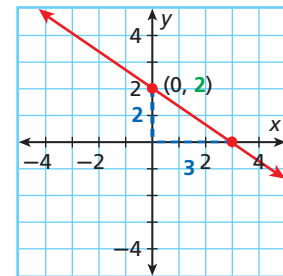
C $3x + 2y = 8$

Step 1 Write the equation in slope-intercept form by solving for y.

$$3x + 2y = 8$$
$$\underline{-3x \qquad\quad -3x} \qquad \textit{Subtract 3x from both sides.}$$
$$2y = 8 - 3x$$
$$\dfrac{2y}{2} = \dfrac{8 - 3x}{2} \qquad \textit{Since y is multiplied by 2, divide both sides by 2.}$$
$$y = 4 - \dfrac{3}{2}x \qquad\qquad \dfrac{3x}{2} = \dfrac{3}{2}x$$
$$y = -\dfrac{3}{2}x + 4 \qquad \textit{Write the equation in the form } y = mx + b.$$

Step 2 Graph the line.

$y = -\dfrac{3}{2}x + 4$ is in the form $y = mx + b$.

slope: $m = -\dfrac{3}{2} = \dfrac{-3}{2}$

y-intercept: $b = 4$

- Plot $(0, 4)$.
- Then count **3 units down** and **2 units right** and plot another point.
- Draw the line connecting the two points.
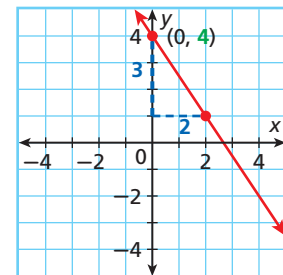
Helpful Hint

To divide $(8 - 3x)$ by 2, you can multiply by $\frac{1}{2}$ and use the Distributive Property.

$\dfrac{8 - 3x}{2} = \dfrac{1}{2}(8 - 3x)$

$= \dfrac{1}{2}(8) + \dfrac{1}{2}(-3x)$

$= 4 - \dfrac{3}{2}x$

CHECK IT OUT! Write each equation in slope-intercept form. Then graph the line described by the equation.

3a. $y = \dfrac{2}{3}x$ **3b.** $6x + 2y = 10$ **3c.** $y = -4$

EXAMPLE 4   *Consumer Application*

Moving Van Costs

To rent a van, a moving company charges $30.00 plus $0.50 per mile. The cost as a function of the number of miles driven is shown in the graph.
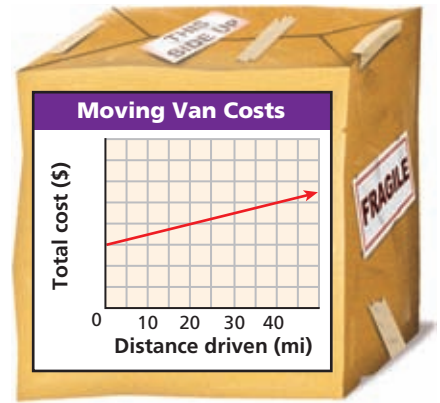
a. Write an equation that represents the cost as a function of the number of miles.

Cost	is	$0.50 per mile	times	miles	plus	$30.00
y	$=$	0.5	$\cdot$	x	$+$	30

An equation is $y = 0.5x + 30$.

b. Identify the slope and y-intercept and describe their meanings.

The y-intercept is 30. This is the cost for 0 miles, or the initial fee of $30.00.

The slope is 0.5. This is the rate of change of the cost: $0.50 per mile.

c. Find the cost of the van for 150 miles.

$y = 0.5x + 30$

$= 0.5(150) + 30 = 105$ *Substitute 150 for x in the equation.*

The cost of the van for 150 miles is $105.

CHECK IT OUT!

4. A caterer charges a $200 fee plus $18 per person served. The cost as a function of the number of guests is shown in the graph.

a. Write an equation that represents the cost as a function of the number of guests.

b. Identify the slope and y-intercept and describe their meanings.

c. Find the cost of catering an event for 200 guests.

Catering Fees
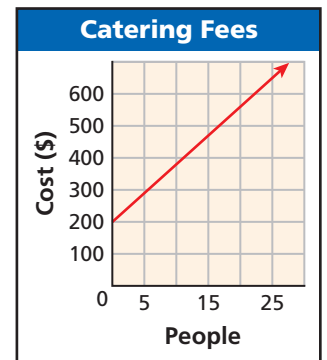
THINK AND DISCUSS

1. If a linear function has a y-intercept of b, at what point does its graph cross the y-axis?

2. Where does the line described by $y = 4.395x - 23.75$ cross the y-axis?

3. GET ORGANIZED Copy and complete the graphic organizer.

Graphing the Line Described by $y = mx + b$

1. Plot the point __?__ .	2. Find a second point on the line by __?__ .	3. Draw __?__ .
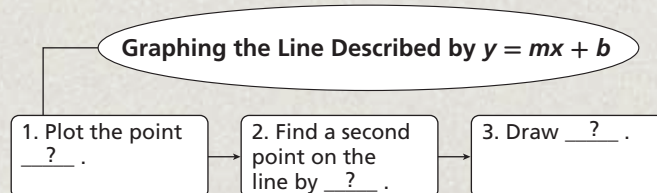
GUIDED PRACTICE

SEE EXAMPLE 1
p. 344

Graph each line given the slope and y-intercept.

1. slope $= \frac{1}{3}$, y-intercept $= -3$

2. slope $= 0.5$, y-intercept $= 3.5$

3. slope $= 5$, y-intercept $= -1$

4. slope $= -2$, y-intercept $= 2$

SEE EXAMPLE 2
p. 345

Write the equation that describes each line in slope-intercept form.

5.
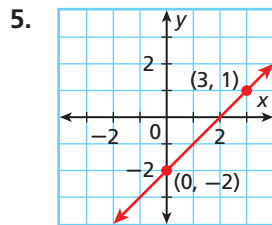
6. slope $= 8$, y-intercept $= 2$

7. slope $= 0$, y-intercept $= -3$

8. slope $= 5$, $(2, 7)$ is on the line.

9. slope $= -2$, $(1, -3)$ is on the line.

SEE EXAMPLE 3
p. 346

Write each equation in slope-intercept form. Then graph the line described by the equation.

10. $y = \frac{2}{5}x - 6$

11. $3x - y = 1$

12. $2x + y = 4$

SEE EXAMPLE 4
p. 347

13. Helen is in a bicycle race. She has already biked 10 miles and is now biking at a rate of 18 miles per hour. Her distance as a function of time is shown in the graph.

   a. Write an equation that represents the distance Helen has biked as a function of time.

   b. Identify the slope and y-intercept and describe their meanings.

   c. How far will Helen have biked after 2 hours?

Distance Biked
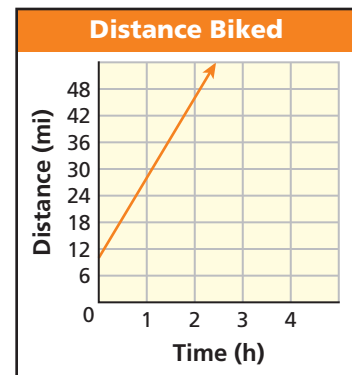
PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14–17	1
19–22	2
23–25	3
26	4

Extra Practice
Skills Practice p. S13
Application Practice p. S32

Graph each line given the slope and y-intercept.

14. slope $= \frac{1}{4}$, y-intercept $= 7$

15. slope $= -6$, y-intercept $= -3$

16. slope $= 1$, y-intercept $= -4$

17. slope $= -\frac{4}{5}$, y-intercept $= 6$

Write the equation that describes each line in slope-intercept form.

18.
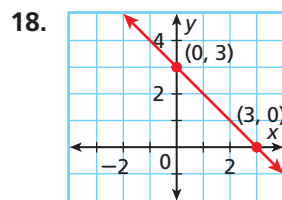
19. slope $= 5$, y-intercept $= -9$

20. slope $= -\frac{2}{3}$, y-intercept $= 2$

21. slope $= -\frac{1}{2}$, $(6, 4)$ is on the line.

22. slope $= 0$, $(6, -8)$ is on the line.

Write each equation in slope-intercept form. Then graph the line described by the equation.

23. $-\dfrac{1}{2}x + y = 4$ **24.** $\dfrac{2}{3}x + y = 2$ **25.** $2x + y = 8$

26. Fitness Pauline's health club has an enrollment fee of $175 and costs $35 per month. Total cost as a function of number of membership months is shown in the graph.
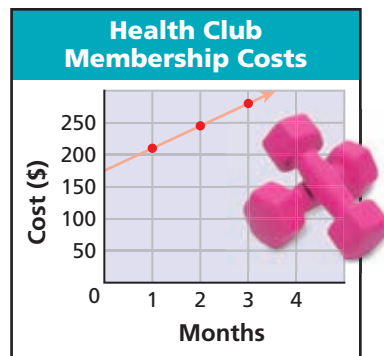
  a. Write an equation that represents the total cost as a function of months.

  b. Identify the slope and y-intercept and describe their meanings.

  c. Find the cost of one year of membership.

Health Club Membership Costs

27. A company rents video games. The table shows the linear relationship between the number of games a customer can rent at one time and the monthly cost of the service.

  a. Graph the relationship.

  b. Write an equation that represents the monthly cost as a function of games rented at one time.

Games Rented at One Time	1	2	3
Monthly Cost ($)	14	18	22

Critical Thinking Tell whether each situation is possible or impossible. If possible, draw a sketch of the graphs. If impossible, explain.

28. Two different lines have the same slope.

29. Two different linear functions have the same y-intercept.

30. Two intersecting lines have the same slope.

31. A linear function does not have a y-intercept.

Match each equation with its corresponding graph.

32.

33.

34.
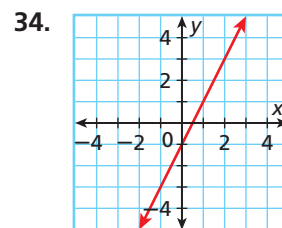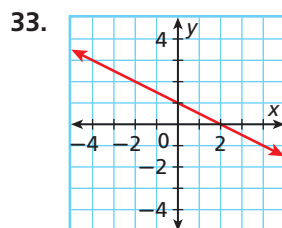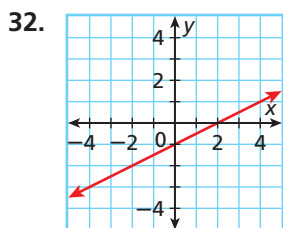
A. $y = 2x - 1$ **B.** $y = \dfrac{1}{2}x - 1$ **C.** $y = -\dfrac{1}{2}x + 1$

35. Write About It Write an equation that describes a vertical line. Can you write this equation in slope-intercept form? Why or why not?

MULTI-STEP TEST PREP

36. This problem will prepare you for the Multi-Step Test Prep on page 376.

  a. Ricardo and Sam walk from Sam's house to school. Sam lives 3 blocks from Ricardo's house. The graph shows their distance from Ricardo's house as they walk to school. Create a table of these values.

  b. Find an equation for the distance as a function of time.

  c. What are the slope and y-intercept? What do they represent in this situation?

Walk to School
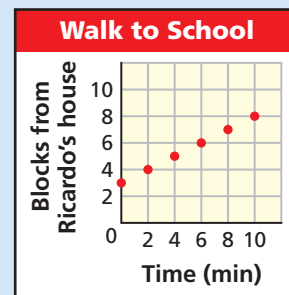
37. Which function has the same y-intercept as $y = \frac{1}{2}x - 2$?

    Ⓐ $2x + 3y = 6$ Ⓑ $x + 4y = -8$ Ⓒ $-\frac{1}{2}x + y = 4$ Ⓓ $\frac{1}{2}x - 2y = -2$

38. What is the slope-intercept form of $x - y = -8$?

    Ⓕ $y = -x - 8$ Ⓖ $y = x - 8$ Ⓗ $y = -x + 8$ Ⓙ $y = x + 8$

39. Which function has a y-intercept of 3?

    Ⓐ $2x - y = 3$ Ⓑ $2x + y = 3$ Ⓒ $2x + y = 6$ Ⓓ $y = 3x$

40. **Gridded Response** What is the slope of the line described by $-6x = -2y + 5$?

41. **Short Response** Write a function whose graph has the same slope as the line described by $3x - 9y = 9$ and the same y-intercept as $8x - 2y = 6$. Show your work.

CHALLENGE AND EXTEND

42. The standard form of a linear equation is $Ax + By = C$. Rewrite this equation in slope-intercept form. What is the slope? What is the y-intercept?

43. What value of n in the equation $nx + 5 = 3y$ would give a line with slope -2?

44. If b is the y-intercept of a linear function whose graph has slope m, then $y = mx + b$ describes the line. Below is an incomplete justification of this statement. Fill in the missing information.

Statements	Reasons
1. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$	1. Slope formula
2. $m = \dfrac{y - b}{x - 0}$	2. By definition, if b is the y-intercept, then $\left(\rule{0.4cm}{0.3cm}, b\right)$ is a point on the line. (x, y) is any other point on the line.
3. $m = \dfrac{y - b}{x}$	3. _____?_____
4. $m\,\rule{0.4cm}{0.3cm} = y - b$	4. Multiplication Property of Equality (Multiply both sides of the equation by x.)
5. $mx + b = y$, or $y = mx + b$	5. _____?_____

SPIRAL REVIEW

Define a variable and write an inequality for each situation. Graph the solutions. *(Lesson 3-1)*

45. Molly has, at most, 2 hours to work out at the gym today.

46. Mishenko is hoping to save at least $300 this month.

Solve each inequality. *(Lesson 3-5)*

47. $3n \leq 2n + 8$ **48.** $4x - 4 > 2(x + 5)$ **49.** $2(2t + 1) > 6t + 8$

50. The amount of water conditioner needed for an aquarium varies directly with the capacity of the aquarium. For every 10 gallons of water, you need 1 teaspoon of conditioner. Write a direct variation equation for the amount of water conditioner y needed for an aquarium that holds x gallons of water. Then graph. *(Lesson 5-6)*

5-8 Point-Slope Form

Why learn this?
You can use point-slope form to represent a cost function, such as the cost of placing a newspaper ad. (See Example 5.)

If you know the slope and any point on the line, you can write an equation of the line by using the slope formula. For example, suppose a line has a slope of **3** and contains $(2, 1)$. Let (x, y) be any other point on the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \longrightarrow 3 = \frac{y - 1}{x - 2}$$ *Substitute into the slope formula.*

Slope formula

$$3(x - 2) = \left(\frac{y - 1}{x - 2}\right)(x - 2)$$ *Multiplication Property of Equality*

$$3(x - 2) = y - 1$$ *Simplify.*

$$y - 1 = 3(x - 2)$$

Know it!
Note

Point-Slope Form of a Linear Equation

The line with slope m that contains the point (x_1, y_1) can be described by the equation $y - y_1 = m(x - x_1)$.

EXAMPLE 1 Writing Linear Equations in Point-Slope Form

Write an equation in point-slope form for the line with the given slope that contains the given point.

A slope $= \frac{5}{2}; (-3, 0)$

$$y - y_1 = m(x - x_1)$$ *Write the point-slope form.*

$$y - 0 = \frac{5}{2}[x - (-3)]$$ *Substitute $\frac{5}{2}$ for m, -3 for x_1, and 0 for y_1.*

$$y - 0 = \frac{5}{2}(x + 3)$$ *Rewrite subtraction of negative numbers as addition.*

B slope $= -7; (4, 2)$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -7(x - 4)$$

C slope $= 0; (-2, -3)$

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = 0[x - (-2)]$$

$$y + 3 = 0(x + 2)$$

CHECK IT OUT! Write an equation in point-slope form for the line with the given slope that contains the given point.

1a. slope $= 2; \left(\frac{1}{2}, 1\right)$

1b. slope $= 0; (3, -4)$

In Lesson 5-7, you graphed a line given its equation in slope-intercept form. You can also graph a line when given its equation in point-slope form. Start by using the equation to identify a point on the line. Then use the slope of the line to identify a second point.

EXAMPLE 2 **Using Point-Slope Form to Graph**

Graph the line described by each equation.

A $y - 1 = 3(x - 1)$

$y - 1 = 3(x - 1)$ is in the form $y - y_1 = m(x - x_1)$.

The line contains the point $(1, 1)$.

slope: $m = 3 = \dfrac{3}{1}$

Step 1 Plot $(1, 1)$.

Step 2 Count **3 units up** and **1 unit right** and plot another point.

Step 3 Draw the line connecting the two points.
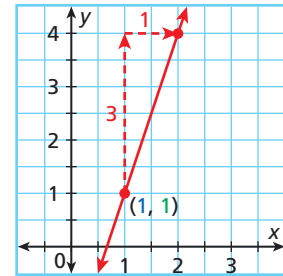
B $y + 2 = -\dfrac{1}{2}(x - 3)$

Step 1 Write the equation in point-slope form: $y - y_1 = m(x - x_1)$.

$y - (-2) = -\dfrac{1}{2}(x - 3)$ *Rewrite addition of 2 as subtraction of −2.*

Step 2 Graph the line.

The line contains the point $(3, -2)$.

slope: $m = -\dfrac{1}{2} = \dfrac{1}{-2}$

- Plot $(3, -2)$.
- Count **1 unit up** and **2 units left** and plot another point.
- Draw the line connecting the two points.
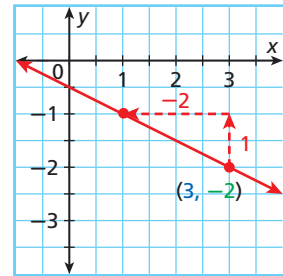
Helpful Hint

For a negative fraction, you can write the negative sign in one of three places.

$-\dfrac{1}{2} = \dfrac{-1}{2} = \dfrac{1}{-2}$

CHECK IT OUT! **Graph the line described by each equation.**

2a. $y + 2 = -(x - 2)$ **2b.** $y + 3 = -2(x - 1)$

EXAMPLE 3 **Writing Linear Equations in Slope-Intercept Form**

Write the equation that describes each line in slope-intercept form.

A slope $= -4$, $(-1, -2)$ is on the line.

Step 1 Write the equation in point-slope form: $y - y_1 = m(x - x_1)$.

$y - (-2) = -4[x - (-1)]$

Step 2 Write the equation in slope-intercept form by solving for y.

$y - (-2) = -4[x - (-1)]$

$y + 2 = -4(x + 1)$ *Rewrite subtraction of negative numbers as addition. Distribute −4 on the right side.*

$y + 2 = -4x - 4$

$\underline{ -2 \qquad\qquad -2}$ *Subtract 2 from both sides.*

$y \qquad = -4x - 6$

B $(1, -4)$ and $(3, 2)$ are on the line.

Step 1 Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-4)}{3 - 1} = \frac{6}{2} = 3$$

Step 2 Substitute the slope and one of the points into the point-slope form. Then write the equation in slope-intercept form.

$y - y_1 = m(x - x_1)$

$y - 2 = 3(x - 3)$ *Use (3, 2).*

$y - 2 = 3x - 9$ *Distribute 3 on the right side.*

$y = 3x - 7$ *Add 2 to both sides.*

C x-intercept = -2, y-intercept = 4

Step 1 Use the intercepts to find two points: $(-2, 0)$ and $(0, 4)$.

Step 2 Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{0 - (-2)} = \frac{4}{2} = 2$$

Step 3 Write the equation in slope-intercept form.

$y = mx + b$ *Write the slope-intercept form.*

$y = 2x + 4$ *Substitute 2 for m and 4 for b.*

CHECK IT OUT! Write the equation that describes each line in slope-intercept form.

3a. slope = $\frac{1}{3}$, $(-3, 1)$ is on the line.

3b. $(1, -2)$ and $(3, 10)$ are on the line.

EXAMPLE 4 **Using Two Points to Find Intercepts**

The points $(4, 8)$ and $(-1, -12)$ are on a line. Find the intercepts.

Step 1 Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-12 - 8}{-1 - 4} = \frac{-20}{-5} = 4$$

Step 2 Write the equation in slope-intercept form.

$y - y_1 = m(x - x_1)$ *Write the point-slope form.*

$y - 8 = 4(x - 4)$ *Substitute (4, 8) for (x_1, y_1) and 4 for m.*

$y - 8 = 4x - 16$ *Distribute 4 on the right side.*

$y = 4x - 8$ *Add 8 to both sides.*

Step 3 Find the intercepts.

x-intercept:

$y = 4x - 8$ *Replace y with*

$0 = 4x - 8$ *0 and solve*

$8 = 4x$ *for x.*

$2 = x$

y-intercept:

$y = 4x - 8$ *Use the slope-*

$b = -8$ *intercept form to identify the y-intercept.*

The x-intercept is 2, and the y-intercept is -8.

CHECK IT OUT! 4. The points $(2, 15)$ and $(-4, -3)$ are on a line. Find the intercepts.

EXAMPLE 5

EXAMPLE 5 *Problem-Solving Application*

PROBLEM
SOLVING

The cost to place an ad in a newspaper for one week is a linear function of the number of lines in the ad. The costs for 3, 5, and 10 lines are shown. Write an equation in slope-intercept form that represents the function. Then find the cost of an ad that is 18 lines long.

City Gazette

Newspaper Ad Costs

Lines	3	5	10
Cost ($)	13.50	18.50	31

1 **Understand the Problem**

- The **answer** will have two parts—an equation in slope-intercept form and the cost of an ad that is 18 lines long.
- The ordered pairs given in the table satisfy the equation.

2 **Make a Plan**

First, find the slope. Then use point-slope form to write the equation. Finally, write the equation in slope-intercept form.

3 **Solve**

Step 1 Choose any two ordered pairs from the table to find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{18.50 - 13.50}{5 - 3} = \frac{5}{2} = 2.5 \quad \textit{Use (3, 13.50) and (5, 18.50).}$$

Step 2 Substitute the slope and any ordered pair from the table into the point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 31 = 2.5(x - 10) \qquad\qquad \textit{Use (10, 31).}$$

Step 3 Write the equation in slope-intercept form by solving for y.

$$y - 31 = 2.5(x - 10)$$
$$y - 31 = 2.5x - 25 \qquad\qquad \textit{Distribute 2.5.}$$
$$y = 2.5x + 6 \qquad\qquad \textit{Add 31 to both sides.}$$

Step 4 Find the cost of an ad containing 18 lines by substituting 18 for x.

$$y = 2.5x + 6$$
$$y = 2.5(18) + 6 = 51$$

The cost of an ad containing 18 lines is $51.

4 **Look Back**

Check the equation by substituting the ordered pairs (3, 13.50) and (5, 18.50).

$y = 2.5x + 6$	
13.50	2.5(3) + 6
13.5	7.5 + 6
13.5	13.5 ✓

$y = 2.5x + 6$	
18.50	2.5(5) + 6
18.5	12.5 + 6
18.5	18.5 ✓

CHECK IT OUT!

5. What if...? At a different newspaper, the costs to place an ad for one week are shown. Write an equation in slope-intercept form that represents this linear function. Then find the cost of an ad that is 21 lines long.

Lines	Cost ($)
3	12.75
5	17.25
10	28.50

THINK AND DISCUSS

1. How are point-slope form and slope-intercept form alike? different?

2. When is point-slope form useful? When is slope-intercept form useful?

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe how to find the equation of a line by using the given method.

Writing the Equation of a Line

| If you know two points on the line | If you know the slope and y-intercept | If you know the slope and a point on the line |

5-8 Exercises
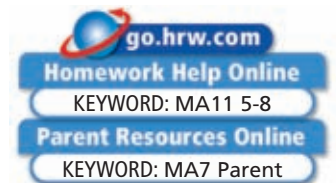
GUIDED PRACTICE

SEE EXAMPLE 1
p. 351

Write an equation in point-slope form for the line with the given slope that contains the given point.

1. slope $= \frac{1}{5}$; $(2, -6)$ **2.** slope $= -4$; $(1, 5)$ **3.** slope $= 0$; $(3, -7)$

SEE EXAMPLE 2
p. 352

Graph the line described by each equation.

4. $y - 1 = -(x - 3)$ **5.** $y + 2 = -2(x + 4)$ **6.** $y + 1 = -\frac{1}{2}(x + 4)$

SEE EXAMPLE 3
p. 352

Write the equation that describes each line in slope-intercept form.

7. slope $= -\frac{1}{3}$, $(-3, 8)$ is on the line.   **8.** slope $= 2$; $(1, 1)$ is on the line.

9. $(-2, 2)$ and $(2, -2)$ are on the line.   **10.** $(1, 1)$ and $(-5, 3)$ are on the line.

11. x-intercept $= 8$, y-intercept $= 4$ **12.** x-intercept $= -2$, y-intercept $= 3$

SEE EXAMPLE 4
p. 353

Each pair of points is on a line. Find the intercepts.

13. $(5, 2)$ and $(7, 4)$ **14.** $(-1, 5)$ and $(-3, -5)$ **15.** $(2, 9)$ and $(-4, -9)$

SEE EXAMPLE 5
p. 354

16. Measurement An oil tank is being filled at a constant rate. The depth of the oil is a function of the number of minutes the tank has been filling, as shown in the table. Write an equation in slope-intercept form that represents this linear function. Then find the depth of the oil after one-half hour.

Time (min)	Depth (ft)
0	3
10	5
15	6

PRACTICE AND PROBLEM SOLVING

Write an equation in point-slope form for the line with the given slope that contains the given point.

17. slope $= \frac{2}{9}$; $(-1, 5)$ **18.** slope $= 0$; $(4, -2)$ **19.** slope $= 8$; $(1, 8)$

Graph the line described by each equation.

20. $y - 4 = -\frac{1}{2}(x + 3)$ **21.** $y + 2 = \frac{3}{5}(x - 1)$ **22.** $y - 0 = 4(x - 1)$

Write the equation that describes each line in slope-intercept form.

23. slope $= -\frac{2}{7}$, $(14, -3)$ is on the line.   **24.** slope $= \frac{4}{5}$, $(-15, 1)$ is on the line.

25. slope $= -6$, $(9, 3)$ is on the line.   **26.** $(7, 8)$ and $(-7, 6)$ are on the line.

27. $(2, 7)$ and $(4, -4)$ are on the line.   **28.** $(-1, 2)$ and $(4, -23)$ are on the line.

29. x-intercept $= 3$, y-intercept $= -6$ **30.** x-intercept $= 4$, y-intercept $= -1$

Each pair of points is on a line. Find the intercepts.

31. $(-1, -4)$ and $(6, 10)$ **32.** $(3, 4)$ and $(-6, 16)$ **33.** $(4, 15)$ and $(-2, 6)$

34. **History** The amount of fresh water left in the tanks of a 19th-century clipper ship is a linear function of the time since the ship left port, as shown in the table. Write an equation in slope-intercept form that represents the function. Then find the amount of water that will be left in the ship's tanks 50 days after leaving port.

Fresh Water Aboard Ship	
Time (days)	Amount (gal)
1	3555
8	3240
15	2925

35. **Science** At higher altitudes, water boils at lower temperatures. This relationship between altitude and boiling point is linear. At an altitude of 1000 feet, water boils at 210 °F. At an altitude of 3000 feet, water boils at 206 °F. Write an equation in slope-intercept form that represents this linear function. Then find the boiling point at 6000 feet.

36. **Consumer Economics** Lora has a gift card from an online music store where all downloads cost the same amount. After downloading 2 songs, the balance on her card was $18.10. After downloading a total of 5 songs, the balance was $15.25.

   a. Write an equation in slope-intercept form that represents the amount in dollars remaining on the card as a function of songs downloaded.

   b. Identify the slope of the line and tell what the slope represents.

   c. Identify the y-intercept of the line and tell what it represents.

   d. How many additional songs can Lora download when there is $15.25 left on the card?

Graph the line with the given slope that contains the given point.

37. slope $= -3$; $(2, 4)$ **38.** slope $= -\frac{1}{4}$; $(0, 0)$ **39.** slope $= \frac{1}{2}$; $(-2, -1)$

Tell whether each statement is sometimes, always, or never true.

40. A line described by the equation $y = mx + b$ contains the point $(0, b)$.

41. The slope of the line that contains the points $(0, 0)$ and (c, d) is negative if both c and d are negative.

42. The y-intercept of the graph of $y - y_1 = m(x - x_1)$ is negative if y_1 is negative.

43. **Meteorology** Snowfall accumulates at an average rate of 2.5 inches per hour during a snowstorm. Two hours after the snowstorm begins, the average depth of snow on the ground is 11 inches.

   a. Write an equation in point-slope form that represents the depth of the snow in inches as a function of hours since the snowstorm began.

   b. How much snow is on the ground when the snowstorm starts?

   c. The snowstorm begins at 2:15 P.M. and continues until 6:30 P.M. How much snow is on the ground at the end of the storm?

LINK

Science

As altitude increases, the amount of breathable oxygen decreases. At elevations above 8000 feet, this can cause altitude sickness. To prevent this, mountain climbers often use tanks containing a mixture of air and pure oxygen.

Write an equation in point-slope form that describes each graph.

44.

45.

46.
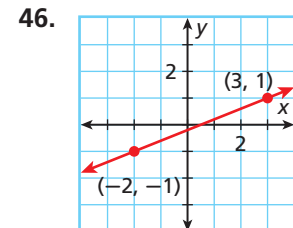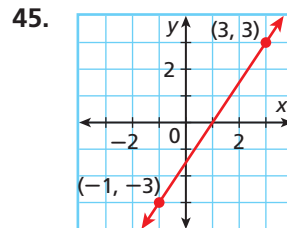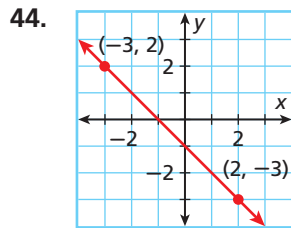
The tables show linear relationships between x and y. Copy and complete the tables.

47.

x	−2	0	■	7
y	−18	■	12	27

48.

x	−4	1	0	■
y	14	4	■	−6

49. **///ERROR ANALYSIS///** Two students used point-slope form to find an equation that describes the line with slope −3 through $(-5, 2)$. Who is incorrect? Explain the error.

A

$y - y_1 = m(x - x_1)$
$y - 2 = -3(x - 5)$

B

$y - y_1 = m(x - x_1)$
$y - 2 = -3[x - (-5)]$
$y - 2 = -3(x + 5)$

50. **Critical Thinking** Compare the methods for finding the equation that describes a line when you know

- a point on the line and the slope of the line.
- two points on the line.

How are the methods alike? How are they different?

51. **Write About It** Explain why the first statement is false but the second is true.

- All linear equations can be written in point-slope form.
- All linear equations that describe functions can be written in point-slope form.

52. **Multi-Step** The table shows the mean scores on a standardized test for several different years.

Years Since 1985	0	5	10	17	21
Mean Combined Score	994	1009	1001	1016	1020

a. Make a scatter plot of the data and add a trend line to your graph.

b. Use your trend line to estimate the slope and y-intercept, and write an equation in slope-intercept form.

c. What do the slope and y-intercept represent in this situation?

MULTI-STEP TEST PREP

53. This problem will prepare you for the Multi-Step Test Prep on page 376.

a. Stephen is walking from his house to his friend Sharon's house. When he is 12 blocks away, he looks at his watch. He looks again when he is 8 blocks away and finds that 6 minutes have passed. Write two ordered pairs for these data in the form (time, blocks).

b. Write a linear equation for these two points.

c. What is the total amount of time it takes Stephen to reach Sharon's house? Explain how you found your answer.

54. Which equation describes the line through $(-5, 1)$ with slope of 1?

    Ⓐ $y + 1 = x - 5$ Ⓒ $y - 1 = -5(x - 1)$

    Ⓑ $y + 5 = x - 1$ Ⓓ $y - 1 = x + 5$

55. A line contains (4, 4) and (5, 2). What are the slope and y-intercept?

    Ⓕ slope $= -2$; y-intercept $= 2$ Ⓗ slope $= -2$; y-intercept $= 12$

    Ⓖ slope $= 1.2$; y-intercept $= -2$ Ⓙ slope $= 12$; y-intercept $= 1.2$

CHALLENGE AND EXTEND

56. A linear function has the same y-intercept as $x + 4y = 8$ and its graph contains the point $(2, 7)$. Find the slope and y-intercept.

57. Write the equation of a line in slope-intercept form that contains $\left(\frac{3}{4}, \frac{1}{2}\right)$ and has the same slope as the line described by $y + 3x = 6$.

58. Write the equation of a line in slope-intercept form that contains $\left(-\frac{1}{2}, -\frac{1}{3}\right)$ and $\left(1\frac{1}{2}, 1\right)$.

SPIRAL REVIEW

Solve each compound inequality and graph the solutions. *(Lesson 3-6)*

59. $-4 \leq x + 2 \leq 1$ **60.** $m - 5 > -7$ AND $m + 1 < 2$

61. A group of hikers is walking the Appalachian Trail. The function $y = 12x$ describes how many miles y the group has traveled in x days. Graph the function. Then use the graph to estimate how many miles the group will hike in 18 days. *(Lesson 4-4)*

Write the equation that describes each line in slope-intercept form. *(Lesson 5-7)*

62. slope $= 3$, y-intercept $= -5$ **63.** slope $= -2$, $(2, 4)$ is on the line

Career Path

Q: What math classes did you take in high school?
A: Algebra 1 and 2, Geometry, and Statistics

Q: What math classes have you taken in college?
A: Applied Statistics, Data Mining Methods, Web Mining, and Artificial Intelligence

Q: How do you use math?
A: Once for a class, I used software to analyze basketball statistics. What I learned helped me develop strategies for our school team.

Michael Raynor
Data mining major

Q: What are your future plans?
A: There are many options for people with data mining skills. I could work in banking, pharmaceuticals, or even the military. But my dream job is to develop game strategies for an NBA team.

Technology LAB

Graph Linear Functions

You can use a graphing calculator to quickly graph lines whose equations are in point-slope form. To enter an equation into your calculator, it must be solved for y, but it does not necessarily have to be in slope-intercept form.

Use with Lesson 5-8

Activity
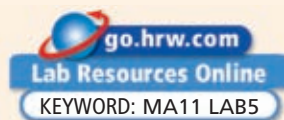
Graph the line with slope 2 that contains the point $(2, 6.09)$.

1 Use point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 6.09 = 2(x - 2)$$

2 Solve for y by adding 6.09 to both sides of the equation.

$$y - 6.09 = 2(x - 2)$$
$$\underline{+\,6.09 \qquad\quad +\,6.09}$$
$$y \qquad\; = 2(x - 2) + 6.09$$

3 Enter this equation into your calculator.

`Y=` `2` `(` `X,T,θ,n` `−` `2` `)` `+` `6.09` `ENTER`

Plot1 Plot2 Plot3
\Y1■2(X−2)+6.09
\Y2=■

4 Graph in the *standard viewing window* by pressing `ZOOM` and selecting **6:ZStandard**. In this window, both the x- and y-axes go from -10 to 10.
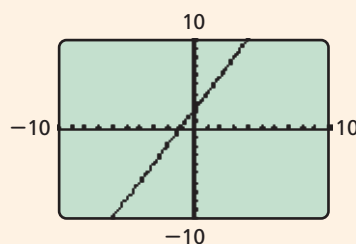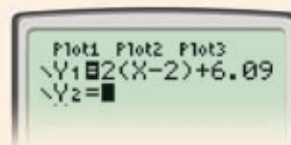
5 Notice that the scale on the y-axis is smaller than the scale on the x-axis. This is because the width of the calculator screen is about 50% greater than its height. To see a more accurate graph of this line, use the *square viewing window*. Press `ZOOM` and select **5:ZSquare**.
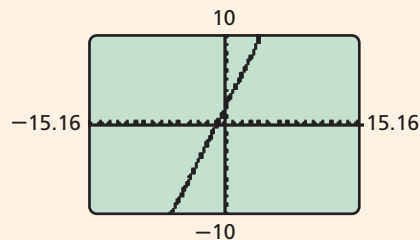
Try This

1. Graph the function represented by the line with slope -1.5 that contains the point $(2.25, -3)$. View the graph in the standard viewing window.

2. Now view the graph in the square viewing window. Press `WINDOW` and write down the minimum and maximum values on the x- and y-axes.

3. In which graph does the line appear steeper? Why?

4. Explain why it might sometimes be useful to look at a graph in a square window.

Interpreting Trend Lines

In Chapter 4 you learned how to draw trend lines on scatter plots. Now you will learn how to find the equations of trend lines and write them in slope-intercept form.

Example

Write an equation for the trend line on the scatter plot.

Two points on the trend line are (30, 75) and (60, 90).

To find the slope of the line that contains (**30**, **75**) and (**60**, **90**), use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \textit{Use the slope formula.}$$

$$m = \frac{90 - 75}{60 - 30} \qquad \textit{Substitute (30, 75) for } (x_1, y_1) \textit{ and}$$
$$\qquad\qquad\qquad\quad (60, 90) \textit{ for } (x_2, y_2).$$

$$m = \frac{15}{30} \qquad \textit{Simplify.}$$

$$m = \frac{1}{2}$$

Test Scores vs. Study Time

Use the slope and the point (30, 75) to find the *y*-intercept of the line.

$$y = mx + b \qquad \textit{Slope-intercept form}$$

$$75 = \frac{1}{2}(30) + b \qquad \textit{Substitute } \frac{1}{2} \textit{ for m, 30 for x, and 75 for y.}$$

$$75 = 15 + b \qquad \textit{Solve for b.}$$

$$60 = b$$

Write the equation.

$$y = \frac{1}{2}x + 60 \qquad \textit{Substitute } \frac{1}{2} \textit{ for m and 60 for b.}$$
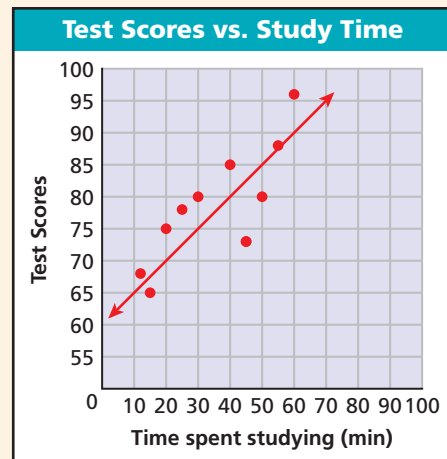
Try This

1. In the example above, what is the meaning of the slope?

2. What does the *y*-intercept represent?

3. Use the equation to predict the test score of a student who spent 25 minutes studying.

4. Use the table to create a scatter plot. Draw a trend line and find the equation of your trend line. Tell the meaning of the slope and *y*-intercept. Then use your equation to predict the race time of a runner who ran 40 miles in training.

Distance Run in Training (mi)	12	15	16	18	21	23	24	25	33
Race Time (min)	65	64	55	58	55	50	50	47	36

5-9 Slopes of Parallel and Perpendicular Lines

Objectives
Identify and graph parallel and perpendicular lines.

Write equations to describe lines parallel or perpendicular to a given line.

Vocabulary
parallel lines
perpendicular lines

Why learn this?
Parallel lines and their equations can be used to model costs, such as the cost of a booth at a farmers' market.

To sell at a particular farmers' market for a year, there is a $100 membership fee. Then you pay $3 for each hour that you sell at the market. However, if you were a member the previous year, the membership fee is reduced to $50.

- The **red** line shows the total cost if you are a new member.

- The **blue** line shows the total cost if you are a returning member.

These two lines are *parallel*. **Parallel lines** are lines in the same plane that have no points in common. In other words, they do not intersect.

Farmers' Market Fees

Know it! Note
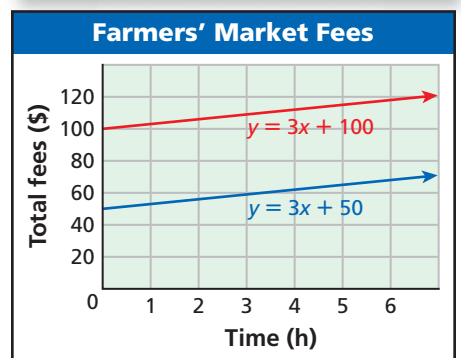
Parallel Lines

WORDS	Two different nonvertical lines are parallel if and only if they have the same slope.	All different vertical lines are parallel.
GRAPH		
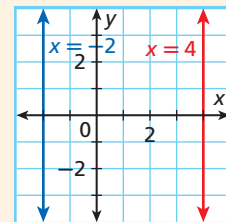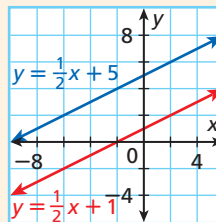
EXAMPLE 1 **Identifying Parallel Lines**

Identify which lines are parallel.

A $y = \frac{4}{3}x + 3$; $y = 2$; $y = \frac{4}{3}x - 5$; $y = -3$

The lines described by $y = \frac{4}{3}x + 3$ and $y = \frac{4}{3}x - 5$ both have slope $\frac{4}{3}$. These lines are parallel. The lines described by $y = 2$ and $y = -3$ both have slope 0. These lines are parallel.
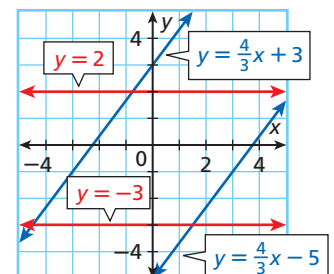
Identify which lines are parallel.

B $y = 3x + 2$; $y = -\dfrac{1}{2}x + 4$; $x + 2y = -4$; $y - 5 = 3(x - 1)$

Write all equations in slope-intercept form to determine the slopes.

$y = 3x + 2$	$y = -\dfrac{1}{2}x + 4$
slope-intercept form ✓	slope-intercept form ✓

$$x + 2y = -4$$
$$\underline{-x \qquad\quad -x}$$
$$2y = -x - 4$$
$$\frac{2y}{2} = \frac{-x - 4}{2}$$
$$y = -\frac{1}{2}x - 2$$

$$y - 5 = 3(x - 1)$$
$$y - 5 = 3x - 3$$
$$\underline{+5 \qquad\quad +5}$$
$$y \qquad = 3x + 2$$

The lines described by $y = 3x + 2$ and $y - 5 = 3(x - 1)$ have the same slope, but they are not parallel lines. They are the same line.

The lines described by $y = -\frac{1}{2}x + 4$ and $x + 2y = -4$ represent parallel lines. They each have slope $-\frac{1}{2}$.
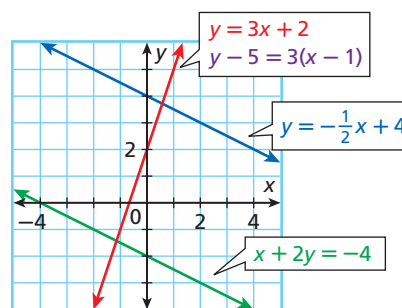
CHECK IT OUT! **Identify which lines are parallel.**

1a. $y = 2x + 2$; $y = 2x + 1$; $y = -4$; $x = 1$

1b. $y = \dfrac{3}{4}x + 8$; $-3x + 4y = 32$; $y = 3x$; $y - 1 = 3(x + 2)$

EXAMPLE 2 *Geometry Application*

Geometry

Show that $ABCD$ is a parallelogram.

Use the ordered pairs and the slope formula to find the slopes of $\overline{AB}$ and $\overline{CD}$.

$$\text{slope of } \overline{AB} = \frac{7 - 5}{4 - (-1)} = \frac{2}{5}$$

$$\text{slope of } \overline{CD} = \frac{3 - 1}{4 - (-1)} = \frac{2}{5}$$

Remember!

In a parallelogram, opposite sides are parallel.

$\overline{AB}$ is parallel to $\overline{CD}$ because they have the same slope.

$\overline{AD}$ is parallel to $\overline{BC}$ because they are both vertical.

Therefore, $ABCD$ is a parallelogram because both pairs of opposite sides are parallel.
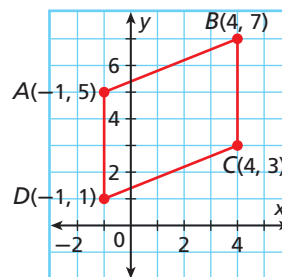
CHECK IT OUT! **2.** Show that the points $A(0, 2)$, $B(4, 2)$, $C(1, -3)$, and $D(-3, -3)$ are the vertices of a parallelogram.

Perpendicular lines are lines that intersect to form right angles (90°).

Know it! Note

Perpendicular Lines

WORDS	Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.	Vertical lines are perpendicular to horizontal lines.
GRAPH	$y = -\frac{2}{3}x + 3$ $y = \frac{3}{2}x - 2$	$y = 3$ $x = 2$

EXAMPLE 3   **Identifying Perpendicular Lines**

Identify which lines are perpendicular: $x = -2$; $y = 1$; $y = -4x$; $y + 2 = \frac{1}{4}(x + 1)$.

The graph described by $x = -2$ is a vertical line, and the graph described by $y = 1$ is a horizontal line. These lines are perpendicular.

The slope of the line described by $y = -4x$ is -4. The slope of the line described by $y + 2 = \frac{1}{4}(x - 1)$ is $\frac{1}{4}$.

$$(-4)\left(\frac{1}{4}\right) = -1$$

These lines are perpendicular because the product of their slopes is -1.

$x = -2$ $y = 1$ $y + 2 = \frac{1}{4}(x - 1)$ $y = -4x$
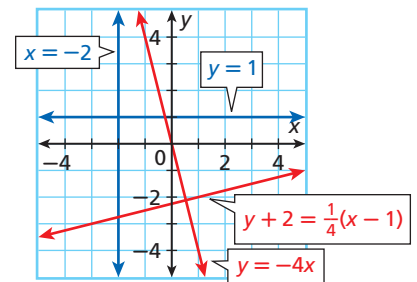
CHECK IT OUT!   **3.** Identify which lines are perpendicular: $y = -4$; $y - 6 = 5(x + 4)$; $x = 3$; $y = -\frac{1}{5}x + 2$.

EXAMPLE 4   *Geometry Application*

Geometry

Show that PQR is a right triangle.

If PQR is a right triangle, $\overline{PQ}$ will be perpendicular to $\overline{QR}$.

$$\text{slope of } \overline{PQ} = \frac{3 - 1}{3 - 0} = \frac{2}{3}$$

$$\text{slope of } \overline{QR} = \frac{3 - 0}{3 - 5} = \frac{3}{-2} = -\frac{3}{2}$$

$\overline{PQ}$ is perpendicular to $\overline{QR}$ because $\frac{2}{3}\left(-\frac{3}{2}\right) = -1$.

Therefore, PQR is a right triangle because it contains a right angle.

$Q(3, 3)$ $P(0, 1)$ $R(5, 0)$

Helpful Hint

A right triangle contains one right angle. In Example 4, $\angle P$ and $\angle R$ are clearly not right angles, so the only possibility is $\angle Q$.
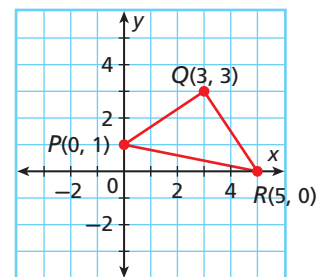
CHECK IT OUT!   **4.** Show that $P(1, 4)$, $Q(2, 6)$, and $R(7, 1)$ are the vertices of a right triangle.

Writing Equations of Parallel and Perpendicular Lines

A Write an equation in slope-intercept form for the line that passes through $(4, 5)$ and is parallel to the line described by $y = 5x + 10$.

Step 1 Find the slope of the line.

$y = \mathbf{5}x + 10$ *The slope is 5.*

The parallel line also has a slope of **5**.

Step 2 Write the equation in point-slope form.

$y - y_1 = \mathbf{m}(x - x_1)$ *Use point-slope form.*

$y - 5 = \mathbf{5}(x - \mathbf{4})$ *Substitute 5 for m, 4 for x_1, and 5 for y_1.*

Step 3 Write the equation in slope-intercept form.

$y - 5 = 5(x - 4)$

$y - 5 = 5x - 20$ *Distributive Property*

$y = 5x - 15$ *Addition Property of Equality*

B Write an equation in slope-intercept form for the line that passes through $(3, 2)$ and is perpendicular to the line described by $y = 3x - 1$.

Step 1 Find the slope of the line.

$y = \mathbf{3}x - 1$ *The slope is 3.*

The perpendicular line has a slope of $-\frac{1}{3}$, because $3\left(-\frac{1}{3}\right) = -1$.

Step 2 Write the equation in point-slope form.

$y - y_1 = \mathbf{m}(x - x_1)$ *Use point-slope form.*

$y - 2 = -\dfrac{1}{3}(x - \mathbf{3})$ *Substitute $-\frac{1}{3}$ for m, 3 for x_1, and 2 for y_1.*

Step 3 Write the equation in slope-intercept form.

$y - 2 = -\dfrac{1}{3}(x - 3)$

$y - 2 = -\dfrac{1}{3}x + 1$ *Distributive Property*

$y = -\dfrac{1}{3}x + 3$ *Addition Property of Equality*

Helpful Hint

If you know the slope of a line, the slope of a perpendicular line will be the "opposite reciprocal."

$\dfrac{2}{3} \rightarrow -\dfrac{3}{2}$

$\dfrac{1}{5} \rightarrow -5$

$-7 \rightarrow \dfrac{1}{7}$

CHECK IT OUT!

5a. Write an equation in slope-intercept form for the line that passes through $(5, 7)$ and is parallel to the line described by $y = \frac{4}{5}x - 6$.

5b. Write an equation in slope-intercept form for the line that passes through $(-5, 3)$ and is perpendicular to the line described by $y = 5x$.

THINK AND DISCUSS

1. Are the lines described by $y = \frac{1}{2}x$ and $y = 2x$ perpendicular? Explain.

2. Describe the slopes and y-intercepts when two nonvertical lines are parallel.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, sketch an example and describe the slopes.

Parallel lines	Perpendicular lines

Know it! Note

GUIDED PRACTICE

1. Vocabulary ____?____ lines have the same slope. (*Parallel* or *Perpendicular*)

SEE EXAMPLE **1**
p. 361

Identify which lines are parallel.

2. $y = 6$; $y = 6x + 5$; $y = 6x - 7$; $y = -8$

3. $y = \frac{3}{4}x - 1$; $y = -2x$; $y - 3 = \frac{3}{4}(x - 5)$; $y - 4 = -2(x + 2)$

SEE EXAMPLE **2**
p. 362

4. Geometry Show that *ABCD* is a trapezoid.
(*Hint:* In a trapezoid, exactly one pair of opposite
sides is parallel.)
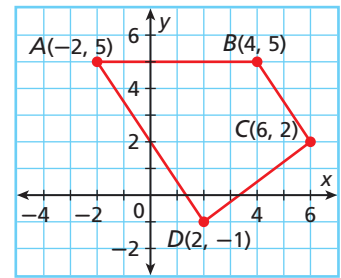
SEE EXAMPLE **3**
p. 363

Identify which lines are perpendicular.

5. $y = \frac{2}{3}x - 4$; $y = -\frac{3}{2}x + 2$; $y = -1$; $x = 3$

6. $y = -\frac{3}{7}x - 4$; $y - 4 = -7(x + 2)$;

$y - 1 = \frac{1}{7}(x - 4)$; $y - 7 = \frac{7}{3}(x - 3)$

SEE EXAMPLE **4**
p. 363

7. Geometry Show that *PQRS* is a rectangle. (*Hint:* In a
rectangle, all four angles are right angles.)
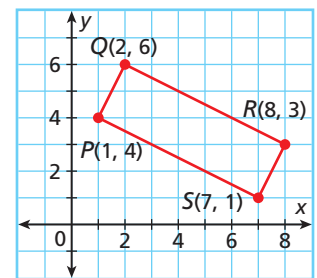
SEE EXAMPLE **5**
p. 364

8. Write an equation in slope-intercept form for the line
that passes through (5, 0) and is perpendicular to the
line described by $y = -\frac{5}{2}x + 6$.

PRACTICE AND PROBLEM SOLVING

Identify which lines are parallel.

9. $x = 7$; $y = -\frac{5}{6}x + 8$; $y = -\frac{5}{6}x - 4$; $x = -9$

10. $y = -x$; $y - 3 = -1(x + 9)$; $y - 6 = \frac{1}{2}(x - 14)$; $y + 1 = \frac{1}{2}x$

11. $y = -3x + 2$; $y = \frac{1}{2}x - 1$; $-x + 2y = 17$; $3x + y = 27$

12. Geometry Show that *LMNP* is a
parallelogram.
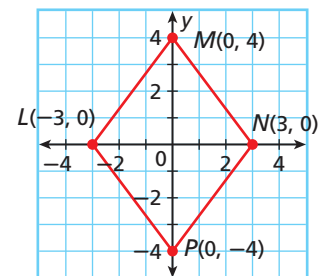
Identify which lines are perpendicular.

13. $y = 6x$; $y = \frac{1}{6}x$; $y = -\frac{1}{6}x$; $y = -6x$

14. $y - 9 = 3(x + 1)$; $y = -\frac{1}{3}x + 5$; $y = 0$; $x = 6$

15. $x - 6y = 15$; $y = 3x - 2$; $y = -3x - 3$; $y = -6x - 8$; $3y = -x - 11$

16. Geometry Show that ABC is a right triangle.

17. Write an equation in slope-intercept form for the line that passes through $(0, 0)$ and is parallel to the line described by $y = -\frac{6}{7}x + 1$.
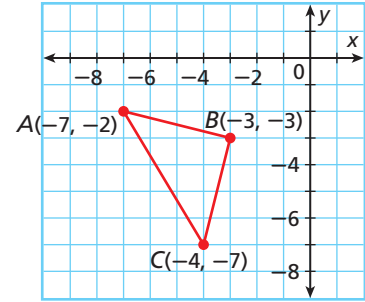
Without graphing, tell whether each pair of lines is parallel, perpendicular, or neither.

18. $x = 2$ and $y = -5$

19. $y = 7x$ and $y - 28 = 7(x - 4)$

20. $y = 2x - 1$ and $y = \frac{1}{2}x + 2$

21. $y - 3 = \frac{1}{4}(x - 3)$ and $y + 13 = \frac{1}{4}(x + 1)$

Write an equation in slope-intercept form for the line that is parallel to the given line and that passes through the given point.

22. $y = 3x - 7;\ (0, 4)$

23. $y = \frac{1}{2}x + 5;\ (4, -3)$

24. $4y = x;\ (4, 0)$

25. $y = 2x + 3;\ (1, 7)$

26. $5x - 2y = 10;\ (3, -5)$

27. $y = 3x - 4;\ (-2, 7)$

28. $y = 7;\ (2, 4)$

29. $x + y = 1;\ (2, 3)$

30. $2x + 3y = 7;\ (4, 5)$

31. $y = 4x + 2;\ (5, -3)$

32. $y = \frac{1}{2}x - 1;\ (0, -4)$

33. $3x + 4y = 8;\ (4, -3)$

Write an equation in slope-intercept form for the line that is perpendicular to the given line and that passes through the given point.

34. $y = -3x + 4;\ (6, -2)$

35. $y = x - 6;\ (-1, 2)$

36. $3x - 4y = 8;\ (-6, 5)$

37. $5x + 2y = 10;\ (3, -5)$

38. $y = 5 - 3x;\ (2, -4)$

39. $-10x + 2y = 8;\ (4, -3)$

40. $2x + 3y = 7;\ (4, 5)$

41. $4x - 2y = -6;\ (3, -2)$

42. $-2x - 8y = 16;\ (4, 5)$

43. $y = -2x + 4;\ (-2, 5)$

44. $y = x - 5;\ (0, 5)$

45. $x + y = 2;\ (8, 5)$

46. Write an equation describing the line that is parallel to the y-axis and that is 6 units to the right of the y-axis.

47. Write an equation describing the line that is perpendicular to the y-axis and that is 4 units below the x-axis.

48. Critical Thinking Is it possible for two linear functions whose graphs are parallel lines to have the same y-intercept? Explain.

49. Estimation Estimate the slope of a line that is perpendicular to the line through $(2.07, 8.95)$ and $(-1.9, 25.07)$.

50. Write About It Explain in words how to write an equation in slope-intercept form that describes a line parallel to $y - 3 = -6(x - 3)$.

MULTI-STEP TEST PREP

WAIT WALK

51. This problem will prepare you for the Multi-Step Test Prep on page 376.

   a. Flora walks from her home to the bus stop at a rate of 50 steps per minute. Write a rule that gives her distance from home (in steps) as a function of time.

   b. Flora's neighbor Dan lives 30 steps closer to the bus stop. He begins walking at the same time and at the same pace as Flora. Write a rule that gives Dan's distance from *Flora's* house as a function of time.

   c. Will Flora meet Dan along the walk? Use a graph to help explain your answer.

52. Which describes a line parallel to the line described by $y = -3x + 2$?

(A) $y = -3x$ (B) $y = \frac{1}{3}x$ (C) $y = 2 - 3x$ (D) $y = \frac{1}{3}x + 2$

53. Which describes a line passing through $(3, 3)$ that is perpendicular to the line described by $y = \frac{3}{5}x + 2$?

(F)

(H)
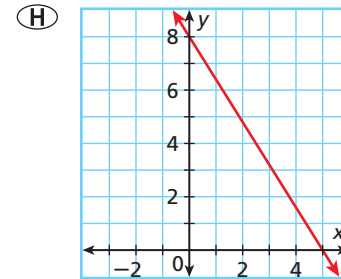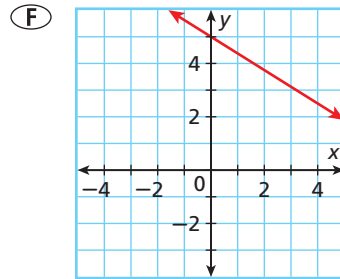
(G) $y = \frac{5}{3}x - 2$ (J) $y = \frac{3}{5}x + \frac{6}{5}$

54. Gridded Response The graph of a linear function $f(x)$ is parallel to the line described by $2x + y = 5$ and contains the point $(6, -2)$. What is the y-intercept of $f(x)$?

CHALLENGE AND EXTEND

55. Three or more points that lie on the same line are called *collinear points*. Explain why the points A, B, and C must be collinear if the line containing A and B has the same slope as the line containing B and C.

56. The lines described by $y = (a + 12)x + 3$ and $y = 4ax$ are parallel. What is the value of a?

57. The lines described by $y = (5a + 3)x$ and $y = -\frac{1}{2}x$ are perpendicular. What is the value of a?

58. Geometry The diagram shows a square in the coordinate plane. Use the diagram to show that the diagonals of a square are perpendicular.
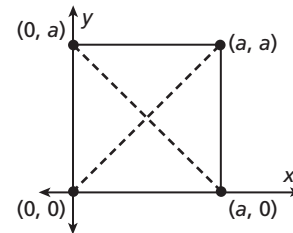
SPIRAL REVIEW

59. The record high temperature for a given city is 112 °F. The morning temperature today was 94 °F and the temperature will increase t degrees. Write and solve an inequality to find all values of t that would break the record for the high temperature. *(Lesson 3-2)*

Graph each function. *(Lesson 4-4)*

60. $y = -3x + 5$ **61.** $y = x - 1$ **62.** $y = x^2 - 3$

Write an equation in slope-intercept form for the line with the given slope that contains the given point. *(Lesson 5-8)*

63. slope $= \frac{2}{3}$; $(6, -1)$ **64.** slope $= -5$; $(2, 4)$ **65.** slope $= -\frac{1}{2}$; $(-1, 0)$

66. slope $= -\frac{1}{3}$; $(2, 7)$ **67.** slope $= 0$; $(-3, 3)$ **68.** slope $= \frac{1}{5}$; $(-4, -2)$

Technology LAB

The Family of Linear Functions

A *family of functions* is a set of functions whose graphs have basic characteristics in common. For example, all linear functions form a family. You can use a graphing calculator to explore families of functions.

Use with Lesson 5-10

go.hrw.com
Lab Resources Online
KEYWORD: MA11 LAB5

Activity

Graph the lines described by $y = x - 2$, $y = x - 1$, $y = x$, $y = x + 1$, $y = x + 2$, $y = x + 3$, and $y = x + 4$. How does the value of b affect the graph described by $y = x + b$?

1 All of the functions are in the form $y = x + b$. Enter them into the **Y=** editor.

Y= **X,T,θ,n** **—** **2** **ENTER**

X,T,θ,n **—** **1** **ENTER**

and so on.

```
Plot1  Plot2  Plot3
\Y1◻X-2
\Y2◻X-1
\Y3◻X
\Y4◻X+1
\Y5◻X+2
\Y6◻X+3
\Y7◻X+4■
```
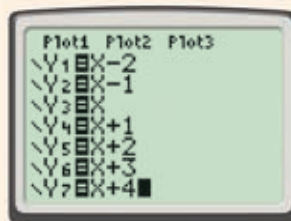
2 Press **ZOOM** and select **6:Zstandard.** Think about the different values of b as you watch the graphs being drawn. Notice that the lines are all parallel.
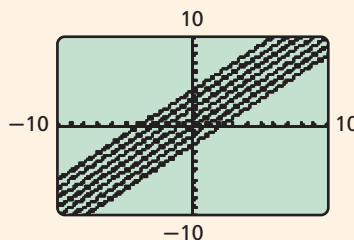
3 It appears that the value of b in $y = x + b$ shifts the graph up or down—up if b is positive and down if b is negative.

Try This

1. Make a prediction about the lines described by $y = 2x - 3$, $y = 2x - 2$, $y = 2x - 1$, $y = 2x$, $y = 2x + 1$, $y = 2x + 2$, and $y = 2x + 3$. Then graph. Was your prediction correct?

2. Now use your calculator to explore what happens to the graph of $y = mx$ when you change the value of m.

   a. Make a Prediction How do you think the lines described by $y = -2x$, $y = -x$, $y = x$, and $y = 2x$ will be related? How will they be alike? How will they be different?

   b. Graph the functions given in part **a**. Was your prediction correct?

   c. How is the effect of m different when m is positive from when m is negative?

Transforming Linear Functions

Objective
Describe how changing slope and *y*-intercept affect the graph of a linear function.

Vocabulary
family of functions
parent function
transformation
translation
rotation
reflection

Who uses this?
Business owners can use transformations to show the effects of price changes, such as the price of trophy engraving. (See Example 5.)

A **family of functions** is a set of functions whose graphs have basic characteristics in common. For example, all linear functions form a family because all of their graphs are the same basic shape.

A **parent function** is the most basic function in a family. For linear functions, the parent function is $f(x) = x$.

The graphs of all other linear functions are *transformations* of the graph of the parent function, $f(x) = x$. A **transformation** is a change in position or size of a figure.

There are three types of transformations—*translations, rotations,* and *reflections.*

Look at the four functions and their graphs below.

> **Remember!**
> Function notation—$f(x)$, $g(x)$, and so on—can be used in place of *y*.
> $$y = f(x)$$
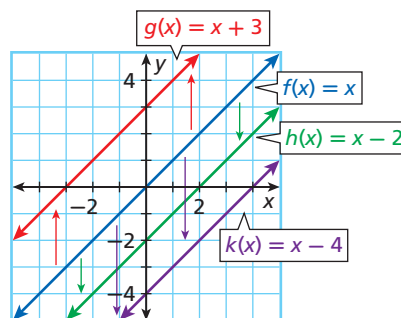
$g(x) = x + 3$
$f(x) = x$
$h(x) = x - 2$
$k(x) = x - 4$

Notice that all of the lines above are parallel. The slopes are the same but the *y*-intercepts are different.

The graphs of $g(x) = x + 3$, $h(x) = x - 2$, and $k(x) = x - 4$ are vertical *translations* of the graph of the parent function, $f(x) = x$. A **translation** is a type of transformation that moves every point the same distance in the same direction. You can think of a translation as a "slide."

> **Know it! Note**
>
> ## Vertical Translation of a Linear Function
> When the *y*-intercept *b* is changed in the function $f(x) = mx + b$, the graph is translated vertically.
> - If *b* increases, the graph is translated up.
> - If *b* decreases, the graph is translated down.

EXAMPLE **1** **Translating Linear Functions**

Graph $f(x) = x$ and $g(x) = x - 5$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.
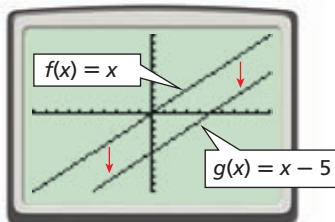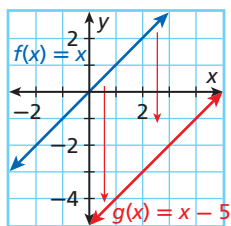
The graph of **$g(x) = x - 5$** is the result of translating the graph of $f(x) = x$ 5 units down.

CHECK IT OUT! **1.** Graph $f(x) = x + 4$ and $g(x) = x - 2$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.

The graphs of **$g(x) = 3x$**, **$h(x) = 5x$**, and **$k(x) = \frac{1}{2}x$** are *rotations* of the graph of $f(x) = x$. A **rotation** is a transformation about a point. You can think of a rotation as a "turn." The y-intercepts are the same, but the slopes are different.
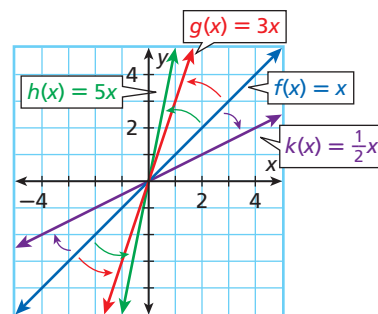
Know it! Note

Rotation of a Linear Function

When the slope m is changed in the function $f(x) = mx + b$ it causes a rotation of the graph about the point $(0, b)$, which changes the line's steepness.

EXAMPLE **2** **Rotating Linear Functions**

Graph $f(x) = x + 2$ and $g(x) = 2x + 2$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.
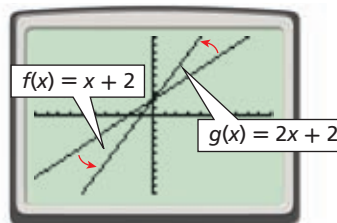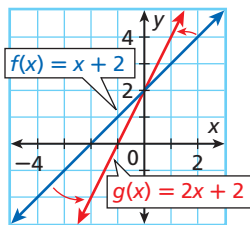
The graph of **$g(x) = 2x + 2$** is the result of rotating the graph of $f(x) = x + 2$ about $(0, 2)$. The graph of $g(x)$ is steeper than the graph of $f(x)$.

CHECK IT OUT! **2.** Graph $f(x) = 3x - 1$ and $g(x) = \frac{1}{2}x - 1$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.

The diagram shows the *reflection* of the graph of $f(x) = 2x$ across the y-axis, producing the graph of $g(x) = -2x$. A **reflection** is a transformation across a line that produces a mirror image. You can think of a reflection as a "flip" over a line.

Know it! Note
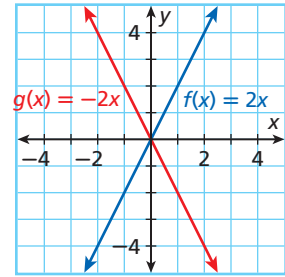
Reflection of a Linear Function

When the slope m is multiplied by -1 in $f(x) = mx + b$, the graph is reflected across the y-axis.

EXAMPLE 3 Reflecting Linear Functions

Graph $f(x)$. Then reflect the graph of $f(x)$ across the y-axis. Write a function $g(x)$ to describe the new graph.

A $f(x) = x$
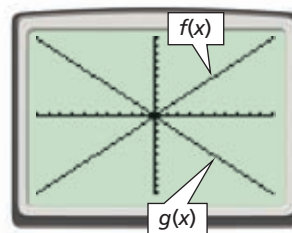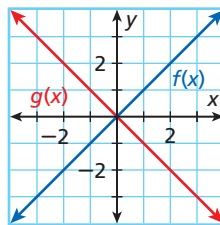
To find $g(x)$, multiply the value of m by -1.
In $f(x) = x$, $m = 1$.
$1(-1) = -1$ *This is the value of m for g(x).*
$g(x) = -x$

B $f(x) = -4x - 1$

To find $g(x)$, multiply the value of m by -1.
In $f(x) = -4x - 1$, $m = -4$.
$-4(-1) = 4$ *This is the value of m for g(x).*
$g(x) = 4x - 1$

CHECK IT OUT!

3. Graph $f(x) = \frac{2}{3}x + 2$. Then reflect the graph of $f(x)$ across the y-axis. Write a function $g(x)$ to describe the new graph.

EXAMPLE 4 — Multiple Transformations of Linear Functions

Graph $f(x) = x$ and $g(x) = 3x + 1$. Then describe the transformations from the graph of $f(x)$ to the graph of $g(x)$.

Find transformations of $f(x) = x$ that will result in $g(x) = 3x + 1$:

- Multiply $f(x)$ by 3 to get $h(x) = 3x$. This rotates the graph about $(0, 0)$ and makes it steeper.

- Then add 1 to $h(x)$ to get $g(x) = 3x + 1$. This translates the graph 1 unit up.

The transformations are a rotation and a translation.

CHECK IT OUT! 4. Graph $f(x) = x$ and $g(x) = -x + 2$. Then describe the transformations from the graph of $f(x)$ to the graph of $g(x)$.

EXAMPLE 5 — *Business Application*

A trophy company charges $175 for a trophy plus $0.20 per letter for the engraving. The total charge for a trophy with x letters is given by the function $f(x) = 0.20x + 175$. How will the graph change if the trophy's cost is lowered to $172? if the charge per letter is raised to $0.50?
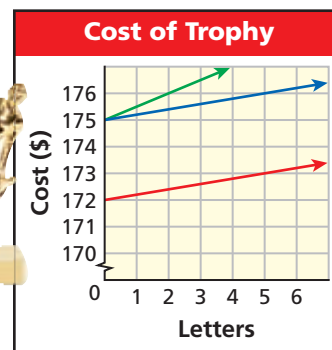
$f(x) = 0.20x + 175$ is graphed in blue.

If the trophy's cost is lowered to $172, the new function is $g(x) = 0.20x + 172$.
The original graph will be translated 3 units down.

If the charge per letter is raised to $0.50, the new function is $h(x) = 0.50x + 175$. The original graph will be rotated about $(0, 175)$ and become steeper.

CHECK IT OUT! 5. **What if...?** How will the graph change if the charge per letter is lowered to $0.15? if the trophy's cost is raised to $180?

THINK AND DISCUSS

1. Describe the graph of $f(x) = x + 3.45$

2. Look at the graphs in Example 5. For each line, is every point on the line a solution in this situation? Explain.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, sketch a graph of the given transformation of $f(x) = x$, and label it with a possible equation.

Know it! Note

Transformations of $f(x) = x$

| Translation | Rotation | Reflection |

5-10 **Exercises**

go.hrw.com
Homework Help Online
KEYWORD: MA11 5-10
Parent Resources Online
KEYWORD: MA7 Parent
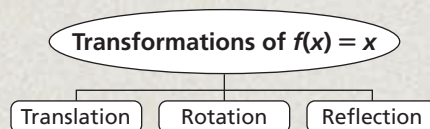
GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

1. Changing the value of b in $f(x) = mx + b$ results in a ___?___ of the graph. (*translation* or *reflection*)

2. Changing the value of m in $f(x) = mx + b$ results in a ___?___ of the graph. (*translation* or *rotation*)

Graph $f(x)$ and $g(x)$. Then describe the transformation from the graph of $f(x)$ to the graph of $g(x)$.

SEE EXAMPLE **1**
p. 370

**3.** $f(x) = x, g(x) = x - 4$

4. $f(x) = x, g(x) = x + 1$

5. $f(x) = x, g(x) = x + 2$

6. $f(x) = x, g(x) = x - 6.5$

SEE EXAMPLE **2**
p. 370

7. $f(x) = x, g(x) = \frac{1}{4}x$

8. $f(x) = \frac{1}{5}x + 3, g(x) = x + 3$

9. $f(x) = 2x - 2, g(x) = 4x - 2$

10. $f(x) = x + 1, g(x) = \frac{1}{2}x + 1$

Graph $f(x)$. Then reflect the graph of $f(x)$ across the y-axis. Write a function $g(x)$ to describe the new graph.

SEE EXAMPLE **3**
p. 371

11. $f(x) = -\frac{1}{5}x$

12. $f(x) = 2x + 4$

13. $f(x) = \frac{1}{3}x - 6$
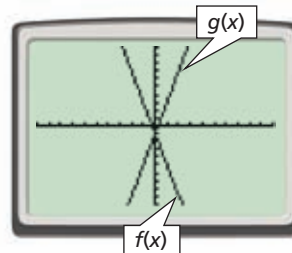
14. $f(x) = 5x - 1$

Graph $f(x)$ and $g(x)$. Then describe the transformations from the graph of $f(x)$ to the graph of $g(x)$.

SEE EXAMPLE **4**
p. 372

15. $f(x) = x, g(x) = 2x - 2$

16. $f(x) = x, g(x) = \frac{1}{3}x + 1$

17. $f(x) = -x - 1, g(x) = -4x$

18. $f(x) = -x, g(x) = -\frac{1}{2}x - 3$

SEE EXAMPLE **5**
p. 372

19. Entertainment For large parties, a restaurant charges a reservation fee of $25, plus $15 per person. The total charge for a party of x people is $f(x) = 15x + 25$. How will the graph of this function change if the reservation fee is raised to $50? if the per-person charge is lowered to $12?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
20–21	1
22–23	2
24–25	3
26–27	4
28	5

Extra Practice

Skills Practice p. S13
Application Practice p. S32

Graph $f(x)$ and $g(x)$. Then describe the transformation(s) from the graph of $f(x)$ to the graph of $g(x)$.

20. $f(x) = x, g(x) = x + \frac{1}{2}$

21. $f(x) = x, g(x) = x - 4$

22. $f(x) = \frac{1}{5}x - 1, g(x) = \frac{1}{10}x - 1$

23. $f(x) = x + 2, g(x) = \frac{2}{3}x + 2$

Graph $f(x)$. Then reflect the graph of $f(x)$ across the y-axis. Write a function $g(x)$ to describe the new graph.

24. $f(x) = 6x$

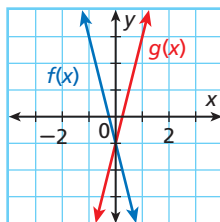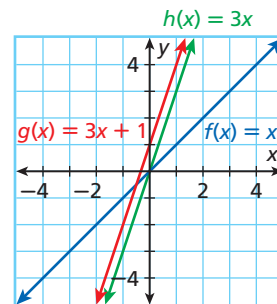**25.** $f(x) = -3x - 2$

Graph $f(x)$ and $g(x)$. Then describe the transformations from the graph of $f(x)$ to the graph of $g(x)$.

26. $f(x) = 2x, g(x) = 4x - 1$

27. $f(x) = -7x + 5, g(x) = -14x$

28. School The number of chaperones on a field trip must include 1 teacher for every 4 students, plus 2 parents total. The function describing the number of chaperones for a trip of x students is $f(x) = \frac{1}{4}x + 2$. How will the graph change if the number of parents is reduced to 0? if the number of teachers is raised to 1 for every 3 students?

Describe the transformation(s) on the graph of $f(x) = x$ that result in the graph of $g(x)$. Graph $f(x)$ and $g(x)$, and compare the slopes and intercepts.

29. $g(x) = -x$

30. $g(x) = x + 8$

31. $g(x) = 3x$

32. $g(x) = -\frac{2}{7}x$

33. $g(x) = 6x - 3$

34. $g(x) = -2x + 1$

Sketch the transformed graph. Then write a function to describe your graph.

35. Rotate the graph of $f(x) = -x + 2$ until it has the same steepness in the opposite direction.

36. Reflect the graph of $f(x) = x - 1$ across the y-axis, and then translate it 4 units down.

37. Translate the graph of $f(x) = \frac{1}{6}x - 10$ six units up.

38. Hobbies A book club charges a membership fee of $20 and then $12 for each book purchased.

   a. Write and graph a function to represent the cost y of membership in the club based on the number of books purchased x.

   b. What if...? Write and graph a second function to represent the cost of membership if the club raises its membership fee to $30.

   c. Describe the relationship between your graphs from parts **a** and **b**.

Describe the transformation(s) on the graph of $f(x) = x$ that result in the graph of $g(x)$.

39. $g(x) = x - 9$

40. $g(x) = -x$

41. $g(x) = 5x$

42. $g(x) = -\frac{2}{3}x + 1$

43. $g(x) = -2x$

44. $g(x) = \frac{1}{5}x$

45. Careers Kelly works as a salesperson. She earns a weekly base salary plus a commission that is a percent of her total sales. Her total weekly pay is described by $f(x) = 0.20x + 300$, where x is total sales in dollars.

   a. What is Kelly's weekly base salary?

   b. What percent of total sales does Kelly receive as commission?

   c. What if...? What is the change in Kelly's salary plan if the weekly pay function changes to $g(x) = 0.25x + 300$? to $h(x) = 0.2x + 400$?

46. Critical Thinking To transform the graph of $f(x) = x$ into the graph of $g(x) = -x$, you can reflect the graph of $f(x)$ across the y-axis. Find another transformation that will have the same result.

47. Write About It Describe how a reflection across the y-axis affects each point on a graph. Give an example to illustrate your answer.

48. This problem will prepare you for the Multi-Step Test Prep on page 376.

   a. Maria is walking from school to the softball field at a rate of 3 feet per second. Write a rule that gives her distance from school (in feet) as a function of time (in seconds). Then graph.

   b. Give a real-world situation that could be described by a line parallel to the one in part **a**.

   c. What does the y-intercept represent in each of these situations?

49. Which best describes the effect on $f(x) = 2x - 5$ if the slope changes to 10?

  (A) Its graph becomes less steep.
  (B) Its graph moves 15 units up.
  (C) Its graph makes 10 complete rotations.
  (D) The x-intercept becomes $\frac{1}{2}$.

50. Given $f(x) = 22x - 182$, which does NOT describe the effect of increasing the y-intercept by 182?

  (F) The new line passes through the origin.
  (G) The new x-intercept is 0.
  (H) The new line is parallel to the original.
  (J) The new line is steeper than the original.

CHALLENGE AND EXTEND

51. You have seen that the graph of $g(x) = x + 3$ is the result of translating the graph of $f(x) = x$ three units up. However, you can also think of this as a *horizontal* translation—that is, a translation left or right. Graph $g(x) = x + 3$. Describe the horizontal translation of the graph of $f(x) = x$ to get the graph of $g(x) = x + 3$.

52. If $c > 0$, how can you describe the translation that transforms the graph of $f(x) = x$ into the graph of $g(x) = x + c$ as a horizontal translation? $g(x) = x - c$ as a horizontal translation?

SPIRAL REVIEW

Give an expression in simplest form for the perimeter of each figure. *(Lesson 1-7)*

53.

5x   5x

5x

54.

y + 2

3y
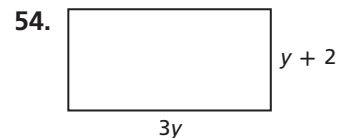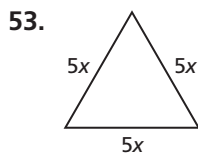
Identify the correlation you would expect to see between each pair of data sets. Explain. *(Lesson 4-5)*

55. the temperature and the number of people at the local ice cream parlor

56. the amount of electricity used and the total electric bill

57. the number of miles driven after a fill-up and the amount of gasoline in the tank

Identify which lines are parallel. *(Lesson 5-9)*

58. $y = -2x + 3$; $y = 2x$; $y = -2$; $y = -2x - 4$; $y = \frac{1}{2}x$; $y - 1 = -\frac{1}{2}(x + 6)$

59. $y = \frac{3}{5}x + 8$; $y = -\frac{3}{5}x$; $y + 1 = -\frac{3}{5}(x - 2)$; $y = \frac{5}{3}x + 9$; $y = 3x + 5$

Identify which lines are perpendicular. *(Lesson 5-9)*

60. $3x - 5y = 5$; $5y = -2x - 15$; $y = 3x + 5$; $5x + 3y = -21$; $y = \frac{5}{2}x - 2$

61. $x = 4$; $2y + x = 6$; $3x - y = 12$; $y = 2x + 3$; $y = -3$

MULTI-STEP TEST PREP

Using Linear Functions

Take a Walk! All intersections in Durango, Colorado, have crossing signals with timers. Once the signal changes to walk, the timer begins at 28 seconds and counts down to show how much time pedestrians have to cross the street.

1. Pauline counted her steps as she crossed the street. She counted 15 steps with 19 seconds remaining. When she reached the opposite side of the street, she had counted a total of 30 steps and had 10 seconds remaining. Copy and complete the table below using these values.

Time Remaining (s)	28		
Steps	0		

2. Find the average rate of change for Pauline's walk.

3. Sketch a graph of the points in the table, or plot them on your graphing calculator.

4. Find an equation that describes the line through the points.

5. How would the graph change if Pauline increased her speed? What if she decreased her speed?

READY TO GO ON?

Quiz for Lessons 5-7 Through 5-10

✓ **5-7** **Slope-Intercept Form**

Graph each line given the slope and *y*-intercept.

1. slope $= \frac{1}{4}$; *y*-intercept $= 2$ **2.** slope $= -3$; *y*-intercept $= 5$ **3.** slope $= -1$; *y*-intercept $= -6$

Write each equation in slope-intercept form. Then graph the line described by the equation.

4. $2x + y = 5$ **5.** $2x - 6y = 6$ **6.** $3x + y = 3x - 4$

7. Entertainment At a chili cook-off, people pay a $3.00 entrance fee and $0.50 for each bowl of chili they taste. The graph shows the total cost per person as a function of the number of bowls of chili tasted.

a. Write an equation that represents the total cost per person as a function of the number of bowls of chili tasted.

b. Identify the slope and *y*-intercept and describe their meanings.

Chili Cook-off

Total cost ($) vs *Bowls of chili*

✓ **5-8** **Point-Slope Form**

Graph the line with the given slope that contains the given point.

8. slope $= -3$; $(0, 3)$ **9.** slope $= -\frac{2}{3}$; $(-3, 5)$ **10.** slope $= 2$; $(-3, -1)$

Write an equation in slope-intercept form for the line through the two points.

11. $(3, 1)$ and $(4, 3)$ **12.** $(-1, -1)$ and $(1, 7)$ **13.** $(1, -4)$ and $(-2, 5)$

✓ **5-9** **Slopes of Parallel and Perpendicular Lines**

Identify which lines are parallel.

14. $y = -2x$; $y = 2x + 1$; $y = 2x$; $y = 2(x + 5)$ **15.** $-3y = x$; $y = -\frac{1}{3}x + 1$; $y = -3x$; $y + 2 = x + 4$

Identify which lines are perpendicular.

16. $y = -4x - 1$; $y = \frac{1}{4}x$; $y = 4x - 6$; $x = -4$ **17.** $y = -\frac{3}{4}x$; $y = \frac{3}{4}x - 3$; $y = \frac{4}{3}x$; $y = 4$; $x = 3$

18. Write an equation in slope-intercept form for the line that passes through $(5, 2)$ and is parallel to the line described by $3x - 5y = 15$.

19. Write an equation in slope-intercept form for the line that passes through $(3, 5)$ and is perpendicular to the line described by $y = -\frac{3}{2}x - 2$.

✓ **5-10** **Transforming Linear Functions**

Graph $f(x)$ and $g(x)$. Then describe the transformation(s) from the graph of $f(x)$ to the graph of $g(x)$.

20. $f(x) = 5x$, $g(x) = -5x$ **21.** $f(x) = \frac{1}{2}x - 1$, $g(x) = \frac{1}{2}x + 4$

22. An attorney charges an initial fee of $250 and then $150 per hour. The total bill after *x* hours is $f(x) = 150x + 250$. How will the graph of this function change if the initial fee is reduced to $200? if the hourly rate is increased to $175?

EXTENSION Absolute-Value Functions

Objectives
Graph absolute-value functions.

Identify characteristics of absolute-value functions and their graphs.

Vocabulary
absolute-value function
axis of symmetry
vertex

An **absolute-value function** is a function whose rule contains an absolute-value expression. To graph an absolute-value function, choose several values of x and generate some ordered pairs.

x	$f(x) = |x|$
-2	2
-1	1
0	0
1	1
2	2
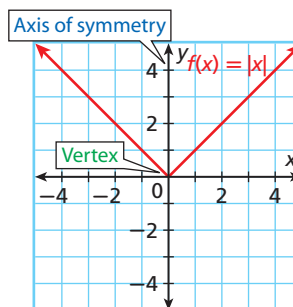
Absolute-value graphs are composed of two linear pieces. The **axis of symmetry** is the line that divides the graph into two congruent halves. The **vertex** is the "corner" point on the graph.

From the table and the graph of $f(x) = |x|$, you can tell that
- the **axis of symmetry** is the y-axis ($x = 0$).
- the **vertex** is $(0, 0)$.
- the domain (x-values) is the set of all real numbers.
- the range (y-values) is described by $y \geq 0$.
- the x-intercept and the y-intercept are both 0.
- the slope of the left linear piece is -1 and the slope of the right linear piece is 1.

EXAMPLE 1 Absolute-Value Functions

Graph $f(x) = |2x| - 1$ and label the axis of symmetry and the vertex. Identify the intercepts, give the domain and range, and find the slope of each piece.

Choose positive, negative, and zero values for x, and find ordered pairs.

x	-2	-1	0	1	2
$f(x) = |2x| - 1$	3	1	-1	1	3

Plot the ordered pairs and connect them.
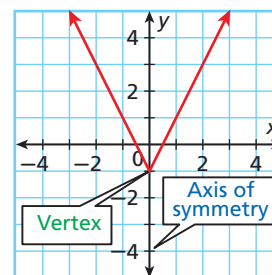
From the table and the graph, you can tell that
- the **axis of symmetry** is the y-axis ($x = 0$).
- the **vertex** is $(0, -1)$.
- the x-intercepts are $\frac{1}{2}$ and $-\frac{1}{2}$.
- the y-intercept is -1.
- the domain is all real numbers.
- the range is described by $y \geq -1$.
- the slope of the left piece is -2 and the slope of the right piece is 2.

 1. Graph $f(x) = 3|x|$ and label the axis of symmetry and the vertex. Identify the intercepts, give the domain and range, and find the slope of each piece.

The parent function for absolute-value functions is $f(x) = |x|$. The graphs of all other absolute-value functions are transformations of the graph of $f(x) = |x|$.

For $f(x) = a|x - b| + c$

WORDS	EXAMPLES	WORDS	EXAMPLES
Opening Direction • If $a > 0$, the graph opens upward. • If $a < 0$, the graph opens downward.		**Horizontal Translation** • If $b > 0$, the graph is translated b units right from $f(x) = \lvert x \rvert$. • If $b < 0$, the graph is translated b units left from $f(x) = \lvert x \rvert$.	
Width • If $\lvert a \rvert > 1$, the graph is narrower than the graph of $f(x) = \lvert x \rvert$. • If $\lvert a \rvert < 1$, the graph is wider than the graph of $f(x) = \lvert x \rvert$. The slopes of the two linear pieces are a and $-a$.		**Vertical Translation** • If $c > 0$, the graph is translated c units up from $f(x) = \lvert x \rvert$. • If $c < 0$, the graph is translated c units down from $f(x) = \lvert x \rvert$.	

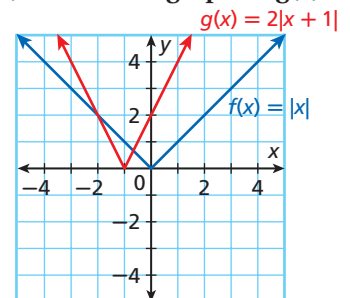**E X A M P L E 2**   **Transforming Absolute-Value Functions**

Describe the transformations from the graph of $f(x) = |x|$ to the graph of $g(x)$. Then graph both functions.

A $g(x) = 2|x + 1|$

Identify a, b, and c.

$g(x) = 2|x + 1| = 2|x - (-1)| + 0$

• $a = 2$: graph is narrower
• $b = -1$: translated 1 unit left
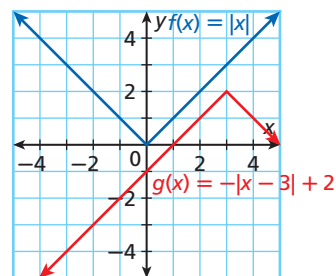• $c = 0$: no vertical translation

Describe the transformations from the graph of $f(x) = |x|$ to the graph of $g(x)$. Then graph both functions.

B $g(x) = -|x - 3| + 2$

Identify a, b, and c.

$g(x) = -|x - 3| + 2 = -1|x - 3| + 2$

- $a = -1$: graph opens downward and width is unchanged
- $b = 3$: translated 3 units right
- $c = 2$: translated 2 units up
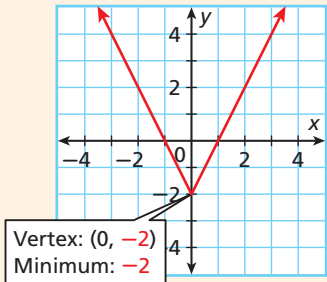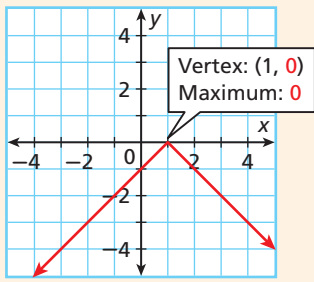
CHECK IT OUT!   Describe the transformations from the graph of $f(x) = |x|$ to the graph of $g(x)$. Then graph both functions.

2a. $g(x) = |x - 4| - 2$ **2b.** $g(x) = \frac{1}{3}|x| + 1$

Minimum and Maximum Values of Absolute-Value Functions

WORDS	If the graph of an absolute-value function opens upward, the y-value of the vertex is the **minimum** value of the function.	If the graph of an absolute-value function opens downward, the y-value of the vertex is the **maximum** value of the function.
EXAMPLES	$f(x) = 2\,|x| - 2$ Vertex: (0, −2) Minimum: −2	$f(x) = -|x - 1|$ Vertex: (1, 0) Maximum: 0

E X A M P L E 3   **Identifying the Minimum or Maximum**

Graph $f(x) = |x + 3| - 5$. Identify the vertex and give the minimum or maximum value of the function.
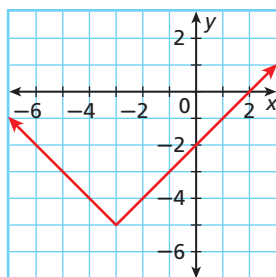
The vertex is (−3, −5).
The graph opens upward, so the function has a minimum.
The minimum is −5.

CHECK IT OUT!   Graph each absolute-value function. Identify the vertex and give the minimum or maximum value of the function.

3a. $f(x) = -|x| + 5$ **3b.** $f(x) = |x + 1| - 6$

EXAMPLE 4 **Sports Application**

Distance From Water Stand

In a charity race, a water stand for the runners is halfway between the start and finish lines. The function $y = \left|\frac{x}{8} - 3\right|$ models Riley's distance y in miles from the water stand x minutes into the race.

A How long is the race?

$y = \left|\frac{x}{8} - 3\right|$ is graphed in blue.

At the start of the race ($x = 0$), Riley is 3 mi from the water stand. The water stand is halfway between the start and finish lines, so the race is 6 mi.

B How much time does it take Riley to reach the water stand?

When Riley reaches the water stand, $y = 0$. This happens when $x = 24$. It takes Riley 24 min to reach the water stand.

C The function $y = \left|\frac{x}{10} - 3\right|$ models Dean's distance from the water stand during the same race. Compare Dean's graph to Riley's graph. What can you conclude about Dean's speed?

$y = \left|\frac{x}{10} - 3\right|$ is graphed in red. Both graphs start at the same point, but Dean's graph is translated to the right. It takes him more time to reach the water stand and to finish the race. Therefore, he is running more slowly than Riley.
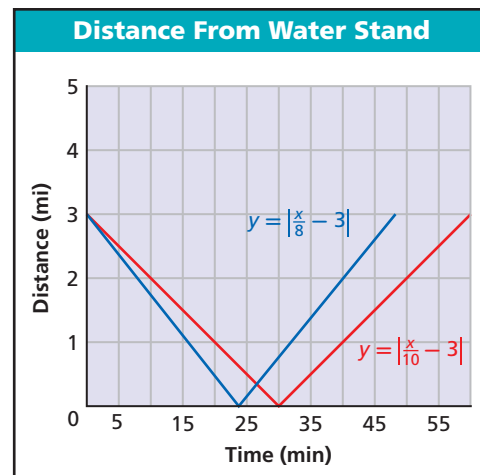
CHECK IT OUT!
4. How would the graph be different for someone who runs faster than Riley?

EXTENSION

Exercises

Graph each absolute-value function and label the axis of symmetry and the vertex. Identify the intercepts, give the domain and range, and find the slope of each piece. Identify the maximum or minimum.

1. $f(x) = |x| + 3$ **2.** $f(x) = |x + 3|$ **3.** $f(x) = \frac{1}{2}|x|$ **4.** $f(x) = |x - 3|$

Tell whether each statement is sometimes, always, or never true.

5. The absolute value of a number is negative.

6. An absolute-value function has an x-intercept.

7. An absolute-value function has two y-intercepts.

Write a function to describe each of the following.

8. The graph of $f(x) = |x|$ is translated 2 units right.

9. The graph of $f(x) = |x|$ is narrowed and reflected across the x-axis.

10. **Critical Thinking** Suppose that an absolute-value function has no x-intercepts. What can you say about the function rule?

Study Guide: Review

Vocabulary

Complete the sentences below with vocabulary words from the list above. Words may be used more than once.

1. A(n) ____?____ is a "slide," a(n) ____?____ is a "turn," and a(n) ____?____ is a "flip."

2. The x-coordinate of the point that contains the ____?____ is always 0.

3. In the equation $y = mx + b$, the value of m is the ____?____ , and the value of b is the ____?____ .

5-1 Identifying Linear Functions (pp. 300–306)

EXAMPLES

Tell whether each function is linear. If so, graph the function.

■ $y = -3x + 2$

$$y = -3x + 2 \qquad \textit{Write the equation in}$$
$$\underline{+3x \quad +3x} \qquad \textit{standard form.}$$
$$3x + y = \qquad 2 \qquad \text{This is a linear function.}$$

Generate ordered pairs.

x	$y = -3x + 2$	(x, y)
−2	$y = -3(-2) + 2 = 8$	(−2, 8)
0	$y = -3(0) + 2 = 2$	(0, 2)
2	$y = -3(2) + 2 = -4$	(2, −4)
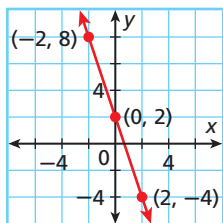
Plot the points and connect them with a straight line.

■ $y = 2x^3$

This is not a linear function because x has an exponent other than 1.

EXERCISES

Tell whether the given ordered pairs satisfy a linear function. Explain.

4.

x	y
−3	3
−1	1
1	1
3	3

5.

x	y
0	−3
1	−1
2	1
3	3

6. $\{(-2, 5), (-1, 3), (0, 1), (1, -1), (2, -3)\}$

7. $\{(1, 7), (3, 6), (6, 5), (9, 4), (13, 3)\}$

Each equation below is linear. Write each equation in standard form and give the values of A, B, and C.

8. $y = -5x + 1$

9. $\dfrac{x + 2}{2} = -3y$

10. $4y = 7x$

11. $9 = y$

12. Helene is selling cupcakes for \$0.50 each. The function $f(x) = 0.5x$ gives the total amount of money Helene makes after selling x cupcakes. Graph this function and give its domain and range.

5-2 Using Intercepts (pp. 307–312)

EXAMPLE

■ Find the x- and y-intercepts of $2x + 5y = 10$.

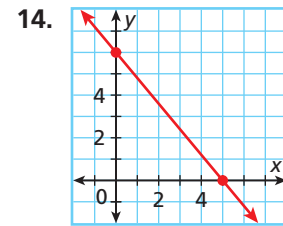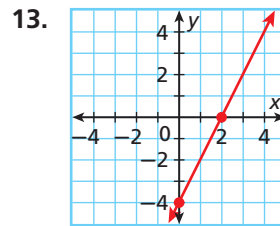| Let $y = 0$.	Let $x = 0$.
$2x + 5(0) = 10$	$2(0) + 5y = 10$
$2x + 0 = 10$	$0 + 5y = 10$
$2x = 10$	$5y = 10$
$\dfrac{2x}{2} = \dfrac{10}{2}$	$\dfrac{5y}{5} = \dfrac{10}{5}$
$x = 5$	$y = 2$

The x-intercept is 5.   The y-intercept is 2.

EXERCISES

Find the x- and y-intercepts.
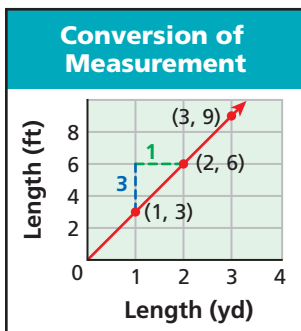
13.

14.

15. $3x - y = 9$

16. $-2x + y = 1$

17. $-x + 6y = 18$

18. $3x - 4y = 1$

5-3 Rate of Change and Slope (pp. 314–321)

EXAMPLE

■ Find the slope of the line.

Conversion of Measurement

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{3}{1} = 3$$

EXERCISES

19. Graph the data and show the rates of change.

Time (s)	Distance (ft)
0	0
1	16
2	64
3	144
4	256

20. Find the slope of the line graphed below.

Casey's Casserole
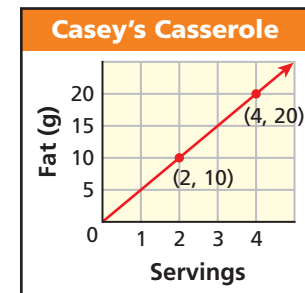
5-4 The Slope Formula (pp. 324–329)

EXAMPLE

■ Find the slope of the line described by $2x - 3y = 6$.

Step 1 Find the x- and y-intercepts.

Let $y = 0$.	Let $x = 0$.
$2x - 3(0) = 6$	$2(0) - 3y = 6$
$2x = 6$	$-3y = 6$
$x = 3$	$y = -2$

The line contains $(3, 0)$ and $(0, -2)$.

Step 2 Use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 0}{0 - 3} = \frac{-2}{-3} = \frac{2}{3}$$

EXERCISES

Find the slope of the line described by each equation.

21. $4x + 3y = 24$

22. $y = -3x + 6$

23. $x + 2y = 10$

24. $3x = y + 3$

25. $y + 2 = 7x$

26. $16x = 4y + 1$

Find the slope of the line that contains each pair of points.

27. $(1, 2)$ and $(2, -3)$

28. $(4, -2)$ and $(-5, 7)$

29. $(-3, -6)$ and $(4, 1)$

30. $\left(\dfrac{1}{2}, 2\right)$ and $\left(\dfrac{3}{4}, \dfrac{5}{2}\right)$

31. $(2, 2)$ and $(2, 7)$

32. $(1, -3)$ and $(5, -3)$

5-5 The Midpoint and Distance Formulas (pp. 330–335)

EXAMPLE

■ Find the coordinates of the midpoint of $\overline{AB}$ with endpoints $A(-4, 3)$ and $B(5, -6)$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \quad \textit{Write the formula.}$$

$$M\left(\frac{-4 + 5}{2}, \frac{3 + (-6)}{2}\right) = M\left(\frac{1}{2}, -\frac{3}{2}\right)$$

■ Use the Distance Formula to find the distance, to the nearest hundredth, from $C(-4, 1)$ to $D(5, -3)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{[5 - (-4)]^2 + (-3 - 1)^2}$$

$$d = \sqrt{9^2 + (-4)^2} \quad \textit{Subtract within}$$
$$\textit{grouping symbols.}$$
$$d = \sqrt{97} \approx 9.85 \quad \textit{Simplify and estimate.}$$

EXERCISES

Find the coordinates of the midpoint of each segment.

33. $\overline{EF}$ with endpoints $E(9, 12)$ and $F(21, 18)$

34. $\overline{GH}$ with endpoints $G(-5, -7)$ and $H(4, -11)$

Use the Distance Formula to find the distance, to the nearest hundredth, between each pair of points.

35. $J(3, 10)$ and $K(-2, 4)$

36. $L(-6, 0)$ and $M(8, -7)$

37. Each unit on a map of a forest represents 1 mile. To the nearest tenth of a mile, what is the distance from a ranger station at $(1, 2)$ on the map to a river crossing at $(2, 4)$?

5-6 Direct Variation (pp. 336–341)

EXAMPLE

■ Tell whether $6x = -4y$ represents a direct variation. If so, identify the constant of variation.

$$6x = -4y$$
$$\frac{6x}{-4} = \frac{-4y}{-4} \quad \textit{Solve the equation for y.}$$
$$-\frac{6}{4}x = y$$
$$y = -\frac{3}{2}x \quad \textit{Simplify.}$$

This equation represents a direct variation because it can be written in the form $y = kx$, where $k = -\frac{3}{2}$.

EXERCISES

Tell whether each equation is a direct variation. If so, identify the constant of variation.

38. $y = -6x$ **39.** $x - y = 0$

40. $y + 4x = 3$ **41.** $2x = -4y$

42. The value of y varies directly with x, and $y = -8$ when $x = 2$. Find y when $x = 3$.

43. Maleka charges $8 per hour for baby-sitting. The amount of money she makes varies directly with the number of hours she baby-sits. Write a direct variation equation for the amount y Maleka earns for baby-sitting x hours. Then graph.

5-7 Slope-Intercept Form (pp. 344–350)

EXAMPLE

■ Graph the line with slope $= -\frac{4}{5}$ and y-intercept $= 8$.

Step 1 Plot $(0, 8)$.

Step 2 For a slope of $\frac{-4}{5}$, count **4 down** and **5 right** from $(0, 8)$. Plot another point.
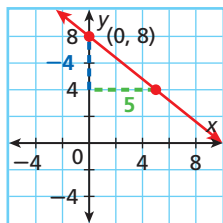
Step 3 Connect the two points with a line.

EXERCISES

Graph each line given the slope and y-intercept.

44. slope $= -\frac{1}{2}$; y-intercept $= 4$

45. slope $= 3$; y-intercept $= -7$

Write the equation that describes each line in slope-intercept form.

46. slope $= \frac{1}{3}$, y-intercept $= 5$

47. slope $= 4$, $(1, -5)$ is on the line.

5-8 Point-Slope Form (pp. 351–358)

EXAMPLE

■ Write an equation in slope-intercept form for the line through $(4, -1)$ and $(-2, 8)$.

Step 1 Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - (-1)}{-2 - 4} = \frac{9}{-6} = -\frac{3}{2}$$

Step 2 Write the point-slope form.

$$y - 8 = -\frac{3}{2}\big[x - (-2)\big] \quad \text{Substitute into the point-slope form.}$$

$$y - 8 = -\frac{3}{2}(x + 2)$$

Step 3 Write the slope-intercept form.

$$y = -\frac{3}{2}x + 5 \qquad \text{Solve for y.}$$

EXERCISES

Graph the line described by each equation.

48. $y + 3 = \frac{1}{2}(x - 4)$ **49.** $y - 1 = -(x + 3)$

Write the equation that describes each line in slope-intercept form.

50. slope $= 2$, $(1, 3)$ is on the line.

51. slope $= -5$, $(-6, 4)$ is on the line.

52. $(1, 4)$ and $(3, 8)$ are on the line.

53. $(-2, 4)$ and $(-1, 6)$ are on the line.

5-9 Slopes of Parallel and Perpendicular Lines (pp. 361–367)

EXAMPLE

■ Write an equation in slope-intercept form for the line that passes through $(4, -2)$ and is perpendicular to the line described by $y = -4x + 3$.

The slope of $y = -4x + 3$ is -4.

The perpendicular line has a slope of $\frac{1}{4}$ and contains $(4, -2)$.

$$y + 2 = \frac{1}{4}(x - 4) \quad \text{Substitute into the point-slope form.}$$

$$y = \frac{1}{4}x - 3 \quad \text{Solve for y.}$$

EXERCISES

Identify which lines are parallel.

54. $y = -\frac{1}{3}x; y = 3x + 2; y = -\frac{1}{3}x - 6; y = 3$

55. $y - 2 = -4(x - 1); y = 4x - 4; y = \frac{1}{4}x; y = -4x - 2$

Identify which lines are perpendicular.

56. $y - 1 = -5(x - 6); y = \frac{1}{5}x + 2; y = 5; y = 5x + 8$

57. $y = 2x; y - 2 = 3(x + 1); y = \frac{2}{3}x - 4; y = -\frac{1}{3}x$

58. Write an equation in slope-intercept form for the line that passes through $(1, -1)$ and is parallel to the line described by $y = 2x - 4$.

5-10 Transforming Linear Functions (pp. 369–375)

EXAMPLE

■ Graph $f(x) = \frac{1}{2}x$ and $g(x) = 4x + 2$. Then describe the transformation(s) from the graph of $f(x)$ to the graph of $g(x)$.

• Multiply $f(x) = \frac{1}{2}x$ by 8 to get $h(x) = 4x$. This rotates the graph about $(0, 0)$, making it steeper.

• Then add 2 to $h(x) = 4x$ to get $g(x) = 4x + 2$. This translates the graph 2 units up.

EXERCISES

Graph $f(x)$ and $g(x)$. Then describe the transformation(s) from the graph of $f(x)$ to the graph of $g(x)$.

59. $f(x) = x, g(x) = x + 4$

60. $f(x) = 4x, g(x) = -4x$

61. $f(x) = \frac{1}{3}x - 2, g(x) = -\frac{1}{3}x - 2$

62. The entrance fee at a carnival is $3 and each ride costs $1. The total cost for x rides is $f(x) = x + 3$. How will the graph of this function change if the entrance fee is increased to $5? if the cost per ride is increased to $2?

CHAPTER TEST

Tell whether each function is linear. If so, graph the function.

1. $3y = 2x + 3$

2. $y = x(4 + x)$

3. Lily plans to volunteer at the tutoring center for 45 hours. She can tutor 3 hours per week. The function $f(x) = 45 - 3x$ gives the number of hours she will have left to tutor after x weeks. Graph the function and find its intercepts. What does each intercept represent?

4. The table shows the number of guppies in an aquarium over time. Graph the data and show the rates of change.

Time (mo)	0	1	2	5	8
Guppies	4	4	10	25	31

Find the slope of each line. Then tell what the slope represents.

5.

Ticket Costs

6.

Water in Tank

7.

Temperature of Specimen
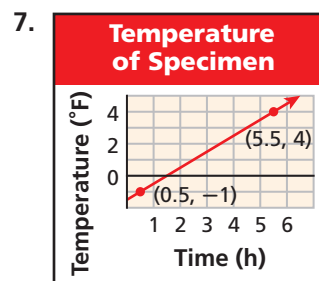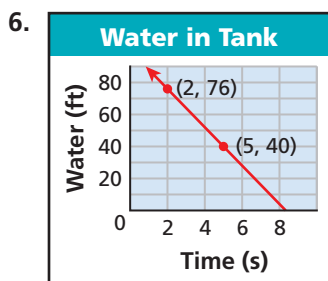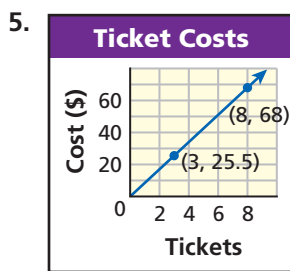
8. Each unit on a map of a bay represents 1 kilometer. Two buoys are located at (1, 5) and (3, 6) on the map. To the nearest tenth of a kilometer, what is the distance between the two buoys?

9. In orbit around Earth, a space shuttle travels at a speed of about 5 miles per second. Write a direct variation equation for the distance y the space shuttle will travel in x seconds. Then graph.

10. Write the equation $2x - 2y = 4$ in slope-intercept form. Then graph the line described by the equation.

Write the equation that describes each line in slope-intercept form.

11. slope $= -2$, y-intercept $= 7$

12. slope $= 4$, (–3, 3) is on the line.

13. (1, 5) and (−2, 8) are on the line.

14. x-intercept $= 3$, y-intercept $= -3$

Write an equation in point-slope form for the line with the given slope that contains the given point.

15. slope $= -1$; (1, 3)

16. slope $= 5$; (−3, 2)

17. Write an equation in slope-intercept form for the line that passes through (0, 6) and is parallel to the line described by $y = 2x + 3$.
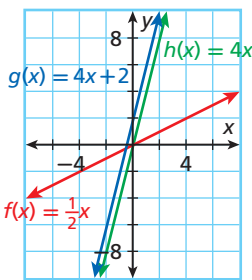
Graph $f(x)$ and $g(x)$. Then describe the transformation(s) from the graph of $f(x)$ to the graph of $g(x)$.

18. $f(x) = 8x$, $g(x) = 4x$

19. $f(x) = -x + 2$, $g(x) = -x - 1$

20. $f(x) = 3x$, $g(x) = 6x - 1$

COLLEGE ENTRANCE EXAM PRACTICE

FOCUS ON SAT

SAT scores are based on the total number of items answered correctly minus a fraction of the number of multiple-choice questions answered incorrectly. No points are subtracted for questions unanswered.

On the SAT, there is a penalty for incorrect answers on multiple-choice items. Guess only when you can eliminate at least one of the answer choices.

You may want to time yourself as you take this practice test. It should take you about 7 minutes to complete.

1. The line through $A(1, -3)$ and $B(-2, d)$ has slope -2. What is the value of d?

   (A) $-\dfrac{3}{2}$

   (B) -1

   (C) $\dfrac{1}{2}$

   (D) 3

   (E) 5

2. The ordered pairs $\{(0, -3), (4, -1), (6, 0), (10, 2)\}$ satisfy a pattern. Which is NOT true?

   (A) The pattern is linear.

   (B) The pattern can be described by $2x - 4y = 12$.

   (C) The ordered pairs lie on a line.

   (D) $(-4, 1)$ satisfies the same pattern.

   (E) The set of ordered pairs is a function.

3. If y varies directly as x, what is the value of x when $y = 72$?

x	7	12	
y	28	48	72

   (A) 17

   (B) 18

   (C) 24

   (D) 28

   (E) 36

4. The line segment between the points $(4, 0)$ and $(2, -2)$ forms one side of a rectangle. Which of the following coordinates could determine another vertex of that rectangle?
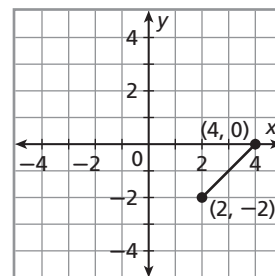
   (A) $(-2, 6)$

   (B) $(-2, -2)$

   (C) $(0, 6)$

   (D) $(1, 2)$

   (E) $(4, 6)$

5. Which of the following has the same slope as the line described by $2x - 3y = 3$?

   (A) $3x - 2y = 2$

   (B) $\dfrac{2}{3}x - y = -2$

   (C) $2x - 2y = 3$

   (D) $\dfrac{1}{3}x - 2y = -2$

   (E) $-2x - 3y = 2$

TEST TACKLER

Standardized Test Strategies

Multiple Choice: Recognize Distracters

In multiple-choice items, the options that are incorrect are called *distracters*. This is an appropriate name, because these incorrect options can distract you from the correct answer.

Test writers create distracters by using common student errors. Beware! Even if the answer you get when you work the problem is one of the options, it may not be the correct answer.

EXAMPLE 1

What is the y-intercept of $4x + 10 = -2y$?

    Ⓐ 10   Ⓒ -2.5

    Ⓑ 5   Ⓓ -5

Look at each option carefully.

    Ⓐ This is a distracter. The y-intercept would be 10 if the function was $4x + 10 = y$. A common error is to ignore the coefficient of y.

    Ⓑ This is a distracter. Another common error is to divide by 2 instead of -2 when solving for y.

    Ⓒ This is a distracter. One of the most common errors students make is confusing the x-intercept and the y-intercept. This distracter is actually the *x-intercept* of the given line.

    Ⓓ This is the correct answer.

EXAMPLE 2

What is the equation of a line with a slope of -4 that contains $(2, -3)$?

    Ⓕ $y - 3 = -4(x - 2)$ Ⓗ $y + 3 = -4(x - 2)$

    Ⓖ $y - 2 = -4(x + 3)$ Ⓙ $y + 4 = -3(x - 2)$

Look at each option carefully.

    Ⓕ This is a distracter. Students often make errors with positive and negative signs. You would get this answer if you simplified $y - (-3)$ as $y - 3$.

    Ⓖ This is a distracter. You would get this answer if you switched the x-coordinate and the y-coordinate.

    Ⓗ This is the correct answer.

    Ⓙ This is a distracter. You would get this answer if you substituted the given values incorrectly in the point-slope equation.

Read each test item and answer the questions that follow.

Item A

A line contains $(1, 2)$ and $(-2, 14)$. What are the slope and y-intercept?

(A) Slope $= -4$; y-intercept $= -2$

(B) Slope $= 4$; y-intercept $= 6$

(C) Slope $= -\frac{1}{4}$; y-intercept $= 1$

(D) Slope $= -4$; y-intercept $= 6$

1. What common error does the slope in choice B represent?

2. The slope given in choice A is correct, but the y-intercept is not. What error was made when finding the y-intercept?

3. What formula can you use to find the slope of a line? How was this formula used incorrectly to get the slope in choice C?
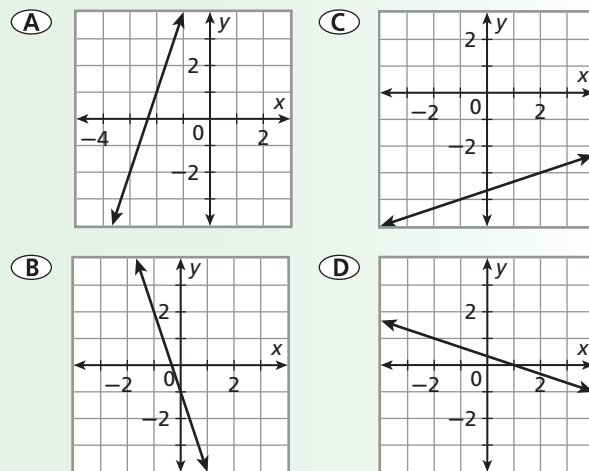
Item B

Which of these functions has a graph that is NOT parallel to the line described by $y = \frac{1}{2}x + 4$?

(F) $y = 6 - \frac{1}{2}x$

(G) $y = \frac{1}{2}x + 6$

(H) $-2y = -x + 1$

(J) $2y = x$

4. When given two linear functions, describe how to determine whether their graphs are parallel.

5. Which is the correct answer? Describe the errors a student might make to get each of the distracters.

Item C

Which of these lines has a slope of -3?

(A)

(C)

(B)

(D)

6. Which two answer choices can be eliminated immediately? Why?

7. Decribe how to find the slope of a line from its graph.

8. What common error does choice A represent?

9. What common error does choice D represent?

10. Which is the correct answer?

Item D

Which is NOT a linear function?

(F) $f(x) = 4 + x$

(G) $f(x) = -x - 4$

(H) $f(x) = 4x^2$

(J) $f(x) = \frac{1}{4}x$

11. When given a function rule, how can you tell if the function is linear?

12. What part of the function given in choice G might make someone think it is not linear?

13. What part of the function given in choice J might make someone think it is not linear?

14. What part of the function given in choice H makes it NOT linear?

STANDARDIZED TEST PREP

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CUMULATIVE ASSESSMENT, CHAPTERS 1–5

Multiple Choice

1. What is the value of $2 - \left[1 - (2 - 1)\right]$?
   - (A) -2
   - (B) 0
   - (C) 2
   - (D) 4

2. Frank borrowed $5000 with an annual simple interest rate. The amount of interest he owed after 6 months was $300. What is the interest rate of the loan?
   - (F) 1%
   - (G) 6%
   - (H) 10%
   - (J) 12%

3. Patty's Pizza charges $5.50 for a large pizza plus $0.30 for each topping. Pizza Town charges $5.00 for a large pizza plus $0.40 for each topping. Which inequality can you use to find the number of toppings x so that the cost of a pizza at Pizza Town is greater than the cost of a pizza at Patty's Pizza?
   - (A) $(5 + 0.4)x > (5.5 + 0.3)x$
   - (B) $5.5x + 0.3 > 5x + 0.4$
   - (C) $5.5 + 0.3x > 5 + 0.4x$
   - (D) $5 + 0.4x > 5.5 + 0.3x$

4. The side length of a square s can be determined by the formula $s = \sqrt{A}$ where A represents the area of the square. What is the side length of a square with area 0.09 square meter?
   - (F) 0.0081 meters
   - (G) 0.81 meters
   - (H) 0.03 meters
   - (J) 0.3 meters

5. What is the value of $f(x) = -3 - x$ when $x = -7$?
   - (A) -10
   - (B) -4
   - (C) 4
   - (D) 10

6. Which relationship is a direct variation?

   (F)
x	1	2	3	4
y	-1	0	1	2

   (G)
x	1	2	3	4
y	0	-1	-2	-3

   (H)
x	1	2	3	4
y	3	5	7	9

   (J)
x	1	2	3	4
y	3	6	9	12

7. Which function has x-intercept -2 and y-intercept 4?
   - (A) $2x - y = 4$
   - (B) $2y - x = 4$
   - (C) $y - 2x = 4$
   - (D) $x - 2y = 4$

8. Which equation describes the relationship between x and y in the table below?

x	-8	-4	0	4	8
y	2	1	0	-1	-2

   - (F) $y = -4x$
   - (G) $y = -\frac{1}{4}x$
   - (H) $y = 4x$
   - (J) $y = \frac{1}{4}x$

9. Which graph is described by $x - 3y = -3$?

10. Which steps could you use to graph the line that has slope 2 and contains the point $(-1, 3)$?

- $\textcircled{F}$ Plot $(-1, 3)$. Move 1 unit up and 2 units right and plot another point.
- $\textcircled{G}$ Plot $(-1, 3)$. Move 2 units up and 1 unit right and plot another point.
- $\textcircled{H}$ Plot $(-1, 3)$. Move 1 unit up and 2 units left and plot another point.
- $\textcircled{J}$ Plot $(-1, 3)$. Move 2 units up and 1 unit left and plot another point.

11. Which line is parallel to the line described by $2x + 3y = 6$?

- $\textcircled{A}$ $3x + 2y = 6$
- $\textcircled{B}$ $3x - 2y = -6$
- $\textcircled{C}$ $2x + 3y = -6$
- $\textcircled{D}$ $2x - 3y = 6$

12. Which function's graph is NOT perpendicular to the line described by $4x - y = -2$?

- $\textcircled{F}$ $y + \frac{1}{4}x = 0$
- $\textcircled{G}$ $\frac{1}{2}x = 10 - 2y$
- $\textcircled{H}$ $3y = \frac{3}{4}x + 3$
- $\textcircled{J}$ $y = -\frac{1}{4}x + \frac{3}{2}$

13. Company A charges $30 plus $0.40 per mile for a car rental. The total charge for m miles is given by $f(m) = 30 + 0.4m$. For a similar car, company B charges $30 plus $0.30 per mile. The total charge for m miles is given by $g(m) = 30 + 0.3m$. Which best describes the transformation from the graph of $f(m)$ to the graph of $g(m)$?

- $\textcircled{A}$ Translation up
- $\textcircled{B}$ Translation down
- $\textcircled{C}$ Rotation
- $\textcircled{D}$ Reflection

Gridded Response

14. What is the value of x in the diagram below?

$$\triangle ABC \sim \triangle DEF$$
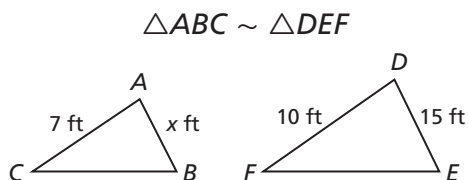
15. What is the 46th term in the arithmetic sequence $-1.5, -1.3, -1.1, -0.9, \ldots$?

**16.** What is the y-intercept of $y - 2 = 3(x + 4)$?

Short Response

17. A video store charges a $10 membership fee plus $2 for each movie rental. The total cost for x movie rentals is given by $f(x) = 2x + 10$.

- **a.** Graph this function.
- **b.** Give a reasonable domain and range.

18. The table below shows the federal minimum wage in different years.

Year	1960	1970	1980	1990	2000
Minimum Wage ($)	1.00	1.60	3.10	3.80	5.15

- **a.** Find the rate of change for each ten-year time period. Show your work.
- **b.** During which time period did the minimum wage increase the fastest? Explain what the rate of change for this time period means.

19. a. Find the slope of the line below.
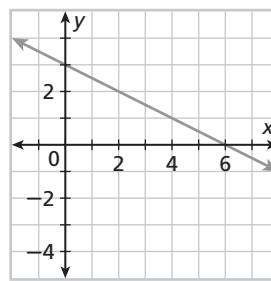
- **b.** Write an equation in slope-intercept form for a line that is perpendicular to the line in part **a** and has the same y-intercept as the function in part **a**. Show your work and explain how you found your answer.

Extended Response

20. The relationship between the wind speed at a given temperature and the apparent temperature is approximately linear. The table shows the apparent temperature at various wind speeds when the actual temperature is t °F.

Wind Speed (mi/h)	5	10	15
Apparent Temperature (°F)	36	34	32

- **a.** Write an equation in slope-intercept form that describes the data in the table. Explain how you found your answer.
- **b.** What does the slope mean in this situation?
- **c.** What is the actual temperature t? Explain.
- **d.** Determine the apparent temperature when the wind speed is 12 miles per hour. Show your work.