CHAPTER 7

Exponents and Polynomials

Chapter Focus

- Use exponents and scientific notation to describe numbers.
- Use laws of exponents to simplify monomials.
- Perform operations with polynomials.

Every Second *Counts*

How many seconds until you graduate? The concepts in this chapter will help you find and use large numbers such as this one.

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ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. Associative Property
2. coefficient
3. Commutative Property
4. exponent
5. like terms

A. a number that is raised to a power
B. a number that is multiplied by a variable
C. a property of addition and multiplication that states you can add or multiply numbers in any order
D. the number of times a base is used as a factor
E. terms that contain the same variables raised to the same powers
F. a property of addition and multiplication that states you can group the numbers in any order

Exponents

Write each expression using a base and an exponent.

6. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ **7.** $5 \cdot 5$ **8.** $(-10)(-10)(-10)(-10)$

9. $x \cdot x \cdot x$ **10.** $k \cdot k \cdot k \cdot k \cdot k$ **11.** 9

Evaluate Powers

Evaluate each expression.

12. 3^4 **13.** -12^2 **14.** 5^3

15. 2^5 **16.** 4^3 **17.** $(-1)^6$

Multiply Decimals

Multiply.

18. 0.006×10 **19.** 25.25×100 **20.** 2.4×6.5

Combine Like Terms

Simplify each expression.

21. $6 + 3p + 14 + 9p$ **22.** $8y - 4x + 2y + 7x - x$

23. $(12 + 3w - 5) + 6w - 3 - 5w$ **24.** $6n - 14 + 5n$

Squares and Square Roots

Tell whether each number is a perfect square. If so, identify its positive square root.

25. 42 **26.** 81 **27.** 36 **28.** 50

29. 100 **30.** 4 **31.** 1 **32.** 12

CHAPTER 7

Study Guide: Preview

Where You've Been

Previously, you

- wrote and evaluated exponential expressions.
- simplified algebraic expressions by combining like terms.

In This Chapter

You will study

- properties of exponents.
- powers of 10 and scientific notation.
- how to add, subtract, and multiply polynomials by using properties of exponents and combining like terms.

Where You're Going

You can use the skills in this chapter

- to model area, perimeter, and volume in geometry.
- to express very small or very large quantities in science classes such as Chemistry, Physics, and Biology.
- in the real world to model business profits and population growth or decline.

Key Vocabulary/Vocabulario

binomial	binomio
degree of a monomial	grado de un monomio
degree of a polynomial	grado de un polinomio
leading coefficient	coeficiente principal
monomial	monomio
perfect-square trinomial	trinomio cuadrado perfecto
polynomial	polinomio
scientific notation	notación científica
standard form of a polynomial	forma estándar de un polinomio
trinomial	trinomio

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. Very large and very small numbers are often encountered in the sciences. If *notation* means a method of writing something, what might **scientific notation** mean?

2. A **polynomial** written in standard form may have more than one algebraic term. What do you think the **leading coefficient** of a polynomial is?

3. A simple definition of **monomial** is "an expression with exactly one term." If the prefix *mono-* means "one" and the prefix *bi-* means "two," define the word **binomial**.

4. What words do you know that begin with the prefix *tri-*? What do they all have in common? Define the word **trinomial** based on the prefix *tri-* and the information given in Problem 3.

Reading and Writing Math

Reading Strategy: Read and Understand the Problem

Follow this strategy when solving word problems.

- Read the problem through once.
- Identify exactly what the problem asks you to do.
- Read the problem again, slowly and carefully, to break it into parts.
- Highlight or underline the key information.
- Make a plan to solve the problem.

From Lesson 6-6

29. **Multi-Step** Linda works at a pharmacy for \$15 an hour. She also baby-sits for \$10 an hour. Linda needs to earn at least \$90 per week, but she does not want to work more than 20 hours per week. Show and describe the number of hours Linda could work at each job to meet her goals. List two possible solutions.

Step 1	**Identify exactly what the problem asks you to do.**	• Show and describe the number of hours Linda can work at each job and earn at least \$90 per week, without working more than 20 hours per week. • List two possible solutions of the system.
Step 2	**Break the problem into parts. Highlight or underline the key information.**	• Linda has two jobs. She makes **\$15 per hour** at one job and **\$10 per hour** at the other job. • She wants to earn **at least \$90 per week.** • She does **not** want to work **more than 20 hours per week.**
Step 3	**Make a plan to solve the problem.**	• Write a system of inequalities. • Solve the system. • Identify two possible solutions of the system.

Try This

For the problem below,

a. identify exactly what the problem asks you to do.

b. break the problem into parts. Highlight or underline the key information.

c. make a plan to solve the problem.

1. The difference between the length and the width of a rectangle is 14 units. The area is 120 square units. Write and solve a system of equations to determine the length and the width of the rectangle. (*Hint:* The formula for the area of a rectangle is $A = \ell w$.)

7-1 Integer Exponents

Objectives

Evaluate expressions containing zero and integer exponents.

Simplify expressions containing zero and integer exponents.

Who uses this?

Manufacturers can use negative exponents to express very small measurements.

In 1930, the Model A Ford was one of the first cars to boast precise craftsmanship in mass production. The car's pistons had a diameter of $3\frac{7}{8}$ inches; this measurement could vary by at most 10^{-3} inch.

You have seen positive exponents. Recall that to simplify 3^2, use 3 as a factor 2 times: $3^2 = 3 \cdot 3 = 9$.

But what does it mean for an exponent to be negative or 0? You can use a table and look for a pattern to figure it out.

Remember!

Base → x^4 ← Exponent

Power	5^5	5^4	5^3	5^2	5^1	5^0	5^{-1}	5^{-2}
Value	3125	625	125	25	5			

$\div 5 \quad \div 5 \quad \div 5 \quad \div 5$

When the exponent decreases by one, the value of the power is divided by 5. Continue the pattern of dividing by 5:

$$5^0 = \frac{5}{5} = 1 \qquad 5^{-1} = \frac{1}{5} = \frac{1}{5^1} \qquad 5^{-2} = \frac{1}{5} \div 5 = \frac{1}{25} = \frac{1}{5^2}$$

Know it! Note

Integer Exponents

WORDS	NUMBERS	ALGEBRA
Zero exponent—Any nonzero number raised to the zero power is 1.	$3^0 = 1 \quad 123^0 = 1$ $(-16)^0 = 1 \quad \left(\frac{3}{7}\right)^0 = 1$	If $x \neq 0$, then $x^0 = 1$.
Negative exponent—A nonzero number raised to a negative exponent is equal to 1 divided by that number raised to the opposite (positive) exponent.	$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$	If $x \neq 0$ and n is an integer, then $x^{-n} = \frac{1}{x^n}$.

Reading Math

2^{-4} is read "2 to the negative fourth power."

Notice the phrase "nonzero number" in the table above. This is because 0^0 and 0 raised to a negative power are both undefined. For example, if you use the pattern given above the table with a base of 0 instead of 5, you would get $0^0 = \frac{0}{0}$. Also, 0^{-6} would be $\frac{1}{0^6} = \frac{1}{0}$. Since division by 0 is undefined, neither value exists.

EXAMPLE 1 Manufacturing Application

The diameter for the Model A Ford piston could vary by at most 10^{-3} inch. Simplify this expression.

$$10^{-3} = \frac{1}{10^3} = \frac{1}{10 \cdot 10 \cdot 10} = \frac{1}{1000}$$

10^{-3} inch is equal to $\frac{1}{1000}$ inch, or 0.001 inch.

CHECK IT OUT! **1.** A sand fly may have a wingspan up to 5^{-3} m. Simplify this expression.

EXAMPLE 2 Zero and Negative Exponents

Simplify.

A 2^{-3}

$$2^{-3} = \frac{1}{2^3} = \frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{8}$$

B 5^0

$5^0 = 1$ *Any nonzero number raised to the zero power is 1.*

C $(-3)^{-4}$

$$(-3)^{-4} = \frac{1}{(-3)^4} = \frac{1}{(-3)(-3)(-3)(-3)} = \frac{1}{81}$$

D -3^{-4}

$$-3^{-4} = -\frac{1}{3^4} = -\frac{1}{3 \cdot 3 \cdot 3 \cdot 3} = -\frac{1}{81}$$

Caution!

In $(-3)^{-4}$, the base is negative because the negative sign is inside the parentheses.

In -3^{-4} the base (3) is positive.

CHECK IT OUT! **Simplify.**

2a. 10^{-4} **2b.** $(-2)^{-4}$ **2c.** $(-2)^{-5}$ **2d.** -2^{-5}

EXAMPLE 3 Evaluating Expressions with Zero and Negative Exponents

Evaluate each expression for the given value(s) of the variable(s).

A x^{-1} **for** $x = 2$

2^{-1} *Substitute 2 for x.*

$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$ *Use the definition $x^{-n} = \frac{1}{x^n}$.*

B $a^0 b^{-3}$ **for** $a = 8$ **and** $b = -2$

$8^0 \cdot (-2)^{-3}$ *Substitute 8 for a and −2 for b.*

$1 \cdot \frac{1}{(-2)^3}$ *Simplify expressions with exponents.*

$1 \cdot \frac{1}{(-2)(-2)(-2)}$ *Write the power in the denominator as a product.*

$1 \cdot \frac{1}{-8}$ *Simplify the power in the denominator.*

$-\frac{1}{8}$ *Simplify.*

CHECK IT OUT! **Evaluate each expression for the given value(s) of the variable(s).**

3a. p^{-3} for $p = 4$ **3b.** $8a^{-2}b^0$ for $a = -2$ and $b = 6$

What if you have an expression with a negative exponent in a denominator, such as $\frac{1}{x^{-8}}$?

$$x^{-n} = \frac{1}{x^n}, \text{ or } \frac{1}{x^n} = x^{-n}$$ *Definition of negative exponent*

$$\frac{1}{x^{-8}} = x^{-(-8)}$$ *Substitute −8 for n.*

$$= x^8$$ *Simplify the exponent on the right side.*

So if a base with a negative exponent is in a denominator, it is equivalent to the same base with the opposite (positive) exponent in the numerator.

An expression that contains negative or zero exponents is not considered to be simplified. Expressions should be rewritten with only positive exponents.

EXAMPLE 4 Simplifying Expressions with Zero and Negative Exponents

Simplify.

A $3y^{-2}$

$$3y^{-2} = 3 \cdot y^{-2}$$
$$= 3 \cdot \frac{1}{y^2}$$
$$= \frac{3}{y^2}$$

B $\frac{-4}{k^{-4}}$

$$\frac{-4}{k^{-4}} = -4 \cdot \frac{1}{k^{-4}}$$
$$= -4 \cdot k^4$$
$$= -4k^4$$

C $\frac{x^{-3}}{a^0 y^5}$

$$\frac{x^{-3}}{a^0y^5} = \frac{1}{x^3 \cdot 1 \cdot y^5}$$ *$a^0 = 1$ and $x^{-3} = \frac{1}{x^3}$.*

$$= \frac{1}{x^3y^5}$$

CHECK IT OUT! **Simplify.**

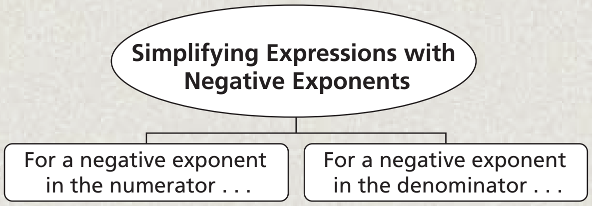
4a. $2r^0m^{-3}$ **4b.** $\frac{r^{-3}}{7}$ **4c.** $\frac{g^4}{h^{-6}}$

THINK AND DISCUSS

1. Complete each equation: $2b^{?} = \frac{2}{b^2}$, $\frac{s^{-3}}{k^{?}} = \frac{1}{s^3}$, $?^{-2} = \frac{1}{t^2}$

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, describe how to simplify, and give an example.



7-1 Exercises

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GUIDED PRACTICE

SEE EXAMPLE 1 p. 461

1. **Medicine** A typical virus is about 10^{-7} m in size. Simplify this expression.

SEE EXAMPLE 2 p. 461

Simplify.

2. 6^{-2}
3. 3^0
4. -5^{-2}
5. 3^{-3}
6. 1^{-8}
7. -8^{-3}
8. 10^{-2}
9. $(4.2)^0$
10. $(-3)^{-3}$
11. 4^{-2}

SEE EXAMPLE 3 p. 461

Evaluate each expression for the given value(s) of the variable(s).

12. b^{-2} for $b = -3$
13. $(2t)^{-4}$ for $t = 2$
14. $(m - 4)^{-5}$ for $m = 6$
15. $2x^0y^{-3}$ for $x = 7$ and $y = -4$

SEE EXAMPLE 4 p. 462

Simplify.

16. $4m^0$
17. $3k^{-4}$
18. $\frac{7}{r^{-7}}$
19. $\frac{x^{10}}{d^{-3}}$
20. $2x^0y^{-4}$
21. $\frac{f^{-4}}{g^{-6}}$
22. $\frac{c^4}{d^{-3}}$
23. p^7q^{-1}

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
24	1
25–36	2
37–42	3
43–57	4

Extra Practice

Skills Practice p. S16
Application Practice p. S34

24. **Biology** One of the smallest bats is the northern blossom bat, which is found from Southeast Asia to Australia. This bat weighs about 2^{-1} ounce. Simplify this expression.

Simplify.

25. 8^0
26. 5^{-4}
27. 3^{-4}
28. -9^{-2}
29. -6^{-2}
30. 7^{-2}
31. $\left(\frac{2}{5}\right)^0$
32. 13^{-2}
33. $(-3)^{-1}$
34. $(-4)^2$
35. $\left(\frac{1}{2}\right)^{-2}$
36. -7^{-1}

Evaluate each expression for the given value(s) of the variable(s).

37. x^{-4} for $x = 4$
38. $\left(\frac{2}{3}v\right)^{-3}$ for $v = 9$
39. $(10 - d)^0$ for $d = 11$
40. $10m^{-1}n^{-5}$ for $m = 10$ and $n = -2$
41. $(3ab)^{-2}$ for $a = \frac{1}{2}$ and $b = 8$
42. $4w^vx^v$ for $w = 3$, $v = 0$, and $x = -5$

Simplify.

43. k^{-4}
44. $2z^{-8}$
45. $\frac{1}{2b^{-3}}$
46. $c^{-2}d$
47. $-5x^{-3}$
48. $4x^{-6}y^{-2}$
49. $\frac{2f^0}{7g^{-10}}$
50. $\frac{r^{-5}}{s^{-1}}$
51. $\frac{s^5}{t^{-12}}$
52. $\frac{3w^{-5}}{x^{-6}}$
53. b^0c^0
54. $\frac{2}{3}m^{-1}n^5$
55. $\frac{q^{-2}r^0}{s^0}$
56. $\frac{a^{-7}b^2}{c^3d^{-4}}$
57. $\frac{h^3k^{-1}}{6m^2}$

Evaluate each expression for $x = 3$, $y = -1$, and $z = 2$.

58. z^{-5} **59.** $(x + y)^{-4}$ **60.** $(yz)^{0}$ **61.** $(xyz)^{-1}$

62. $(xy - 3)^{-2}$ **63.** x^{-y} **64.** $(yz)^{-x}$ **65.** xy^{-4}

66. **ERROR ANALYSIS** Look at the two equations below. Which is incorrect? Explain the error.

A $5x^{-3} = \frac{1}{5x^3}$

B $5x^{-3} = \frac{5}{x^3}$

Simplify.

67. a^3b^{-2} **68.** $c^{-4}d^3$ **69.** $v^0w^2y^{-1}$ **70.** $(a^2b^{-7})^0$ **71.** $-5y^{-6}$

72. $\frac{2a^{-5}}{b^{-6}}$ **73.** $\frac{2a^3}{b^{-1}}$ **74.** $\frac{m^2}{n^{-3}}$ **75.** $\frac{x^{-8}}{3y^{12}}$ **76.** $-\frac{20p^{-1}}{5q^{-3}}$

LINK

Biology

When bleeding occurs, platelets (which appear green in the image above) help to form a clot to reduce blood loss. Calcium and vitamin K are also necessary for clot formation.

77. **Biology** Human blood contains red blood cells, white blood cells, and platelets. The table shows the sizes of these components. Simplify each expression.

Blood Components

Part	Size (m)
Red blood cell	$125{,}000^{-1}$
White blood cell	$3(500)^{-2}$
Platelet	$3(1000)^{-2}$

Tell whether each statement is sometimes, always, or never true.

78. If n is a positive integer, then $x^{-n} = \frac{1}{x^n}$.

79. If x is positive, then x^{-n} is negative.

80. If n is zero, then x^{-n} is 1.

81. If n is a negative integer, then $x^{-n} = 1$.

82. If x is zero, then x^{-n} is 1.

83. If n is an integer, then $x^{-n} > 1$.

84. **Critical Thinking** Find the value of $2^3 \cdot 2^{-3}$. Then find the value of $3^2 \cdot 3^{-2}$. Make a conjecture about the value of $a^n \cdot a^{-n}$.

85. **Write About It** Explain in your own words why 2^{-3} is the same as $\frac{1}{2^3}$.

Find the missing value.

86. $\frac{1}{4} = 2^{\square}$ **87.** $9^{-2} = \frac{1}{\square}$ **88.** $\frac{1}{64} = \square^{-2}$ **89.** $\frac{\square}{3} = 3^{-1}$

90. $7^{-2} = \frac{1}{\square}$ **91.** $10^{\square} = \frac{1}{1000}$ **92.** $3 \cdot 4^{-2} = \frac{3}{\square}$ **93.** $2 \cdot \frac{1}{5} = 2 \cdot 5^{\square}$

MULTI-STEP TEST PREP

94. This problem will prepare you for the Multi-Step Test Prep on page 494.

a. The product of the frequency f and the wavelength w of light in air is a constant v. Write an equation for this relationship.

b. Solve this equation for wavelength. Then write this equation as an equation with f raised to a negative exponent.

c. The units for frequency are hertz (Hz). One hertz is one cycle per second, which is often written as $\frac{1}{s}$. Rewrite this expression using a negative exponent.

Test Prep

95. Which is NOT equivalent to the other three?

(A) $\frac{1}{25}$ (B) 5^{-2} (C) 0.04 (D) -25

96. Which is equal to 6^{-2}?

(F) $6(-2)$ (G) $(-6)(-6)$ (H) $-\frac{1}{6 \cdot 6}$ (J) $\frac{1}{6 \cdot 6}$

97. Simplify $\frac{a^3b^{-2}}{c^{-1}}$.

(A) $\frac{a^3c}{b^2}$ (B) $\frac{a^3b^2}{-c}$ (C) $\frac{a^3}{-b^2c}$ (D) $\frac{c}{a^3b^2}$

98. **Gridded Response** Simplify $\left[2^{-2} + (6 + 2)^0\right]$.

99. **Short Response** If a and b are real numbers and n is a positive integer, write a simplified expression for the product $a^{-n} \cdot b^0$ that contains only positive exponents. Explain your answer.

CHALLENGE AND EXTEND

100. **Multi-Step** Copy and complete the table of values below. Then graph the ordered pairs and describe the shape of the graph.

x	-4	-3	-2	-1	0	1	2	3	4
$y = 2^x$	■	■	■	■	■	■	■	■	■

101. **Multi-Step** Copy and complete the table. Then write a rule for the values of 1^n and $(-1)^n$ when n is any negative integer.

n	-1	-2	-3	-4	-5
1^n	■	■	■	■	■
$(-1)^n$	■	■	■	■	■

SPIRAL REVIEW

Solve each equation. *(Lesson 2-3)*

102. $6x - 4 = 8$

103. $-9 = 3(p - 1)$

104. $\frac{y}{5} - 8 = -12$

105. $1.5h - 5 = 1$

106. $2w + 6 - 3w = -10$

107. $-12 = \frac{1}{2}n + 2 - n$

Identify the independent and dependent variables. Write a rule in function notation for each situation. *(Lesson 4-3)*

108. Pink roses cost $1.50 per stem.

109. For dog-sitting, Beth charges a $30 flat fee plus $10 a day.

Write the equation that describes each line in slope-intercept form. *(Lesson 5-7)*

110. slope = 3, y-intercept = -4

111. slope = $\frac{1}{3}$, y-intercept = 5

112. slope = 0, y-intercept = $\frac{2}{3}$

113. slope = -4, the point $(1, 5)$ is on the line.

7-2 Powers of 10 and Scientific Notation

Nucleus of a silicon atom

Objectives
Evaluate and multiply by powers of 10.

Convert between standard notation and scientific notation.

Vocabulary
scientific notation

Why learn this?
Powers of 10 can be used to read and write very large and very small numbers, such as the masses of atomic particles. (See Exercise 44.)

The table shows relationships between several powers of 10.

÷ 10 ÷ 10 ÷ 10 ÷ 10 ÷ 10 ÷ 10

Power	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
Value	1000	100	10	1	$\frac{1}{10} = 0.1$	$\frac{1}{100} = 0.01$	$\frac{1}{1000} = 0.001$

× 10 × 10 × 10 × 10 × 10 × 10

- Each time you **divide by 10**, the exponent decreases by 1 and the decimal point moves one place to the left.
- Each time you **multiply by 10**, the exponent increases by 1 and the decimal point moves one place to the right.

Know it! Note

Powers of 10

WORDS	NUMBERS
Positive Integer Exponent	
If n is a positive integer, find the value of 10^n by starting with 1 and moving the decimal point n places to the right.	$10^4 = 1\ 0,\ 0\ 0\ 0$ (4 places)
Negative Integer Exponent	
If n is a positive integer, find the value of 10^{-n} by starting with 1 and moving the decimal point n places to the left.	$10^{-6} = \frac{1}{10^6} = 0.0\ 0\ 0\ 0\ 0\ 1$ (6 places)

EXAMPLE 1

Evaluating Powers of 10

Find the value of each power of 10.

A 10^{-3}

Start with 1 and move the decimal point three places to the left.

0. 0 0 1

0.001

B 10^2

Start with 1 and move the decimal point two places to the right.

1 0 0

100

C 10^0

Start with 1 and move the decimal point zero places.

1

Writing Math

You may need to add zeros to the right or left of a number in order to move the decimal point in that direction.

CHECK IT OUT! Find the value of each power of 10.

1a. 10^{-2} **1b.** 10^{5} **1c.** 10^{10}

EXAMPLE 2 Writing Powers of 10

Reading Math

If you do not see a decimal point in a number, it is understood to be at the end of the number.

Write each number as a power of 10.

A 10,000,000

The decimal point is seven places to the right of 1, so the exponent is 7.

10^{7}

B 0.001

The decimal point is three places to the left of 1, so the exponent is –3.

10^{-3}

C 10

The decimal point is one place to the right of 1, so the exponent is 1.

10^{1}

CHECK IT OUT! Write each number as a power of 10.

2a. 100,000,000 **2b.** 0.0001 **2c.** 0.1

You can also move the decimal point to find the product of any number and a power of 10. You start with the number instead of starting with 1.

Know it! Note

Multiplying by Powers of 10	
If the exponent is a positive integer, move the decimal point to the right.	$125 \times 10^{5} = 12{,}500{,}000$ (5 places)
If the exponent is a negative integer, move the decimal point to the left.	$36.2 \times 10^{-3} = 0.0362$ (3 places)

EXAMPLE 3 Multiplying by Powers of 10

Find the value of each expression.

A 97.86×10^{6}

97.8 6 0 0 0 0 *Move the decimal point 6 places to the right.*

97,860,000

B 19.5×10^{-4}

0 0 1 9.5 *Move the decimal point 4 places to the left.*

0.00195

CHECK IT OUT! Find the value of each expression.

3a. 853.4×10^{5} **3b.** 0.163×10^{-2}

Scientific notation is a method of writing numbers that are very large or very small. A number written in scientific notation has two parts that are multiplied.

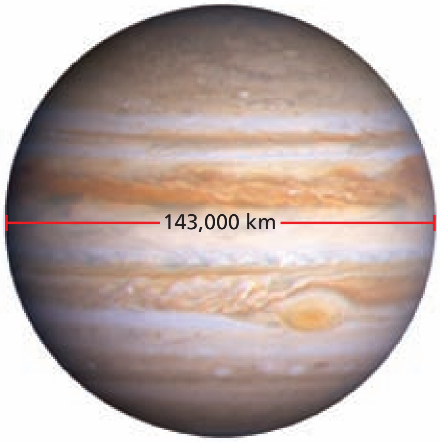
The first part is a number that is greater than or equal to 1 and less than 10.

3.5×10^{11} 9.98×10^{-2}

The second part is a power of 10.

EXAMPLE 4 Astronomy Application

Jupiter has a diameter of about 143,000 km. Its shortest distance from Earth is about 5.91×10^8 km, and its average distance from the Sun is about 778,400,000 km. Jupiter's orbital speed is approximately 1.3×10^4 m/s.



Reading Math

Standard form refers to the usual way that numbers are written.

A **Write Jupiter's shortest distance from Earth in standard form.**

5.91×10^8

5.9 1 0 0 0 0 0 0 — *Move the decimal point 8 places to the right.*

591,000,000 km

B **Write Jupiter's average distance from the Sun in scientific notation.**

778,400,000

7 7 8, 4 0 0, 0 0 0 — *Count the number of places you need to move the decimal point to get a number between 1 and 10.*

8 places

7.784×10^8 km — *Use that number as the exponent of 10.*

CHECK IT OUT!

4a. Use the information above to write Jupiter's diameter in scientific notation.

4b. Use the information above to write Jupiter's orbital speed in standard form.

EXAMPLE 5 Comparing and Ordering Numbers in Scientific Notation

Order the list of numbers from least to greatest.

1.2×10^{-1}, 8.2×10^4, 6.2×10^5, 2.4×10^5, 1×10^{-1}, 9.9×10^{-4}

Step 1 List the numbers in order by powers of 10.

9.9×10^{-4}, 1.2×10^{-1}, 1×10^{-1}, 8.2×10^4, 6.2×10^5, 2.4×10^5

Step 2 Order the numbers that have the same power of 10.

9.9×10^{-4}, 1×10^{-1}, 1.2×10^{-1}, 8.2×10^4, 2.4×10^5, 6.2×10^5

CHECK IT OUT!

5. Order the list of numbers from least to greatest.
5.2×10^{-3}, 3×10^{14}, 4×10^{-3}, 2×10^{-12}, 4.5×10^{30}, 4.5×10^{14}

THINK AND DISCUSS

1. Tell why 34.56×10^4 is not correctly written in scientific notation.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer.

Powers of 10 and Scientific Notation

- A negative exponent corresponds to moving the decimal point __?__.
- A positive exponent corresponds to moving the decimal point __?__.

7-2 Exercises

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GUIDED PRACTICE

1. **Vocabulary** Explain how you can tell whether a number is written in *scientific notation*.

SEE EXAMPLE 1 p. 466

Find the value of each power of 10.

2. 10^6
3. 10^{-5}
4. 10^{-4}
5. 10^8

SEE EXAMPLE 2 p. 467

Write each number as a power of 10.

6. 10,000
7. 0.000001
8. 100,000,000,000,000,000

SEE EXAMPLE 3 p. 467

Find the value of each expression.

9. 650.3×10^6
10. 48.3×10^{-4}
11. 92×10^{-3}

SEE EXAMPLE 4 p. 468

12. **Astronomy** A light-year is the distance that light travels in a year and is equivalent to 9.461×10^{12} km. Write this distance in standard form.

SEE EXAMPLE 5 p. 468

13. Order the list of numbers from least to greatest.
8.5×10^{-1}, 3.6×10^8, 5.85×10^{-3}, 2.5×10^{-1}, 8.5×10^8

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14–17	1
18–20	2
21–24	3
25–26	4
27	5

Extra Practice

Skills Practice p. S16
Application Practice p. S34

Find the value of each power of 10.

14. 10^3
15. 10^{-9}
16. 10^{-12}
17. 10^{14}

Write each number as a power of 10.

18. 0.01
19. 1,000,000
20. 0.000000000000001

Find the value of each expression.

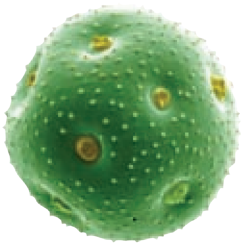
21. 9.2×10^4
22. 1.25×10^{-7}
23. 42×10^{-5}
24. 0.05×10^7

25. **Biology** The human body is made of about 1×10^{13} cells. Write this number in standard form.

26. **Statistics** At the beginning of the twenty-first century, the population of China was about 1,287,000,000. Write this number in scientific notation.

27. Order the list of numbers from least to greatest.
2.13×10^{-1}, 3.12×10^2, 1.23×10^{-3}, 2.13×10^1, 1.32×10^{-3}, 3.12×10^{-3}

28. **Health** Donnell is allergic to pollen. The diameter of a grain of pollen is between 1.2×10^{-5} m and 9×10^{-5} m. Donnell's air conditioner has a filter that removes particles larger than 3×10^{-7} m. Will the filter remove pollen? Explain.



Grain of pollen, enlarged 1300 times

29. **Entertainment** In the United States, a CD is certified platinum if it sells 1,000,000 copies. A CD that has gone 2 times platinum has sold 2,000,000 copies. How many copies has a CD sold if it has gone 27 times platinum? Write your answer in scientific notation.

Write each number in scientific notation.

30. 40,080,000
31. 235,000
32. 170,000,000,000
33. 0.0000006
34. 0.000077
35. 0.0412

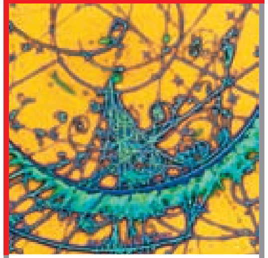
State whether each number is written in scientific notation. If not, write it in scientific notation.

36. 50×10^{-5} **37.** 8.1×10^{-2} **38.** 1,200,000 **39.** 0.25×10^{3}

40. 0.1 **41.** 7×10^{8} **42.** 48,000 **43.** 3.5×10^{-6}

LINK

Chemistry



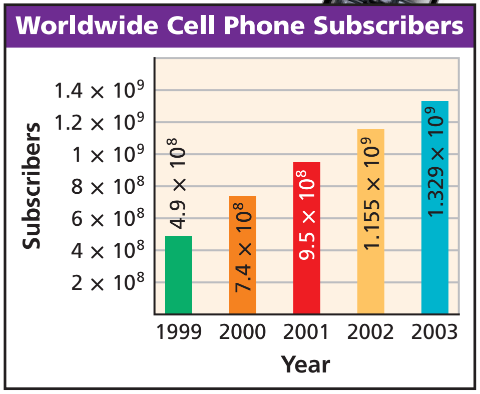
The image above is a colored bubble-chamber photograph. It shows the tracks left by subatomic particles in a particle accelerator.

44. Chemistry Atoms are made of three elementary particles: protons, electrons, and neutrons. The mass of a proton is about 1.67×10^{-27} kg. The mass of an electron is about 0.000000000000000000000000000000911 kg. The mass of a neutron is about 1.68×10^{-27} kg. Which particle has the least mass? (*Hint:* Compare the numbers after they are written in scientific notation.)

45. Communication This bar graph shows the increase of cellular telephone subscribers worldwide.

a. Write the number of subscribers for the following years in standard form: 1999, 2000, and 2003.

b. Zorah looks at the bar graph and says, "It looks like the number of cell phone subscribers nearly doubled from 2000 to 2003." Do you agree with Zorah? Use scientific notation to explain your answer.



46. Measurement In the metric system, the basic unit for measuring length is the meter (m). Other units for measuring length are based on the meter and powers of 10, as shown in the table.

Selected Metric Lengths	
1 millimeter (mm) = 10^{-3} m	1 dekameter (dam) = 10^{1} m
1 centimeter (cm) = 10^{-2} m	1 hectometer (hm) = 10^{2} m
1 decimeter (dm) = 10^{-1} m	1 kilometer (km) = 10^{3} m

a. Which lengths in the table are longer than a meter? Which are shorter than a meter? How do you know?

b. Evaluate each power of 10 in the table to check your answers to part **a.**

47. Critical Thinking Recall that $\frac{1}{10^3} = 10^{-3}$. Based on this information, complete the following statement: Dividing a number by 10^3 is equivalent to multiplying by ▢.

48. Write About It When you change a number from scientific notation to standard form, explain how you know which way to move the decimal point and how many places to move it.

MULTI-STEP TEST PREP

49. This problem will prepare you for the Multi-Step Test Prep on page 494.

a. The speed of light is approximately 3×10^{8} m/s. Write this number in standard form.

b. Why do you think it would be better to express this number in scientific notation rather than standard form?

c. The wavelength of a shade of red light is 0.00000068 meters. Write this number in scientific notation.

TEST PREP

50. There are about 3.2×10^7 seconds in one year. What is this number in standard form?
    - (A) 0.000000032
    - (B) 0.00000032
    - (C) 32,000,000
    - (D) 320,000,000

51. Which expression is the scientific notation for 82.35?
    - (F) 8.235×10^1
    - (G) 823.5×10^{-1}
    - (H) 8.235×10^{-1}
    - (J) 0.8235×10^2

52. Which statement is correct for the list of numbers below?
    2.35×10^{-8}, 0.000000029, 1.82×10^8, 1,290,000,000, 1.05×10^9
    - (A) The list is in increasing order.
    - (B) If 0.000000029 is removed, the list will be in increasing order.
    - (C) If 1,290,000,000 is removed, the list will be in increasing order.
    - (D) The list is in decreasing order.

CHALLENGE AND EXTEND

53. **Technology** The table shows estimates of computer storage. A CD-ROM holds 700 MB. A DVD-ROM holds 4.7 GB. Estimate how many times more storage a DVD has than a CD. Explain how you found your answer.

Computer Storage
1 kilobyte (KB) ≈ 1000 bytes
1 megabyte (MB) ≈ 1 million bytes
1 gigabyte (GB) ≈ 1 billion bytes

54. For parts **a–d**, use what you know about multiplying by powers of 10 and the Commutative and Associative Properties of Multiplication to find each product. Write each answer in scientific notation.
    - **a.** $(3 \times 10^2)(2 \times 10^3)$
    - **b.** $(5 \times 10^8)(1.5 \times 10^{-6})$
    - **c.** $(2.2 \times 10^{-8})(4 \times 10^{-3})$
    - **d.** $(2.5 \times 10^{-12})(2 \times 10^6)$
    - **e.** Based on your answers to parts **a–d**, write a rule for multiplying numbers in scientific notation.
    - **f.** Does your rule work when you multiply $(6 \times 10^3)(8 \times 10^5)$? Explain.

SPIRAL REVIEW

Define a variable and write an inequality for each situation. Graph the solutions. *(Lesson 3-1)*

55. Melanie must wait at least 45 minutes for the results of her test.

56. Ulee's dog can lose no more than 8 pounds to stay within a healthy weight range.

57. Charlene must spend more than $50 to get the advertised discount.

Solve each system by elimination. *(Lesson 6-3)*

58. $\begin{cases} x + y = 8 \\ x - y = 2 \end{cases}$

59. $\begin{cases} 2x + y = -3 \\ 2x + 3y = -1 \end{cases}$

60. $\begin{cases} x - 6y = -3 \\ 3x + 4y = 13 \end{cases}$

Evaluate each expression for the given value(s) of the variable(s). *(Lesson 7-1)*

61. t^{-4} for $t = 2$

62. $(-8m)^0$ for $m = -5$

63. $3a^{-3}b^0$ for $a = 5$ and $b = 6$

7-3 Algebra LAB

Explore Properties of Exponents

You can use patterns to find some properties of exponents.

Use with Lesson 7-3

Activity 1

1 Copy and complete the table below.

$3^2 \cdot 3^3 = (3 \cdot 3)(3 \cdot 3 \cdot 3) = 3^{\blacksquare}$
$5^4 \cdot 5^2 = (\blacksquare \cdot \blacksquare \cdot \blacksquare \cdot \blacksquare)(\blacksquare \cdot \blacksquare) = 5^{\blacksquare}$
$4^3 \cdot 4^3 = (\blacksquare \cdot \blacksquare \cdot \blacksquare)(\blacksquare \cdot \blacksquare \cdot \blacksquare) = \blacksquare^{\blacksquare}$
$2^3 \cdot 2^2 = (\blacksquare \cdot \blacksquare \cdot \blacksquare)(\blacksquare \cdot \blacksquare) = \blacksquare^{\blacksquare}$
$6^3 \cdot 6^4 = (\quad)(\quad) =$

2 Examine your completed table. Look at the two exponents in each factor and the exponent in the final answer. What pattern do you notice?

3 Use your pattern to make a conjecture: $a^m \cdot a^n = a^{\blacksquare}$.

Try This

Use your conjecture to write each product below as a single power.

1. $5^3 \cdot 5^5$ **2.** $7^2 \cdot 7^2$ **3.** $10^8 \cdot 10^4$ **4.** $8^7 \cdot 8^3$

5. Make a table similar to the one above to explore what happens when you multiply more than two powers that have the same base. Then write a conjecture in words to summarize what you find.

Activity 2

1 Copy and complete the table below.

$(2^3)^2 = 2^3 \cdot 2^3 = (\blacksquare \cdot \blacksquare \cdot \blacksquare)(\blacksquare \cdot \blacksquare \cdot \blacksquare) = 2^{\blacksquare}$
$(2^2)^3 = \blacksquare \cdot \blacksquare \cdot \blacksquare = (\blacksquare \cdot \blacksquare)(\blacksquare \cdot \blacksquare)(\blacksquare \cdot \blacksquare) = \blacksquare^{\blacksquare}$
$(4^2)^4 = \blacksquare \cdot \blacksquare \cdot \blacksquare \cdot \blacksquare = (\blacksquare \cdot \blacksquare)(\blacksquare \cdot \blacksquare)(\blacksquare \cdot \blacksquare)(\blacksquare \cdot \blacksquare) = \blacksquare^{\blacksquare}$
$(3^4)^2 = \blacksquare \cdot \blacksquare = (\blacksquare \cdot \blacksquare \cdot \blacksquare \cdot \blacksquare)(\blacksquare \cdot \blacksquare \cdot \blacksquare \cdot \blacksquare) = \blacksquare^{\blacksquare}$
$(6^3)^4 =$

2 Examine your completed table. Look at the two exponents in the original expression and the exponent in the final answer. What pattern do you notice?

3 Use your pattern to make a conjecture: $(a^m)^n = a^{\blacksquare}$.

Try This

Use your conjecture to write each product below as a single power.

6. $(5^3)^2$ **7.** $(7^2)^2$ **8.** $(3^3)^4$ **9.** $(9^7)^3$

10. Make a table similar to the one in Activity 2 to explore what happens when you raise a power to two powers, for example, $\left[(4^2)^3\right]^3$. Then write a conjecture in words to summarize what you find.

Activity 3

1 Copy and complete the table below.

$(ab)^3 = (ab)(ab)(ab) = (a \cdot a \cdot a)(b \cdot b \cdot b) = a^{\square} b^{\square}$
$(mn)^4 = (\square)(\square)(\square)(\square) = (\square \cdot \square \cdot \square \cdot \square)(\square \cdot \square \cdot \square \cdot \square) = \square^{\square}\square^{\square}$
$(xy)^2 = (\square)(\square) = (\square \cdot \square)(\square \cdot \square) = \square^{\square}\square^{\square}$
$(cd)^5 = (\square)(\square)(\square)(\square)(\square) = (\square \cdot \square \cdot \square \cdot \square \cdot \square)(\square \cdot \square \cdot \square \cdot \square \cdot \square) = \square^{\square}\square^{\square}$
$(pq)^6 =$

2 Examine your completed table. Look at the original expression and the final answer. What pattern do you notice?

3 Use your pattern to make a conjecture: $(ab)^n = a^{\square} b^{\square}$.

Try This

Use your conjecture to write each power below as a product.

11. $(rs)^8$ **12.** $(yz)^9$ **13.** $(ab)^7$ **14.** $(xz)^{12}$

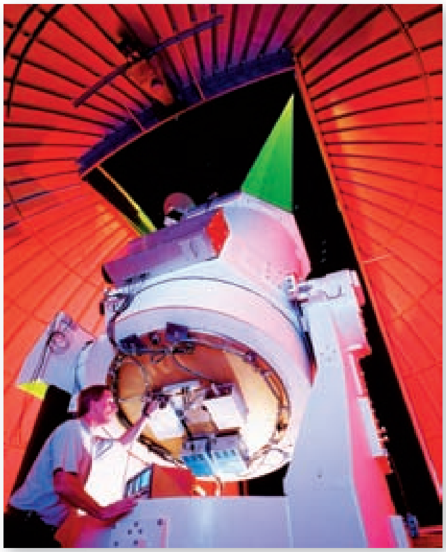
15. Look at the first row of your table. What property or properties allow you to write $(ab)(ab)(ab)$ as $(a \cdot a \cdot a)(b \cdot b \cdot b)$?

16. Make a table similar to the one above to explore what happens when you raise a product containing more than two factors to a power, for example, $(xyz)^7$. Then write a conjecture in words to summarize what you find.

7-3 Multiplication Properties of Exponents

Objective
Use multiplication properties of exponents to evaluate and simplify expressions.

Who uses this?
Astronomers can multiply expressions with exponents to find the distance between objects in space. (See Example 2.)



You have seen that exponential expressions are useful when writing very small or very large numbers. To perform operations on these numbers, you can use properties of exponents. You can also use these properties to simplify your answer.

In this lesson, you will learn some properties that will help you simplify exponential expressions containing multiplication.

Know it! Note

Simplifying Exponential Expressions

An exponential expression is completely simplified if…

- There are no negative exponents.
- The same base does not appear more than once in a product or quotient.
- No powers are raised to powers.
- No products are raised to powers.
- No quotients are raised to powers.
- Numerical coefficients in a quotient do not have any common factor other than 1.

Examples	Nonexamples
$\frac{b}{a}$ x^3 z^{12} a^4b^4 $\frac{s^5}{t^5}$ $\frac{5a^2}{2b}$	$a^{-2}ba$ $x \cdot x^2$ $(z^3)^4$ $(ab)^4$ $\left(\frac{s}{t}\right)^5$ $\frac{10a^2}{4b}$

Products of powers with the same base can be found by writing each power as repeated multiplication.

$$3^5 \cdot 3^2 = (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3) = 3^7$$

Notice the relationship between the exponents in the factors and the exponent in the product: $5 + 2 = 7$.

Know it! Note

Product of Powers Property

WORDS	NUMBERS	ALGEBRA
The product of two powers with the same base equals that base raised to the sum of the exponents.	$6^7 \cdot 6^4 = 6^{7+4} = 6^{11}$	If a is any nonzero real number and m and n are integers, then $a^m \cdot a^n = a^{m+n}$.

EXAMPLE 1 Finding Products of Powers

Simplify.

A $2^5 \cdot 2^6$

$2^5 \cdot 2^6$

2^{5+6} — *Since the powers have the same base, keep the base and add the exponents.*

2^{11}

B $4^2 \cdot 3^{-2} \cdot 4^5 \cdot 3^6$

$4^2 \cdot 3^{-2} \cdot 4^5 \cdot 3^6$

$(4^2 \cdot 4^5) \cdot (3^{-2} \cdot 3^6)$ — *Group powers with the same base together.*

$4^{2+5} \cdot 3^{-2+6}$ — *Add the exponents of powers with the same base.*

$4^7 \cdot 3^4$

C $a^4 \cdot b^5 \cdot a^2$

$a^4 \cdot b^5 \cdot a^2$

$(a^4 \cdot a^2) \cdot b^5$ — *Group powers with the same base together.*

$a^6 \cdot b^5$ — *Add the exponents of powers with the same base.*

$a^6 b^5$

D $y^2 \cdot y \cdot y^{-4}$

$(y^2 \cdot y^1) \cdot y^{-4}$ — *Group the first two powers.*

$y^3 \cdot y^{-4}$ — *The first two powers have the same base, so add the exponents.*

y^{-1} — *The two remaining powers have the same base, so add the exponents.*

$\frac{1}{y}$ — *Write with a positive exponent.*

Remember!

A number or variable written without an exponent actually has an exponent of 1.

$10 = 10^1$

$y = y^1$

CHECK IT OUT! Simplify.

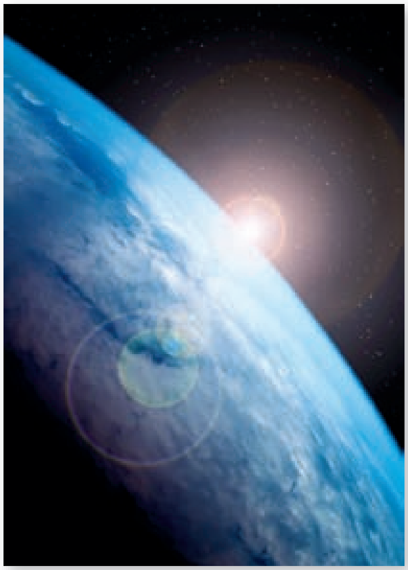
1a. $7^8 \cdot 7^4$

1b. $3^{-3} \cdot 5^8 \cdot 3^4 \cdot 5^2$

1c. $m \cdot n^{-4} \cdot m^4$

1d. $x \cdot x^{-1} \cdot x^{-3} \cdot x^{-4}$

EXAMPLE 2 *Astronomy Application*



Light from the Sun travels at about 1.86×10^5 miles per second. It takes about 500 seconds for the light to reach Earth. Find the approximate distance from the Sun to Earth. Write your answer in scientific notation.

distance = rate × time

$= (1.86 \times 10^5) \times 500$

$= (1.86 \times 10^5) \times (5 \times 10^2)$ — *Write 500 in scientific notation.*

$= (1.86 \times 5) \times (10^5 \times 10^2)$ — *Use the Commutative and Associative Properties to group.*

$= 9.3 \times 10^7$ — *Multiply within each group.*

The Sun is about 9.3×10^7 miles from Earth.

CHECK IT OUT!

2. Light travels at about 1.86×10^5 miles per second. Find the approximate distance that light travels in one hour. Write your answer in scientific notation.

To find a power of a power, you can use the meaning of exponents.

$$(4^{3})^{2} = 4^{3} \cdot 4^{3} = (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4) = 4^{6}$$

Notice the relationship between the exponents in the original power and the exponent in the final power: $3 \cdot 2 = 6$.

Know it! Note

Power of a Power Property

WORDS	NUMBERS	ALGEBRA
A power raised to another power equals that base raised to the product of the exponents.	$(6^{7})^{4} = 6^{7 \cdot 4} = 6^{28}$	If a is any nonzero real number and m and n are integers, then $(a^{m})^{n} = a^{mn}$.

EXAMPLE 3 Finding Powers of Powers

Simplify.

A $(7^{4})^{3}$

$7^{4 \cdot 3}$	*Use the Power of a Power Property.*
7^{12}	*Simplify.*

B $(3^{6})^{0}$

$3^{6 \cdot 0}$	*Use the Power of a Power Property.*
3^{0}	*Zero multiplied by any number is zero.*
1	*Any number raised to the zero power is 1.*

C $(x^{2})^{-4} \cdot x^{5}$

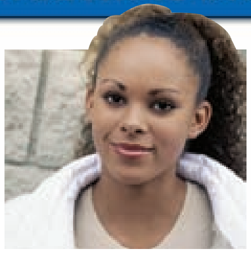
$x^{2 \cdot (-4)} \cdot x^{5}$	*Use the Power of a Power Property.*
$x^{-8} \cdot x^{5}$	*Simplify the exponent of the first term.*
x^{-8+5}	*Since the powers have the same base, add the exponents.*
x^{-3}	
$\frac{1}{x^{3}}$	*Write with a positive exponent.*

CHECK IT OUT! **Simplify.**

3a. $(3^{4})^{5}$ **3b.** $(6^{0})^{3}$ **3c.** $(a^{3})^{4} \cdot (a^{-2})^{-3}$

Student to Student

Multiplication Properties of Exponents



Briana Tyler
Memorial High School

Sometimes I can't remember when to add exponents and when to multiply them. When this happens, I write everything in expanded form.

For example, I would write $x^{2} \cdot x^{3}$ as $(x \cdot x)(x \cdot x \cdot x) = x^{5}$. Then $x^{2} \cdot x^{3} = x^{2+3} = x^{5}$.

I would write $(x^{2})^{3}$ as $x^{2} \cdot x^{2} \cdot x^{2}$, which is $(x \cdot x)(x \cdot x)(x \cdot x) = x^{6}$.

Then $(x^{2})^{3} = x^{2 \cdot 3} = x^{6}$.

This way I get the right answer even if I forget the properties.

Powers of products can be found by using the meaning of an exponent.

$$(8x)^3 = 8x \cdot 8x \cdot 8x = 8 \cdot 8 \cdot 8 \cdot x \cdot x \cdot x = 8^3x^3 = 512x^3$$

Know it! Note

Power of a Product Property

WORDS	NUMBERS	ALGEBRA
A product raised to a power equals the product of each factor raised to that power.	$(2 \cdot 4)^3 = 2^3 \cdot 4^3$ $= 8 \cdot 64$ $= 512$	If a and b are any nonzero real numbers and n is any integer, then $(ab)^n = a^nb^n$.

EXAMPLE 4

Finding Powers of Products

Simplify.

A $(-3x)^2$

$(-3)^2 \cdot x^2$ — *Use the Power of a Product Property.*

$9x^2$ — *Simplify.*

B $-(3x)^2$

$-(3^2 \cdot x^2)$ — *Use the Power of a Product Property.*

$-(9 \cdot x^2)$ — *Simplify.*

$-9x^2$

C $(x^{-2} \cdot y^0)^3$

$(x^{-2})^3 \cdot (y^0)^3$ — *Use the Power of a Product Property.*

$x^{-2 \cdot 3} \cdot y^{0 \cdot 3}$ — *Use the Power of a Power Property.*

$x^{-6} \cdot y^0$ — *Simplify.*

$x^{-6} \cdot 1$ — *Write y^0 as 1.*

$\frac{1}{x^6}$ — *Write with a positive exponent.*

Caution!

In Example 4B, the negative sign is not part of the base.
$-(3x)^2 = -1 \cdot (3x)^2$

CHECK IT OUT! **Simplify.**

4a. $(4p)^3$ **4b.** $(-5t^2)^2$ **4c.** $(x^2y^3)^4 \cdot (x^2y^4)^{-4}$

THINK AND DISCUSS

Know it! Note

1. Explain why $(a^2)^3$ and $a^2 \cdot a^3$ are not equivalent expressions.

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, supply the missing exponents. Then give an example for each property.

Multiplication Properties of Exponents		
Product of Powers Property	Power of a Power Property	Power of a Product Property
$a^m \cdot a^n = a^{\square}$	$(a^m)^n = a^{\square}$	$(ab)^n = a^{\square}b^{\square}$

7-3 Exercises

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Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

SEE EXAMPLE 1 p. 475

Simplify.

1. $2^2 \cdot 2^3$
2. $5^3 \cdot 5^3$
3. $n^6 \cdot n^2$
4. $x^2 \cdot x^{-3} \cdot x^4$

SEE EXAMPLE 2 p. 475

5. **Science** If you traveled in space at a speed of 1000 miles per hour, how far would you travel in 7.5×10^5 hours? Write your answer in scientific notation.

Simplify.

SEE EXAMPLE 3 p. 476

6. $(x^2)^5$
7. $(y^4)^8$
8. $(p^3)^3$
9. $(3^{-2})^2$
10. $(a^{-3})^4 \cdot (a^7)^2$
11. $xy \cdot (x^2)^3 \cdot (y^3)^4$

SEE EXAMPLE 4 p. 477

12. $(2t)^5$
13. $(6k)^2$
14. $(r^2s)^7$
15. $(-2x^5)^3$
16. $-(2x^5)^3$
17. $(a^2b^2)^5 \cdot (a^{-5})^2$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
18–21	1
22	2
23–28	3
29–34	4

Extra Practice
Skills Practice p. S16
Application Practice p. S34

Simplify.

18. $3^3 \cdot 2^3 \cdot 3$
19. $6 \cdot 6^2 \cdot 6^3 \cdot 6^2$
20. $a^5 \cdot a^0 \cdot a^{-5}$
21. $x^7 \cdot x^{-6} \cdot y^{-3}$
22. **Geography** Rhode Island is the smallest state in the United States. Its land area is about 2.9×10^{10} square feet. Alaska, the largest state, is about 5.5×10^2 times as large as Rhode Island. What is the land area of Alaska in square feet? Write your answer in scientific notation.

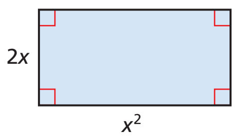
Simplify.

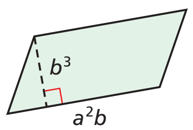
23. $(2^3)^3$
24. $(3^6)^0$
25. $(x^2)^{-1}$
26. $(b^4)^6 \cdot b$
27. $b \cdot (a^3)^4 \cdot (b^{-2})^3$
28. $(x^4)^2 \cdot (x^{-1})^{-4}$
29. $(3x)^3$
30. $(5w^8)^2$
31. $(p^4q^2)^7$
32. $(-4x^3)^4$
33. $-(4x^3)^4$
34. $(x^3y^4)^3 \cdot (xy^3)^{-2}$

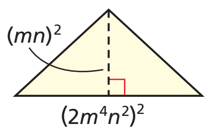
Find the missing exponent in each expression.

35. $a^{\square}a^4 = a^{10}$
36. $(a^{\square})^4 = a^{12}$
37. $(a^2b^{\square})^4 = a^8b^{12}$
38. $(a^3b^6)^{\square} = \dfrac{1}{a^9b^{18}}$
39. $(b^2)^{-4} = \dfrac{1}{b^{\square}}$
40. $a^{\square} \cdot a^6 = a^6$

Geometry Write an expression for the area of each figure.

41.


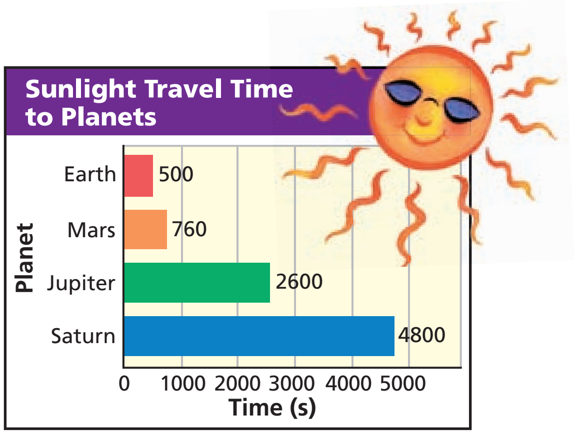
42.


43.


Simplify, if possible.

44. x^6y^5
45. $(2x^2)^2 \cdot (3x^3)^3$
46. $x^2 \cdot y^{-3} \cdot x^{-2} \cdot y^{-3}$
47. $(5x^2)(5x^2)^2$
48. $-(x^2)^4(-x^2)^4$
49. $a^3 \cdot a^0 \cdot 3a^3$
50. $(ab)^3(ab)^{-2}$
51. $10^2 \cdot 10^{-4} \cdot 10^5$
52. $(x^2y^2)^2(x^2y)^{-2}$

53. Astronomy The graph shows the approximate time it takes light from the Sun, which travels at a speed of 1.86×10^5 miles per second, to reach several planets. Find the approximate distance from the Sun to each planet in the graph. Write your answers in scientific notation. (*Hint:* Remember $d = rt$.)



54. Geometry The volume of a rectangular prism can be found by using the formula $V = \ell wh$ where ℓ, w, and h represent the length, width, and height of the prism. Find the volume of a rectangular prism whose dimensions are $3a^2$, $4a^5$, and $4a^2b^2$.

55. ERROR ANALYSIS Explain the error in each simplification below. What is the correct answer in each case?

a. $x^2 \cdot x^4 = x^8$ **b.** $(x^4)^5 = x^9$ **c.** $(x^2)^3 = x^{2^3} = x^8$

Simplify.

56. $(-3x^2)(5x^{-3})$ **57.** $(a^4b)(a^3b^{-6})$ **58.** $(6w^5)(2v^2)(w^6)$

59. $(3m^7)(m^2n)(5m^3n^8)$ **60.** $(b^2)^{-2}(b^4)^5$ **61.** $(3st)^2t^5$

62. $(2^2)^2(x^5y)^3$ **63.** $(-t)(-t)^2(-t^4)$ **64.** $(2m^2)(4m^4)(8n)^2$

65. Estimation Estimate the value of each expression. Explain how you estimated.

a. $\left[(-3.031)^2\right]^3$ **b.** $(6.2085 \times 10^2) \times (3.819 \times 10^{-5})$

66. Physical Science The speed of sound at sea level is about 344 meters per second. The speed of light is about 8.7×10^5 times faster than the speed of sound. What is the speed of light in meters per second? Write your answer in scientific notation and in standard form.

67. Write About It Is $(x^2)^3$ equal to $(x^3)^2$? Explain.

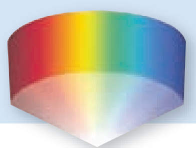
68. Biology A newborn baby has about 26,000,000,000 cells. An adult has about 1.9×10^3 times as many cells as a baby. About how many cells does an adult have? Write your answer in scientific notation.

Simplify.

69. $(-4k)^2 + k^2$ **70.** $-3z^3 + (-3z)^3$ **71.** $(2x^2)^2 + 2(x^2)^2$

72. $(2r)^2s^2 + 6(rs)^2 + 1$ **73.** $(3a)^2b^3 + 3(ab)^2(2b)$ **74.** $(x^2)(x^2)(x^2) + 3x^2$

MULTI-STEP TEST PREP



75. This problem will prepare you for the Multi-Step Test Prep on page 494.

a. The speed of light v is the product of the frequency f and the wavelength w $(v = fw)$. Wavelengths are often measured in *nanometers*. *Nano* means 10^{-9}, so 1 nanometer $= 10^{-9}$ meters. What is 600 nanometers in meters? Write your answer in scientific notation.

b. Use your answer from part ***a*** to find the speed of light in meters per second if $f = 5 \times 10^{14}$ Hz.

c. Explain why you can rewrite $(6 \times 10^{-7})(5 \times 10^{14})$ as $(6 \times 5)(10^{-7})(10^{14})$.

Critical Thinking Rewrite each expression so that it has only one exponent. (*Hint:* You may use parentheses.)

76. c^3d^3 **77.** $36a^2b^2$ **78.** $\frac{8a^3}{b^3}$ **79.** $\frac{k^{-2}}{4m^2n^2}$

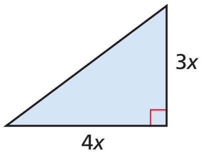
TEST PREP

80. Which of the following is equivalent to $x^2 \cdot x^0$?
Ⓐ 0 Ⓑ 1 Ⓒ x^2 Ⓓ x^{20}

81. Which of the following is equivalent to $(3 \times 10^5)^2$?
Ⓕ 9×10^7 Ⓖ 9×10^{10} Ⓗ 6×10^7 Ⓙ 6×10^{10}

82. What is the value of n^3 when $n = 4 \times 10^5$?
Ⓐ 1.2×10^9 Ⓑ 1.2×10^{16} Ⓒ 6.4×10^9 Ⓓ 6.4×10^{16}

83. Which represents the area of the triangle?
Ⓕ $6x^2$ Ⓗ $7x^2$
Ⓖ $12x^2$ Ⓙ $24x^2$



CHALLENGE AND EXTEND

Simplify.

84. $3^2 \cdot 3^x$ **85.** $(3^2)^x$ **86.** $(x^yz)^2$
87. $(x+1)^{-2}(x+1)^3$ **88.** $(x+1)^2(x+1)^{-3}$ **89.** $(x^y \cdot x^z)^3$
90. $(4^x)^x$ **91.** $(x^x)^x$ **92.** $(3x)^{2y}$

Find the value of *x*.

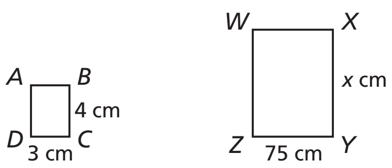
93. $5^x \cdot 5^4 = 5^8$ **94.** $7^3 \cdot 7^x = 7^{12}$ **95.** $(4^x)^3 = 4^{12}$ **96.** $(6^2)^x = 6^{16}$

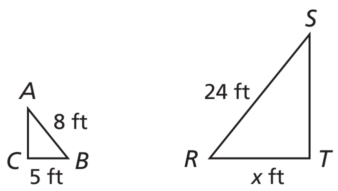
97. **Multi-Step** The edge of a cube measures 1.2×10^{-2} m. What is the volume of the cube in cubic centimeters?

SPIRAL REVIEW

Find the value of *x* in each diagram. *(Lesson 2-8)*

98. $\square ABCD \sim \square WXYZ$ **99.** $\triangle ABC \sim \triangle RST$





Determine whether each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms. *(Lesson 4-6)*

100. 5, 1, −3, −7, … **101.** −3, −2, 0, 3, … **102.** 0.4, 1.0, 1.6, 2.2, …

Write each number in standard form. *(Lesson 7-2)*

103. 7.8×10^6 **104.** 4.95×10^{-4} **105.** 983×10^{-1} **106.** 0.06×10^8

7-4 Division Properties of Exponents

Objective
Use division properties of exponents to evaluate and simplify expressions.

Who uses this?
Economists can use expressions with exponents to calculate national debt statistics. (See Example 3.)

A quotient of powers with the same base can be found by writing the powers in factored form and dividing out common factors.

$$\frac{3^5}{3^3} = \frac{\cancel{3}\cdot\cancel{3}\cdot\cancel{3}\cdot 3\cdot 3}{\cancel{3}\cdot\cancel{3}\cdot\cancel{3}} = 3\cdot 3 = 3^2$$

Notice the relationship between the exponents in the original quotient and the exponent in the final answer: $5 - 3 = 2$.

Know it! Note

Quotient of Powers Property

WORDS	NUMBERS	ALGEBRA
The quotient of two nonzero powers with the same base equals the base raised to the difference of the exponents.	$\frac{6^7}{6^4} = 6^{7-4} = 6^3$	If a is a nonzero real number and m and n are integers, then $\frac{a^m}{a^n} = a^{m-n}$.

EXAMPLE 1 Finding Quotients of Powers

Simplify.

A $\frac{3^8}{3^2}$

$$\frac{3^8}{3^2} = 3^{8-2}$$
$$= 3^6 = 729$$

B $\frac{x^5}{x^5}$

$$\frac{x^5}{x^5} = x^{5-5}$$
$$= x^0 = 1$$

C $\frac{a^5b^9}{(ab)^4}$

$$\frac{a^5b^9}{(ab)^4} = \frac{a^5b^9}{a^4b^4}$$
$$= a^{5-4}\cdot b^{9-4}$$
$$= a^1\cdot b^5$$
$$= ab^5$$

D $\frac{2^3\cdot 3^2\cdot 5^7}{2\cdot 3^4\cdot 5^5}$

$$\frac{2^3\cdot 3^2\cdot 5^7}{2\cdot 3^4\cdot 5^5} = 2^{3-1}\cdot 3^{2-4}\cdot 5^{7-5}$$
$$= 2^2\cdot 3^{-2}\cdot 5^2$$
$$= \frac{2^2\cdot 5^2}{3^2}$$
$$= \frac{4\cdot 25}{9} = \frac{100}{9}$$

Helpful Hint

$3^6 = 729$
Both 3^6 and 729 are considered to be simplified.

CHECK IT OUT! Simplify.

1a. $\frac{2^9}{2^7}$ **1b.** $\frac{y}{y^4}$ **1c.** $\frac{m^5n^4}{(m^5)^2n}$ **1d.** $\frac{3^5\cdot 2^4\cdot 4^3}{3^4\cdot 2^2\cdot 4^6}$

EXAMPLE 2 Dividing Numbers in Scientific Notation

Simplify $(2 \times 10^8) \div (8 \times 10^5)$ and write the answer in scientific notation.

$$(2 \times 10^8) \div (8 \times 10^5) = \frac{2 \times 10^8}{8 \times 10^5}$$

$$= \frac{2}{8} \times \frac{10^8}{10^5}$$ *Write as a product of quotients.*

$$= 0.25 \times 10^{8-5}$$ *Simplify each quotient.*

$$= 0.25 \times 10^3$$ *Simplify the exponent.*

$$= 2.5 \times 10^{-1} \times 10^3$$ *Write 0.25 in scientific notation as 2.5×10^{-1}.*

$$= 2.5 \times 10^{-1+3}$$ *The second two terms have the same base, so add the exponents.*

$$= 2.5 \times 10^2$$ *Simplify the exponent.*

Writing Math

You can "split up" a quotient of products into a product of quotients:

$$\frac{a \times c}{b \times d} = \frac{a}{b} \times \frac{c}{d}$$

Example:

$$\frac{3 \times 4}{5 \times 7} = \frac{3}{5} \times \frac{4}{7} = \frac{12}{35}$$

CHECK IT OUT!

2. Simplify $(3.3 \times 10^6) \div (3 \times 10^8)$ and write the answer in scientific notation.

EXAMPLE 3 *Economics Application*

In the year 2000, the United States public debt was about 5.6×10^{12} dollars. The population of the United States in that year was about 2.8×10^8 people. What was the average debt per person? Give your answer in standard form.

To find the average debt per person, divide the total debt by the number of people.

$$\frac{\text{total debt}}{\text{number of people}} = \frac{5.6 \times 10^{12}}{2.8 \times 10^8}$$

$$= \frac{5.6}{2.8} \times \frac{10^{12}}{10^8}$$ *Write as a product of quotients.*

$$= 2 \times 10^{12-8}$$ *Simplify each quotient.*

$$= 2 \times 10^4$$ *Simplify the exponent.*

$$= 20{,}000$$ *Write in standard form.*

The average debt per person was about $20,000.

CHECK IT OUT!

3. In 1990, the United States public debt was about 3.2×10^{12} dollars. The population of the United States in 1990 was about 2.5×10^8 people. What was the average debt per person? Write your answer in standard form.

A power of a quotient can be found by first writing factors and then writing the numerator and denominator as powers.

$$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3} = \frac{2^3}{3^3}$$

Notice that the exponents in the final answer are the same as the exponent in the original expression.

Know it! Note

Positive Power of a Quotient Property

WORDS	NUMBERS	ALGEBRA
A quotient raised to a positive power equals the quotient of each base raised to that power.	$\left(\frac{3}{5}\right)^4 = \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5} = \frac{3 \cdot 3 \cdot 3 \cdot 3}{5 \cdot 5 \cdot 5 \cdot 5} = \frac{3^4}{5^4}$	If a and b are nonzero real numbers and n is a positive integer, then $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

EXAMPLE 4 **Finding Positive Powers of Quotients**

Simplify.

A $\left(\frac{3}{4}\right)^3$

$$\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3}$$ *Use the Power of a Quotient Property.*

$$= \frac{27}{64}$$ *Simplify.*

B $\left(\frac{2x^3}{yz}\right)^3$

$$\left(\frac{2x^3}{yz}\right)^3 = \frac{(2x^3)^3}{(yz)^3}$$ *Use the Power of a Quotient Property.*

$$= \frac{2^3(x^3)^3}{y^3z^3}$$ *Use the Power of a Product Property: $(2x^3)^3 = 2^3(x^3)^3$ and $(yz)^3 = y^3z^3$.*

$$= \frac{8x^9}{y^3z^3}$$ *Simplify 2^3 and use the Power of a Power Property: $(x^3)^3 = x^{3 \cdot 3} = x^9$.*

CHECK IT OUT! Simplify.

4a. $\left(\frac{2^3}{3^2}\right)^2$ **4b.** $\left(\frac{ab^4}{c^2d^3}\right)^5$ **4c.** $\left(\frac{a^3b}{a^2b^2}\right)^3$

Remember that $x^{-n} = \frac{1}{x^n}$. What if x is a fraction?

$$\left(\frac{a}{b}\right)^{-n} = \frac{1}{\left(\frac{a}{b}\right)^n} = 1 \div \left(\frac{a}{b}\right)^n$$ *Write the fraction as division.*

$$= 1 \div \frac{a^n}{b^n}$$ *Use the Power of a Quotient Property.*

$$= 1 \cdot \frac{b^n}{a^n}$$ *Multiply by the reciprocal.*

$$= \frac{b^n}{a^n}$$ *Simplify.*

$$= \left(\frac{b}{a}\right)^n$$ *Use the Power of a Quotient Property.*

Therefore, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$.

Know it! Note

Negative Power of a Quotient Property

WORDS	NUMBERS	ALGEBRA
A quotient raised to a negative power equals the reciprocal of the quotient raised to the opposite (positive) power.	$\left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^{4} = \frac{3^4}{2^4}$	If a and b are nonzero real numbers and n is a positive integer, then $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n} = \frac{b^n}{a^n}$.

EXAMPLE 5 Finding Negative Powers of Quotients

Simplify.

A $\left(\frac{2}{5}\right)^{-3}$

$\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^{3}$ *Rewrite with a positive exponent.*

$= \frac{5^3}{2^3}$ *Use the Power of a Quotient Property.*

$= \frac{125}{8}$ *$5^3 = 125$ and $2^3 = 8$.*

B $\left(\frac{3x}{y^2}\right)^{-3}$

$\left(\frac{3x}{y^2}\right)^{-3} = \left(\frac{y^2}{3x}\right)^{3}$ *Rewrite with a positive exponent.*

$= \frac{(y^2)^3}{(3x)^3}$ *Use the Power of a Quotient Property.*

$= \frac{y^6}{3^3x^3}$ *Use the Power of a Power Property: $(y^2)^3 = y^{2\cdot 3} = y^6$. Use the Power of a Product Property: $(3x)^3 = 3^3x^3$.*

$= \frac{y^6}{27x^3}$ *Simplify the denominator.*

C $\left(\frac{3}{4}\right)^{-1}\left(\frac{2x}{3y}\right)^{-2}$

$\left(\frac{3}{4}\right)^{-1}\left(\frac{2x}{3y}\right)^{-2} = \left(\frac{4}{3}\right)^{1}\left(\frac{3y}{2x}\right)^{2}$ *Rewrite each fraction with a positive exponent.*

$= \frac{4}{3} \cdot \frac{(3y)^2}{(2x)^2}$ *Use the Power of a Quotient Property.*

$= \frac{4}{3} \cdot \frac{3^2y^2}{2^2x^2}$ *Use the Power of a Product Property: $(3y)^2 = 3^2y^2$ and $(2x)^2 = 2^2x^2$.*

$= \frac{{}^{1}\cancel{4}}{{}_{1}\cancel{3}} \cdot \frac{\cancel{9}^{3}y^2}{{}_{1}\cancel{4}x^2}$ *Divide out common factors.*

$= \frac{3y^2}{x^2}$

Helpful Hint

Whenever all of the factors in the numerator or the denominator divide out, replace them with 1.

CHECK IT OUT! **Simplify.**

5a. $\left(\frac{4}{3^2}\right)^{-3}$ **5b.** $\left(\frac{2a}{b^2c^3}\right)^{-4}$ **5c.** $\left(\frac{s}{3}\right)^{-2}\left(\frac{9s^2}{t}\right)^{-1}$

THINK AND DISCUSS

1. Compare the Quotient of Powers Property and the Product of Powers Property. Then compare the Power of a Quotient Property and the Power of a Product Property.

Know it! Note

2. **GET ORGANIZED** Copy and complete the graphic organizer. In each cell, supply the missing information. Then give an example for each property.

If a and b are nonzero real numbers and m and n are integers, then…		
$\frac{a^m}{a^n} = \square$	$\left(\frac{a}{b}\right)^n = \frac{\square}{\square}$	$\left(\frac{a}{b}\right)^{-n} = \left(\frac{\square}{\square}\right)^{\square}$

7-4 Exercises

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GUIDED PRACTICE

SEE EXAMPLE 1 p. 481

Simplify.

1. $\frac{5^8}{5^6}$
2. $\frac{2^2 \cdot 3^4 \cdot 4^4}{2^9 \cdot 3^5}$
3. $\frac{15x^6}{5x^6}$
4. $\frac{a^5b^6}{a^3b^7}$

SEE EXAMPLE 2 p. 482

Simplify each quotient and write the answer in scientific notation.

5. $(2.8 \times 10^{11}) \div (4 \times 10^8)$
6. $(5.5 \times 10^3) \div (5 \times 10^8)$
7. $(1.9 \times 10^4) \div (1.9 \times 10^4)$

SEE EXAMPLE 3 p. 482

8. **Sports** A star baseball player earns an annual salary of $\$8.1 \times 10^6$. There are 162 games in a baseball season. How much does this player earn per game? Write your answer in standard form.

Simplify.

SEE EXAMPLE 4 p. 483

9. $\left(\frac{2}{5}\right)^2$
10. $\left(\frac{x^2}{xy^3}\right)^3$
11. $\left(\frac{a^3}{(a^3b)^2}\right)^2$
12. $\frac{y^{10}}{y}$

SEE EXAMPLE 5 p. 484

13. $\left(\frac{3}{4}\right)^{-2}$
14. $\left(\frac{2x}{y^3}\right)^{-4}$
15. $\left(\frac{2}{3}\right)^{-1}\left(\frac{3a}{2b}\right)^{-2}$
16. $\left(\frac{x^3}{y^2}\right)^{-4}$

PRACTICE AND PROBLEM SOLVING

Simplify.

17. $\frac{3^9}{3^6}$
18. $\frac{5^4 \cdot 3^3}{5^2 \cdot 3^2}$
19. $\frac{x^8y^3}{x^3y^3}$
20. $\frac{x^8y^4}{x^9yz}$

Simplify each quotient and write the answer in scientific notation.

21. $(4.7 \times 10^{-3}) \div (9.4 \times 10^3)$
22. $(8.4 \times 10^9) \div (4 \times 10^{-5})$
23. $(4.2 \times 10^{-5}) \div (6 \times 10^{-3})$
24. $(2.1 \times 10^2) \div (8.4 \times 10^5)$

Independent Practice	
For Exercises	See Example
17–20	1
21–24	2
25	3
26–29	4
30–33	5

Extra Practice

Skills Practice p. S16
Application Practice p. S34

25. **Astronomy** The mass of Earth is about 3×10^{-3} times the mass of Jupiter. The mass of Earth is about 6×10^{24} kg. What is the mass of Jupiter? Give your answer in scientific notation.

Simplify.

26. $\left(\frac{2}{3}\right)^4$ **27.** $\left(\frac{a^4}{b^2}\right)^3$ **28.** $\left(\frac{a^3b^2}{ab^3}\right)^6$ **29.** $\left(\frac{xy^2}{x^3y}\right)^3$

30. $\left(\frac{1}{7}\right)^{-3}$ **31.** $\left(\frac{x^2}{y^5}\right)^{-5}$ **32.** $\left(\frac{8w^7}{16}\right)^{-1}$ **33.** $\left(\frac{1}{4}\right)^{-2}\left(\frac{6x}{7}\right)^{-2}$

Simplify, if possible.

34. $\frac{x^6}{x^5}$ **35.** $\frac{8d^5}{4d^3}$ **36.** $\frac{x^2y^3}{a^2b^3}$ **37.** $\frac{(3x^3)^3}{(6x^2)^2}$

38. $\frac{(5x^2)^3}{5x^2}$ **39.** $\left(\frac{c^2a^3}{a^5}\right)^2$ **40.** $\left(\frac{3a}{a^3 \cdot a^0}\right)^3$ **41.** $\left(\frac{-p^4}{-5p^3}\right)^{-2}$

42. $\left(\frac{b^{-2}}{b^3}\right)^2$ **43.** $\left(\frac{10^2}{10^{-5} \cdot 10^5}\right)^{-1}$ **44.** $\left(\frac{x^2y^2}{x^2y}\right)^{-3}$ **45.** $\frac{(-x^2)^4}{-(x^2)^4}$

46. **Critical Thinking** How can you use the Quotient of a Power Property to explain the definition of x^{-n}? ($Hint:$ Think of $\frac{1}{x^n}$ as $\frac{x^0}{x^n}$.)

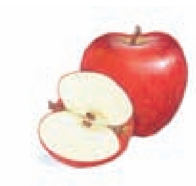
47. **Geography** *Population density* is the number of people per unit of area. The area of the United States is approximately 9.37×10^6 square kilometers. The table shows population data from the U. S. Census Bureau.

United States Population	
Year	Population (to nearest million)
2000	2.81×10^8
1995	2.66×10^8
1990	2.48×10^8

Write the approximate population density (people per square kilometer) for each of the given years in scientific notation. Round decimals to the nearest hundredth.

48. **Chemistry** The pH of a solution is a number that describes the concentration of hydrogen ions in that solution. For example, if the concentration of hydrogen ions in a solution is 10^{-4}, that solution has a pH of 4.


Lemon juice
pH 2


Apples
pH 3


Water
pH 7


Ammonia
pH 11

a. What is the concentration of hydrogen ions in lemon juice?

b. What is the concentration of hydrogen ions in water?

c. How many times more concentrated are the hydrogen ions in lemon juice than in water?

49. **Write About It** Explain how to simplify $\frac{4^5}{4^2}$. How is it different from simplifying $\frac{4^2}{4^5}$?

Find the missing exponent(s).

50. $\frac{x^{\square}}{x^4} = x^2$ **51.** $\frac{x^7}{x^{\square}} = x^4$ **52.** $\left(\frac{a^2}{b^{\square}}\right)^4 = \frac{a^8}{b^{12}}$ **53.** $\left(\frac{x^4}{y^{\square}}\right)^{-1} = \frac{y^3}{x^{\square}}$

MULTI-STEP TEST PREP

54. This problem will prepare you for the Multi-Step Test Prep on page 494.

a. Yellow light has a wavelength of 589 nm. A nanometer (nm) is 10^{-9} m. What is 589 nm in meters? Write your answer in scientific notation.

b. The speed of light in air, v, is 3×10^8 m/s, and $v = fw$, where f represents the frequency in hertz (Hz) and w represents the wavelength in meters. What is the frequency of yellow light?

TEST PREP

55. Which of the following is equivalent to $(8 \times 10^6) \div (4 \times 10^2)$?

Ⓐ 2×10^3 Ⓑ 2×10^4 Ⓒ 4×10^3 Ⓓ 4×10^4

56. Which of the following is equivalent to $\left(\frac{x^{12}}{3xy^4}\right)^{-2}$?

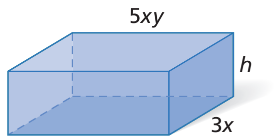
Ⓕ $\frac{9y^8}{x^{22}}$ Ⓖ $\frac{3y^8}{x^{22}}$ Ⓗ $\frac{3y^6}{x^{12}}$ Ⓙ $\frac{6y^8}{x^{26}}$

57. Which of the following is equivalent to $\frac{(-3x)^4}{-(3x)^4}$?

Ⓐ -1 Ⓑ 1 Ⓒ $-81x^4$ Ⓓ $\frac{1}{81x^4}$

CHALLENGE AND EXTEND

58. Geometry The volume of the prism at right is $V = 30x^4y^3$. Write and simplify an expression for the prism's height in terms of x and y.



59. Simplify $\frac{3^{2x}}{3^{2x-1}}$.

60. Simplify $\frac{(x+1)^2}{(x+1)^3}$.

61. Copy and complete the table below to show how the Quotient of Powers Property can be found by using the Product of Powers Property.

Statements	Reasons
1. $a^{m-n} = a^{\square + \square}$	**1.** Subtraction is addition of the opposite.
2. $= a^{\square} \cdot a^{\square}$	**2.** Product of Powers Property
3. $= a^m \cdot \frac{1}{a^n}$	**3.** ______?______
4. $= \frac{a^m}{\square}$	**4.** Multiplication can be written as division.

SPIRAL REVIEW

Find each square root. *(Lesson 1-5)*

62. $\sqrt{36}$ **63.** $\sqrt{1}$ **64.** $-\sqrt{49}$ **65.** $\sqrt{144}$

Solve each equation. *(Lesson 2-4)*

66. $-2(x-1) + 4x = 5x + 3$ **67.** $x - 1 - (4x + 3) = 5x$

Simplify. *(Lesson 7-3)*

68. $3^2 \cdot 3^3$ **69.** $k^5 \cdot k^{-2} \cdot k^{-3}$ **70.** $(4t^5)^2$ **71.** $-(5x^4)^3$

7-5 Rational Exponents

Objective
Evaluate and simplify expressions containing rational exponents.

Vocabulary
index

Why learn this?
You can use rational exponents to find the number of Calories animals need to consume each day to maintain health. (See Example 3.)

Recall that the radical symbol $\sqrt{}$ is used to indicate roots. The **index** is the small number to the left of the radical symbol that tells which root to take. For example, $\sqrt[3]{}$ represents a cube root. Since $2^3 = 2 \cdot 2 \cdot 2 = 8$, $\sqrt[3]{8} = 2$.

Another way to write nth roots is by using exponents that are fractions. For example, for $b > 1$, suppose $\sqrt{b} = b^k$.

$$\sqrt{b} = b^k$$

$$\left(\sqrt{b}\right)^2 = \left(b^k\right)^2 \quad \textit{Square both sides.}$$

$$b^1 = b^{2k} \quad \textit{Power of a Power Property}$$

$$1 = 2k \quad \textit{If } b^m = b^n\textit{, then } m = n.$$

$$\frac{1}{2} = k \quad \textit{Divide both sides by 2.}$$

So for all $b > 1$, $\sqrt{b} = b^{\frac{1}{2}}$.

Helpful Hint
When $b = 0$, $\sqrt[n]{b} = 0$.
When $b = 1$, $\sqrt[n]{b} = 1$.

Know it! Note

Definition of $b^{\frac{1}{n}}$

WORDS	NUMBERS	ALGEBRA
A number raised to the power of $\frac{1}{n}$ is equal to the nth root of that number.	$3^{\frac{1}{2}} = \sqrt{3}$ $5^{\frac{1}{4}} = \sqrt[4]{5}$ $2^{\frac{1}{7}} = \sqrt[7]{2}$	If $b > 1$ and n is an integer, where $n \geq 2$, then $b^{\frac{1}{n}} = \sqrt[n]{b}$. $b^{\frac{1}{2}} = \sqrt{b}$, $b^{\frac{1}{3}} = \sqrt[3]{b}$, $b^{\frac{1}{4}} = \sqrt[4]{b}$, and so on.

EXAMPLE 1

Simplifying $b^{\frac{1}{n}}$

Simplify each expression.

A $125^{\frac{1}{3}}$

$$125^{\frac{1}{3}} = \sqrt[3]{125} = \sqrt[3]{5^3} = 5 \quad \textit{Use the definition of } b^{\frac{1}{n}}.$$

B $64^{\frac{1}{6}} + 25^{\frac{1}{2}}$

$$64^{\frac{1}{6}} + 25^{\frac{1}{2}} = \sqrt[6]{64} + \sqrt{25} \quad \textit{Use the definition of } b^{\frac{1}{n}}.$$

$$= \sqrt[6]{2^6} + \sqrt{5^2}$$

$$= 2 + 5 = 7$$

Remember!
$\sqrt{}$ is equivalent to $\sqrt[2]{}$. See Lesson 1-5.

CHECK IT OUT! Simplify each expression.

1a. $81^{\frac{1}{4}}$ **1b.** $121^{\frac{1}{2}} + 256^{\frac{1}{4}}$

A fractional exponent can have a numerator other than 1, as in the expression $b^{\frac{2}{3}}$. You can write the exponent as a product in two different ways.

$$b^{\frac{2}{3}} = b^{\frac{1}{3}\cdot 2} \qquad\qquad b^{\frac{2}{3}} = b^{2\cdot\frac{1}{3}}$$

$$= \left(b^{\frac{1}{3}}\right)^2 \quad \textit{Power of a Power Property} \quad = \left(b^2\right)^{\frac{1}{3}}$$

$$= \left(\sqrt[3]{b}\right)^2 \quad \textit{Definition of } b^{\frac{1}{n}} \quad = \sqrt[3]{b^2}$$

Know it! Note

Definition of $b^{\frac{m}{n}}$

WORDS	NUMBERS	ALGEBRA
A number raised to the power of $\frac{m}{n}$ is equal to the nth root of the number raised to the mth power.	$8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$ $8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$	If $b > 1$ and m and n are integers, where $m \geq 1$ and $n \geq 2$, then $b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m = \sqrt[n]{b^m}$.

EXAMPLE 2 Simplifying Expressions with Fractional Exponents

Simplify each expression.

A $216^{\frac{2}{3}}$

$$216^{\frac{2}{3}} = \left(\sqrt[3]{216}\right)^2 \quad \textit{Definition of } b^{\frac{m}{n}}$$
$$= \left(\sqrt[3]{6^3}\right)^2$$
$$= (6)^2 = 36$$

B $32^{\frac{4}{5}}$

$$32^{\frac{4}{5}} = \left(\sqrt[5]{32}\right)^4$$
$$= \left(\sqrt[5]{2^5}\right)^4$$
$$= (2)^4 = 16$$

CHECK IT OUT! Simplify each expression.

2a. $16^{\frac{3}{4}}$ **2b.** $1^{\frac{2}{5}}$ **2c.** $27^{\frac{4}{3}}$

EXAMPLE 3 *Biology Application*

The approximate number of Calories C that an animal needs each day is given by $C = 72m^{\frac{3}{4}}$, where m is the animal's mass in kilograms. Find the number of Calories that a 16 kg dog needs each day.

$$C = 72m^{\frac{3}{4}}$$
$$= 72(16)^{\frac{3}{4}} \quad \textit{Substitute 16 for m.}$$
$$= 72 \cdot \left(\sqrt[4]{16}\right)^3 \quad \textit{Definition of } b^{\frac{m}{n}}$$
$$= 72 \cdot \left(\sqrt[4]{2^4}\right)^3$$
$$= 72 \cdot (2)^3$$
$$= 72 \cdot 8 = 576$$

The dog needs 576 Calories per day to maintain health.

CHECK IT OUT! **3.** Find the number of Calories that an 81 kg panda needs each day.

Remember that $\sqrt{}$ always indicates a nonnegative square root. When you simplify variable expressions that contain $\sqrt{}$, such as $\sqrt{x^2}$, the answer cannot be negative. But x may be negative. Therefore you simplify $\sqrt{x^2}$ as $|x|$ to ensure the answer is nonnegative.

When x is...	and n is...	x^n is...	and $\sqrt[n]{x^n}$ is...
Positive	Even	Positive	Positive
Negative	Even	Positive	Positive
Positive	Odd	Positive	Positive
Negative	Odd	Negative	Negative

When n is even, you must simplify $\sqrt[n]{x^n}$ to $|x|$, because you do not know whether x is positive or negative. When n is odd, simplify $\sqrt[n]{x^n}$ to x.

EXAMPLE 4 Using Properties of Exponents to Simplify Expressions

Simplify. All variables represent nonnegative numbers.

A $\sqrt[3]{x^9y^3}$

$\sqrt[3]{x^9y^3} = (x^9y^3)^{\frac{1}{3}}$ — *Definition of $b^{\frac{1}{n}}$*

$= (x^9)^{\frac{1}{3}} \cdot (y^3)^{\frac{1}{3}}$ — *Power of a Product Property*

$= \left(x^{9\cdot\frac{1}{3}}\right) \cdot \left(y^{3\cdot\frac{1}{3}}\right)$ — *Power of a Power Property*

$= (x^3) \cdot (y^1) = x^3y$ — *Simplify exponents.*

B $\left(x^2y^{\frac{1}{2}}\right)^4 \sqrt[3]{y^3}$

$\left(x^2y^{\frac{1}{2}}\right)^4 \sqrt[3]{y^3} = \left(x^2y^{\frac{1}{2}}\right)^4 \cdot y$ — *$\sqrt[3]{y^3} = y$*

$= (x^{2\cdot4}) \cdot \left(y^{\frac{1}{2}\cdot4}\right) \cdot y$ — *Power of a Product Property*

$= (x^8) \cdot (y^2) \cdot y$ — *Simplify exponents.*

$= x^8 \cdot y^{2+1} = x^8y^3$ — *Product of Powers Property*

Helpful Hint

When you are told that all variables represent non-negative numbers, you do not need to use absolute values in your answers.

CHECK IT OUT! **Simplify. All variables represent nonnegative numbers.**

4a. $\sqrt[4]{x^4y^{12}}$

4b. $\dfrac{\left(xy^{\frac{1}{2}}\right)^2}{\sqrt[5]{x^5}}$

THINK AND DISCUSS

1. Explain how to find the value of $\left(\sqrt[10]{25}\right)^5$.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each cell, provide the definition and a numerical example of each type of exponent.

Exponent	Definition	Numerical Example
$b^{\frac{1}{n}}$		
$b^{\frac{m}{n}}$		

7-5 Exercises

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Homework Help Online
KEYWORD: MA11 7-5
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

1. **Vocabulary** In the expression $\sqrt[5]{3x}$, what is the *index*?

SEE EXAMPLE 1 p. 488

Simplify each expression.

2. $8^{\frac{1}{3}}$
3. $16^{\frac{1}{2}}$
4. $0^{\frac{1}{6}}$
5. $27^{\frac{1}{3}}$
6. $81^{\frac{1}{2}}$
7. $216^{\frac{1}{3}}$
8. $1^{\frac{1}{9}}$
9. $625^{\frac{1}{4}}$
10. $36^{\frac{1}{2}} + 1^{\frac{1}{3}}$
11. $8^{\frac{1}{3}} + 64^{\frac{1}{2}}$
12. $81^{\frac{1}{4}} + 8^{\frac{1}{3}}$
13. $25^{\frac{1}{2}} - 1^{\frac{1}{4}}$

SEE EXAMPLE 2 p. 489

14. $81^{\frac{3}{4}}$
15. $8^{\frac{5}{3}}$
16. $125^{\frac{2}{3}}$
17. $25^{\frac{3}{2}}$
18. $36^{\frac{3}{2}}$
19. $64^{\frac{4}{3}}$
20. $1^{\frac{3}{4}}$
21. $0^{\frac{2}{3}}$

SEE EXAMPLE 3 p. 489

22. **Geometry** Given a square with area a, you can use the formula $P = 4a^{\frac{1}{2}}$ to find the perimeter P of the square. Find the perimeter of a square that has an area of 64 m^2.

SEE EXAMPLE 4 p. 490

Simplify. All variables represent nonnegative numbers.

23. $\sqrt{x^4y^2}$
24. $\sqrt[4]{z^4}$
25. $\sqrt{x^6y^6}$
26. $\sqrt[3]{a^{12}b^6}$
27. $\left(a^{\frac{1}{2}}\right)^2\sqrt{a^2}$
28. $\left(x^{\frac{1}{3}}\right)^6\sqrt[4]{y^4}$
29. $\dfrac{\left(z^{\frac{1}{3}}\right)^3}{\sqrt{z^2}}$
30. $\dfrac{\sqrt[3]{x^6y^9}}{x^2}$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
31–42	1
43–50	2
51	3
52–59	4

Extra Practice

Skills Practice p. S17
Application Practice p. S34

Simplify each expression.

31. $100^{\frac{1}{2}}$
32. $1^{\frac{1}{5}}$
33. $512^{\frac{1}{3}}$
34. $729^{\frac{1}{2}}$
35. $32^{\frac{1}{5}}$
36. $196^{\frac{1}{2}}$
37. $256^{\frac{1}{8}}$
38. $400^{\frac{1}{2}}$
39. $125^{\frac{1}{3}} + 81^{\frac{1}{2}}$
40. $25^{\frac{1}{2}} - 81^{\frac{1}{4}}$
41. $121^{\frac{1}{2}} - 243^{\frac{1}{5}}$
42. $256^{\frac{1}{4}} + 0^{\frac{1}{3}}$
43. $4^{\frac{3}{2}}$
44. $27^{\frac{2}{3}}$
45. $256^{\frac{3}{4}}$
46. $64^{\frac{5}{6}}$
47. $100^{\frac{3}{2}}$
48. $1^{\frac{5}{3}}$
49. $9^{\frac{5}{2}}$
50. $243^{\frac{2}{5}}$

51. **Biology** Biologists use a formula to estimate the mass of a mammal's brain. For a mammal with a mass of m grams, the approximate mass B of the brain, also in grams, is given by $B = \frac{1}{8}m^{\frac{2}{3}}$. Find the approximate mass of the brain of a mouse that has a mass of 64 grams.

Simplify. All variables represent nonnegative numbers.

52. $\sqrt[3]{a^6c^9}$
53. $\sqrt[3]{8m^3}$
54. $\sqrt[4]{x^{16}y^4}$
55. $\sqrt[3]{27x^6}$
56. $\left(x^{\frac{1}{2}}y^3\right)^2\sqrt{x^2}$
57. $(a^2b^4)^{\frac{1}{2}}\sqrt[3]{b^6}$
58. $\dfrac{\sqrt[3]{x^6y^6}}{yx^2}$
59. $\dfrac{\left(a^2b^{\frac{1}{2}}\right)^4}{\sqrt{b^2}}$

Fill in the boxes to make each statement true.

60. $256^{\frac{\square}{4}} = 4$
61. $\square^{\frac{1}{5}} = 1$
62. $225^{\frac{1}{\square}} = 15$
63. $\square^{\frac{1}{6}} = 0$
64. $64^{\frac{\square}{3}} = 16$
65. $\square^{\frac{3}{4}} = 125$
66. $27^{\frac{4}{\square}} = 81$
67. $36^{\frac{\square}{2}} = 216$

Simplify each expression.

68. $\left(\frac{81}{169}\right)^{\frac{1}{2}}$ **69.** $\left(\frac{8}{27}\right)^{\frac{1}{3}}$ **70.** $\left(\frac{256}{81}\right)^{\frac{1}{4}}$

71. $\left(\frac{1}{16}\right)^{\frac{1}{2}}$ **72.** $\left(\frac{9}{16}\right)^{\frac{3}{2}}$ **73.** $\left(\frac{8}{27}\right)^{\frac{2}{3}}$

74. $\left(\frac{16}{81}\right)^{\frac{3}{4}}$ **75.** $\left(\frac{4}{49}\right)^{\frac{3}{2}}$ **76.** $\left(\frac{4}{25}\right)^{\frac{3}{2}}$

77. $\left(\frac{1}{81}\right)^{\frac{3}{4}}$ **78.** $\left(\frac{27}{64}\right)^{\frac{2}{3}}$ **79.** $\left(\frac{8}{125}\right)^{\frac{4}{3}}$

80. Multi-Step Scientists have found that the life span of a mammal living in captivity is related to the mammal's mass. The life span in years L can be approximated by the formula $L = 12m^{\frac{1}{5}}$, where m is the mammal's mass in kilograms. How much longer is the life span of a lion compared with that of a wolf?

Typical Mass of Mammals

Mammal	Mass (kg)
Koala	8
Wolf	32
Lion	243
Giraffe	1024

81. Geometry Given a sphere with volume V, the formula $r = 0.62V^{\frac{1}{3}}$ may be used to approximate the sphere's radius r. Find the approximate radius of a sphere that has a volume of 27 in^3.

82. Critical Thinking Show that a number raised to the power $\frac{1}{3}$ is the same as the cube root of that number. (*Hint:* Use properties of exponents to find the cube of $b^{\frac{1}{3}}$. Then compare this with the cube of $\sqrt[3]{b}$. Use the fact that if two numbers have the same cube, then they are equal.)

83. Critical Thinking Compare $n^{\frac{2}{3}}$ and $n^{\frac{3}{2}}$ for values of n greater than 1. When simplifying each of these expressions, will the result be greater than n or less than n? Explain.

84. ERROR ANALYSIS Two students simplified $64^{\frac{3}{2}}$. Which solution is incorrect? Explain the error.

A

$$64^{\frac{3}{2}} = \left(\sqrt[3]{64}\right)^2$$
$$= (4)^2$$
$$= 16$$

B

$$64^{\frac{3}{2}} = \left(\sqrt{64}\right)^3$$
$$= (8)^3$$
$$= 512$$

MULTI-STEP TEST PREP

85. This problem will prepare you for the Multi-Step Test Prep on page 494.
You can estimate an object's distance in inches from a light source by using the formula $d = \left(0.8\frac{L}{B}\right)^{\frac{1}{2}}$, where L is the light's luminosity in lumens and B is the light's brightness in lumens per square inch.

a. Find an object's distance to a light source with a luminosity of 4000 lumens and a brightness of 32 lumens per square inch.

b. Suppose the brightness of this light source decreases to 8 lumens per square inch. How does the object's distance from the source change?

86. **Write About It** You can write $4^{\frac{3}{2}}$ as $4^{3 \cdot \frac{1}{2}}$ or as $4^{\frac{1}{2} \cdot 3}$. Use the Power of a Power Property to show that both expressions are equal. Is one method easier than the other? Explain.

TEST PREP

87. What is $9^{\frac{1}{2}} + 8^{\frac{1}{3}}$?

Ⓐ 4 Ⓑ 5 Ⓒ 6 Ⓓ 10

88. Which expression is equal to 8?

Ⓕ $4^{\frac{3}{2}}$ Ⓖ $16^{\frac{1}{2}}$ Ⓗ $32^{\frac{4}{5}}$ Ⓙ $64^{\frac{3}{2}}$

89. Which expression is equivalent to $\sqrt[3]{a^9b^3}$?

Ⓐ a^2b Ⓑ a^3 Ⓒ a^3b Ⓓ a^3b^3

90. Which of the following is NOT equal to $16^{\frac{3}{2}}$?

Ⓕ $\left(\sqrt{16}\right)^3$ Ⓖ 4^3 Ⓗ $\left(\sqrt[3]{16}\right)^2$ Ⓙ $\sqrt{16^3}$

CHALLENGE AND EXTEND

Use properties of exponents to simplify each expression.

91. $\left(a^{\frac{1}{3}}\right)\left(a^{\frac{1}{3}}\right)\left(a^{\frac{1}{3}}\right)$

92. $\left(x^{\frac{1}{2}}\right)^5\left(x^{\frac{3}{2}}\right)$

93. $\left(x^{\frac{1}{3}}\right)^4\left(x^5\right)^{\frac{1}{3}}$

You can use properties of exponents to help you solve equations. For example, to solve $x^3 = 64$, raise both sides to the $\frac{1}{3}$ power to get $\left(x^3\right)^{\frac{1}{3}} = 64^{\frac{1}{3}}$. Simplifying both sides gives $x = 4$. Use this method to solve each equation. Check your answer.

94. $y^5 = 32$

95. $27x^3 = 729$

96. $1 = \frac{1}{8}x^3$

97. **Geometry** The formula for the surface area of a sphere S in terms of its volume V is $S = (4\pi)^{\frac{1}{3}}(3V)^{\frac{2}{3}}$. What is the surface area of a sphere that has a volume of 36π cm^3? Leave the symbol π in your answer. What do you notice?

SPIRAL REVIEW

Solve each equation. *(Lesson 2-6)*

98. $|x + 6| = 2$

99. $|5x + 5| = 0$

100. $|2x - 1| = 3$

Solve each inequality and graph the solutions. *(Lesson 3-4)*

101. $3n + 5 < 14$

102. $4 \le \frac{1}{2}x + 3$

103. $7 \ge 2y + 11$

Give the domain and range of each relation. Tell whether the relation is a function. Explain. *(Lesson 4-2)*

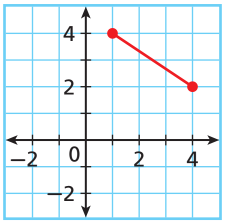
104. $\{(2, 3), (2, 4), (2, 5), (2, 6)\}$

105. $\{(-2, 0), (-1, 1), (0, 2), (1, 3)\}$

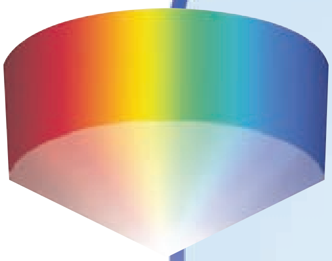
106.

x	y
5	2
7	2
9	2
11	2

107.



MULTI-STEP TEST PREP



Exponents

I See the Light! The speed of light is the product of its frequency *f* and its wavelength *w*. In air, the speed of light is 3×10^8 m/s.

1. Write an equation for the relationship described above, and then solve this equation for frequency. Write this equation as an equation with *w* raised to a negative exponent.

2. Wavelengths of visible light range from 400 to 700 nanometers (10^{-9} meters). Use a graphing calculator and the relationship you found in Problem 1 to graph frequency as a function of wavelength. Sketch the graph with the axes clearly labeled. Describe your graph.

3. The speed of light in water is $\frac{3}{4}$ of its speed in air. Find the speed of light in water.

4. When light enters water, some colors bend more than others. How much the light bends depends on its wavelength. This is what creates a rainbow. The frequency of green light is about 5.9×10^{14} cycles per second. Find the wavelength of green light in water.

5. When light enters water, colors with shorter wavelengths bend more than colors with longer wavelengths. Violet light has a frequency of 7.5×10^{14} cycles per second, and red light has a frequency of 4.6×10^{14} cycles per second. Which of these colors of light will bend more when it enters water? Justify your answer.

READY TO GO ON?

Quiz for Lessons 7-1 Through 7-5

7-1 Integer Exponents

Evaluate each expression for the given value(s) of the variable(s).

1. t^{-6} for $t = 2$ **2.** n^{-3} for $n = -5$ **3.** $r^0 s^{-2}$ for $r = 8$ and $s = 10$

Simplify.

4. $5k^{-3}$ **5.** $\frac{x^4}{y^{-6}}$ **6.** $8f^{-4} g^0$ **7.** $\frac{a^{-3}}{b^{-2}}$

8. **Measurement** Metric units can be written in terms of a base unit. The table shows some of these equivalencies. Simplify each expression.

Selected Metric Prefixes					
Milli-	Centi-	Deci-	Deka-	Hecto-	Kilo-
10^{-3}	10^{-2}	10^{-1}	10^1	10^2	10^3

7-2 Powers of 10 and Scientific Notation

9. Find the value of 10^4.

10. Write 0.0000001 as a power of 10.

11. Write 100,000,000,000 as a power of 10.

12. Find the value of 82.1×10^4.

13. **Measurement** The lead in a mechanical pencil has a diameter of 0.5 mm. Write this number in scientific notation.

7-3 Multiplication Properties of Exponents

Simplify.

14. $2^2 \cdot 2^5$ **15.** $3^5 \cdot 3^{-3}$ **16.** $p^4 \cdot p^5$ **17.** $a^3 \cdot a^{-6} \cdot a^{-2}$

18. **Biology** A swarm of locusts was estimated to contain 2.8×10^{10} individual insects. If each locust weighs about 2.5 grams, how much did this entire swarm weigh? Write your answer in scientific notation.

Simplify.

19. $(3x^4)^3$ **20.** $(m^3 n^2)^5$ **21.** $(-4d^7)^2$ **22.** $(cd^6)^3 \cdot (c^5 d^2)^2$

7-4 Division Properties of Exponents

Simplify.

23. $\frac{6^9}{6^7}$ **24.** $\frac{12a^5}{3a^2}$ **25.** $\left(\frac{3}{5}\right)^3$ **26.** $\left(\frac{4p^3}{2pq^4}\right)^2$

Simplify each quotient and write the answer in scientific notation.

27. $(8 \times 10^9) \div (2 \times 10^6)$ **28.** $(3.5 \times 10^5) \div (7 \times 10^8)$ **29.** $(1 \times 10^4) \div (4 \times 10^4)$

7-5 Rational Exponents

Simplify each expression. All variables represent nonnegative numbers.

30. $81^{\frac{1}{2}}$ **31.** $125^{\frac{1}{3}}$ **32.** $4^{\frac{3}{2}}$ **33.** $0^{\frac{2}{9}}$

34. $\sqrt{x^8 y^4}$ **35.** $\sqrt[3]{r^9}$ **36.** $\sqrt[6]{z^{12}}$ **37.** $\sqrt[3]{p^3 q^{12}}$

7-6 Polynomials

Objectives
Classify polynomials and write polynomials in standard form.

Evaluate polynomial expressions.

Vocabulary
monomial
degree of a monomial
polynomial
degree of a polynomial
standard form of a polynomial
leading coefficient
quadratic
cubic
binomial
trinomial

Who uses this?
Pyrotechnicians can use polynomials to plan complex fireworks displays. (See Example 5.)

A **monomial** is a number, a variable, or a product of numbers and variables with whole-number exponents.

Monomials	Not Monomials
$5 \quad x \quad -7xy \quad 0.5x^4$	$-0.3x^{-2} \quad 4x - y \quad \frac{2}{x^3}$

The **degree of a monomial** is the sum of the exponents of the variables. A constant has degree 0.

EXAMPLE 1
Finding the Degree of a Monomial

Find the degree of each monomial.

A $-2a^2b^4$

The degree is 6. *Add the exponents of the variables: 2 + 4 = 6*

B 4

$4x^0$ *There is no variable, but you can write 4 as $4x^0$.*

The degree is 0.

C $8y$

$8y^1$ *A variable written without an exponent has exponent 1.*

The degree is 1.

Remember!
The *terms* of an expression are the parts being added or subtracted. See Lesson 1-7.

CHECK IT OUT! **Find the degree of each monomial.**

1a. $1.5k^2m$ **1b.** $4x$ **1c.** $2c^3$

A **polynomial** is a monomial or a sum or difference of monomials. The **degree of a polynomial** is the degree of the term with the greatest degree.

EXAMPLE 2
Finding the Degree of a Polynomial

Find the degree of each polynomial.

A $4x - 18x^5$

$4x$: degree 1 $\quad -18x^5$: degree 5 *Find the degree of each term.*

The degree of the polynomial is the greatest degree, 5.

Find the degree of each polynomial.

B $0.5x^2y + 0.25xy + 0.75$

$0.5x^2y$: degree 3 $0.25xy$: degree 2   0.75: degree 0

The degree of the polynomial is the greatest degree, 3.

C $6x^4 + 9x^2 - x + 3$

$6x^4$: degree 4 $9x^2$: degree 2 $-x$: degree 1   3: degree 0

The degree of the polynomial is the greatest degree, 4.

CHECK IT OUT! Find the degree of each polynomial.

2a. $5x - 6$ **2b.** $x^3y^2 + x^2y^3 - x^4 + 2$

The terms of a polynomial may be written in any order. However, polynomials that contain only one variable are usually written in *standard form*.

The **standard form of a polynomial** that contains one variable is written with the terms in order from greatest degree to least degree. When written in standard form, the coefficient of the first term is called the **leading coefficient**.

EXAMPLE 3 Writing Polynomials in Standard Form

Write each polynomial in standard form. Then give the leading coefficient.

A $20x - 4x^3 + 2 - x^2$

Find the degree of each term. Then arrange them in descending order.

$$\underbrace{20x}_{1}\ \underbrace{-4x^3}_{3}\ \underbrace{+2}_{0}\ \underbrace{-x^2}_{2} \rightarrow \underbrace{-4x^3}_{3}\ \underbrace{-x^2}_{2}\ \underbrace{+20x}_{1}\ \underbrace{+2}_{0}$$

Degree: 1 3 0 2 → 3 2 1 0

The standard form is $-4x^3 - x^2 + 20x + 2$. The leading coefficient is -4.

B $y^3 + y^5 + 4y$

Find the degree of each term. Then arrange them in descending order.

$$\underbrace{y^3}_{3}\ \underbrace{+y^5}_{5}\ \underbrace{+4y}_{1} \rightarrow \underbrace{y^5}_{5}\ \underbrace{+y^3}_{3}\ \underbrace{+4y}_{1}$$

Degree: 3 5 1 → 5 3 1

The standard form is $y^5 + y^3 + 4y$. The leading coefficient is 1.

Remember!

A variable written without a coefficient has a coefficient of 1.

$y^5 = 1y^5$

CHECK IT OUT! Write each polynomial in standard form. Then give the leading coefficient.

3a. $16 - 4x^2 + x^5 + 9x^3$ **3b.** $18y^5 - 3y^8 + 14y$

Some polynomials have special names based on their degree and the number of terms they have.

Degree	Name
0	Constant
1	Linear
2	**Quadratic**
3	**Cubic**
4	Quartic
5	Quintic
6 or more	6th degree, 7th degree, and so on

Terms	Name
1	Monomial
2	**Binomial**
3	**Trinomial**
4 or more	Polynomial

EXAMPLE 4 Classifying Polynomials

Classify each polynomial according to its degree and number of terms.

A $5x - 6$

Degree: 1 Terms: 2 $5x - 6$ is a **linear binomial**.

B $y^2 + y + 4$

Degree: 2 Terms: 3 $y^2 + y + 4$ is a **quadratic trinomial**.

C $6x^7 + 9x^2 - x + 3$

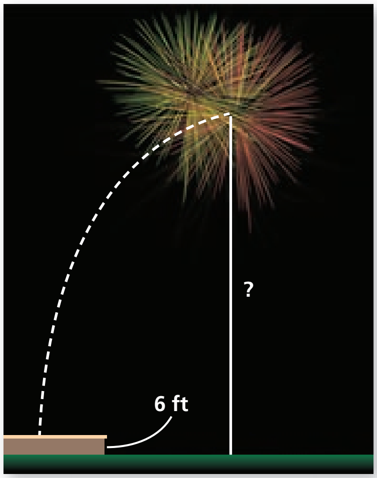
Degree: 7 Terms: 4 $6x^7 + 9x^2 - x + 3$ is a **7th-degree polynomial**.

CHECK IT OUT! **Classify each polynomial according to its degree and number of terms.**

4a. $x^3 + x^2 - x + 2$ **4b.** 6 **4c.** $-3y^8 + 18y^5 + 14y$

EXAMPLE 5 *Physics Application*

A firework is launched from a platform 6 feet above the ground at a speed of 200 feet per second. The firework has a 5-second fuse. The height of the firework in feet is given by the polynomial $-16t^2 + 200t + 6$, where t is the time in seconds. How high will the firework be when it explodes?



Substitute the time for t to find the firework's height.

$-16t^2 + 200t + 6$

$-16(5)^2 + 200(5) + 6$ *The time is 5 seconds.*

$-16(25) + 200(5) + 6$ *Evaluate the polynomial by using the order of operations.*

$-400 + 1000 + 6$

606

When the firework explodes, it will be 606 feet above the ground.

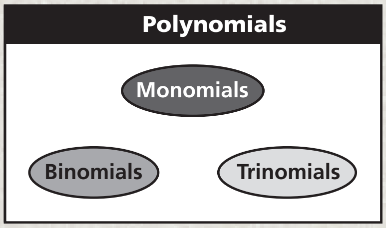
CHECK IT OUT! **5. What if...?** Another firework with a 5-second fuse is launched from the same platform at a speed of 400 feet per second. Its height is given by $-16t^2 + 400t + 6$. How high will this firework be when it explodes?

THINK AND DISCUSS

1. Explain why each expression is not a polynomial: $2x^2 + 3x^{-3}$; $1 - \frac{a}{b}$.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each oval, write an example of the given type of polynomial.



7-6 Exercises

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Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

Vocabulary **Match each polynomial on the left with its classification on the right.**

1. $2x^3 + 6$ **a.** quartic polynomial

2. $3x^3 + 4x^2 - 7$ **b.** quadratic polynomial

3. $5x^2 - 2x + 3x^4 - 6$ **c.** cubic trinomial

d. cubic binomial

SEE EXAMPLE 1 p. 496

Find the degree of each monomial.

4. 10^6 **5.** $-7xy^2$ **6.** $0.4n^8$ **7.** 2

SEE EXAMPLE 2 p. 496

Find the degree of each polynomial.

8. $x^2 - 2x + 1$ **9.** $0.75a^2b - 2a^3b^5$ **10.** $15y - 84y^3 + 100 - 3y^2$

11. $r^3 + r^2 - 5$ **12.** $a^3 + a^2 - 2a$ **13.** $3k^4 + k^3 - 2k^2 + k$

SEE EXAMPLE 3 p. 497

Write each polynomial in standard form. Then give the leading coefficient.

14. $-2b + 5 + b^2$ **15.** $9a^8 - 8a^9$ **16.** $5s^2 - 3s + 3 - s^7$

17. $2x + 3x^2 - 1$ **18.** $5g - 7 + g^2$ **19.** $3c^2 + 5c^4 + 5c^3 - 4$

SEE EXAMPLE 4 p. 498

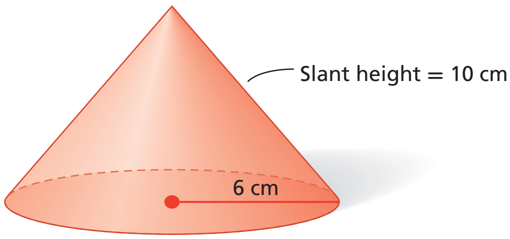
Classify each polynomial according to its degree and number of terms.

20. $x^2 + 2x + 3$ **21.** $x - 7$ **22.** $8 + k + 5k^4$

23. $q^2 + 6 - q^3 + 3q^4$ **24.** $5k^2 + 7k^3$ **25.** $2a^3 + 4a^2 - a^4$

SEE EXAMPLE 5 p. 498

26. **Geometry** The surface area of a cone is approximated by the polynomial $3.14r^2 + 3.14r\ell$, where r is the radius and ℓ is the slant height. Find the approximate surface area of this cone.



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
27–34	1
35–40	2
41–49	3
50–57	4
58	5

Extra Practice

Skills Practice p. S17
Application Practice p. S34

Find the degree of each monomial.

27. $3y^4$ **28.** $6k$ **29.** $2a^3b^2c$ **30.** 325

31. $2y^4z^3$ **32.** $9m^5$ **33.** p **34.** 5

Find the degree of each polynomial.

35. $a^2 + a^4 - 6a$ **36.** $3^2b - 5$ **37.** $3.5y^2 - 4.1y - 6$

38. $-5f^4 + 2f^6 + 10f^8$ **39.** $4n^3 - 2n$ **40.** $4r^3 + 4r^6$

Write each polynomial in standard form. Then give the leading coefficient.

41. $2.5 + 4.9t^3 - 4t^2 + t$ **42.** $8a - 10a^2 + 2$ **43.** $x^7 - x + x^3 - x^5 + x^{10}$

44. $-m + 7 - 3m^2$ **45.** $3x^2 + 5x - 4 + 5x^3$ **46.** $-2n + 1 - n^2$

47. $4d + 3d^2 - d^3 + 5$ **48.** $3s^2 + 12s^3 + 6$ **49.** $4x^2 - x^5 - x^3 + 1$

Link: Transportation

Hybrid III is the crash test dummy used by the Insurance Institute for Highway Safety. During a crash test, sensors in the dummy's head, neck, chest, legs, and feet measure and record forces. Engineers study this data to help design safer cars.

Classify each polynomial according to its degree and number of terms.

50. 12 **51.** $6k$ **52.** $3.5x^3 - 4.1x - 6$ **53.** $4g + 2g^2 - 3$

54. $2x^2 - 6x$ **55.** $6 - s^3 - 3s^4$ **56.** $c^2 + 7 - 2c^3$ **57.** $-y^2$

58. Transportation The polynomial $3.675v + 0.096v^2$ is used by transportation officials to estimate the stopping distance in feet for a car whose speed is v miles per hour on flat, dry pavement. What is the stopping distance for a car traveling at 30 miles per hour?

Tell whether each statement is sometimes, always, or never true.

59. A monomial is a polynomial.

60. A trinomial is a 3rd-degree polynomial.

61. A binomial is a trinomial.

62. A polynomial has two or more terms.

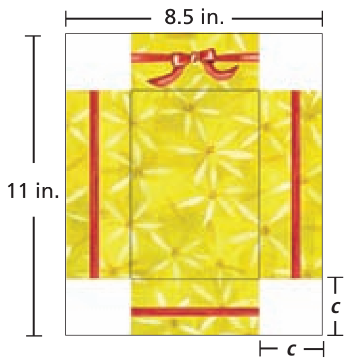
63. Geometry A piece of 8.5-by-11-inch cardboard has identical squares cut from its corners. It is then folded into a box with no lid. The volume of the box in cubic inches is $4c^3 - 39c^2 + 93.5c$, where c is the side length of the missing squares in inches.

a. What is the volume of the box if $c = 1$ in.?

b. What is the volume of the box if $c = 1.5$ in.?

c. What is the volume of the box if $c = 4.25$ in.?

d. Critical Thinking Does your answer to part **c** make sense? Explain why or why not.



Copy and complete the table by evaluating each polynomial for the given values of x.

	Polynomial	$x = -2$	$x = 0$	$x = 5$
64.	$5x - 6$	$5(-2) - 6 = -16$	$5(0) - 6 = -6$	
65.	$x^5 + x^3 + 4x$			
66.	$-10x^2$			

Give one example of each type of polynomial.

67. quadratic trinomial **68.** linear binomial **69.** constant monomial

70. cubic monomial **71.** quintic binomial **72.** 12th-degree trinomial

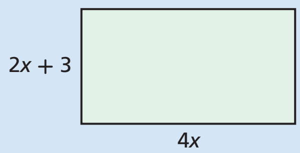
73. Write About It Explain the steps you would follow to write the polynomial $4x^3 - 3 + 5x^2 - 2x^4 - x$ in standard form.

Multi-Step Test Prep

74. This problem will prepare you for the Multi-Step Test Prep on page 528.

a. The perimeter of the rectangle shown is $12x + 6$. What is the degree of this polynomial?

b. The area of the rectangle is $8x^2 + 12x$. What is the degree of this polynomial?



75. ERROR ANALYSIS Two students evaluated $4x - 3x^5$ for $x = -2$. Which is incorrect? Explain the error.

A

$4(-2) - 3(-2)^5$

$-8 + 6^5$

$-8 + 7776$

7768

B

$4(-2) - 3(-2)^5$

$-8 - 3(-32)$

$-8 + 96$

88

TEST PREP

76. Which polynomial has the highest degree?

Ⓐ $3x^8 - 2x^7 + x^6$ Ⓑ $5x - 100$ Ⓒ $25x^{10} + 3x^5 - 15$ Ⓓ $134x^2$

77. What is the value of $-3x^3 + 4x^2 - 5x + 7$ when $x = -1$?

Ⓕ 3 Ⓖ 13 Ⓗ 9 Ⓙ 19

78. **Short Response** A toy rocket is launched from the ground at 75 feet per second. The polynomial $-16t^2 + 75t$ gives the rocket's height in feet after t seconds. Make a table showing the rocket's height after 1 second, 2 seconds, 3 seconds, and 4 seconds. At which of these times will the rocket be the highest?

CHALLENGE AND EXTEND

79. **Medicine** Doctors and nurses use growth charts and formulas to tell whether a baby is developing normally. The polynomial $0.016m^3 - 0.390m^2 + 4.562m + 50.310$ gives the average length in centimeters of a baby boy between 0 and 10 months of age, where m is the baby's age in months.

a. What is the average length of a 2-month-old baby boy? a 5-month-old baby boy? Round your answers to the nearest centimeter.

b. What is the average length of a newborn (0-month-old) baby boy?

c. How could you find the answer to part **b** without doing any calculations?

80. Consider the binomials $4x^5 + x$, $4x^4 + x$, and $4x^3 + x$.

a. Without calculating, which binomial has the greatest value for $x = 5$?

b. Are there any values of x for $4x^3 + x$ which will have the greatest value? Explain.

SPIRAL REVIEW

81. Jordan is allowed 90 minutes of screen time per day. Today, he has already used m minutes. Write an expression for the remaining number of minutes Jordan has today. *(Lesson 1-1)*

82. Pens cost \$0.50 each. Giselle bought p pens. Write an expression for the total cost of Giselle's pens. *(Lesson 1-1)*

Classify each system. Give the number of solutions. *(Lesson 6-4)*

83. $\begin{cases} y = -4x + 5 \\ 4x + y = 2 \end{cases}$ 84. $\begin{cases} 2x + 8y = 10 \\ 4y = -x + 5 \end{cases}$ 85. $\begin{cases} y = 3x + 2 \\ y = -5x - 6 \end{cases}$

Simplify. *(Lesson 7-4)*

86. $\dfrac{4^7}{4^4}$ 87. $\dfrac{x^6y^4}{x^4y^9}$ 88. $\left(\dfrac{2v^4}{vw^5}\right)^2$ 89. $\left(\dfrac{2p}{p^3}\right)^{-4}$

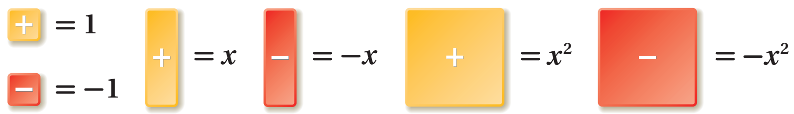
7-7 Algebra LAB

Model Polynomial Addition and Subtraction

You can use algebra tiles to model polynomial addition and subtraction.

Use with Lesson 7-7

KEY



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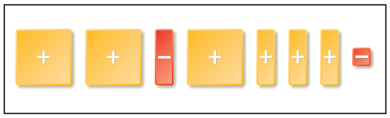
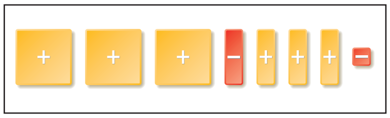
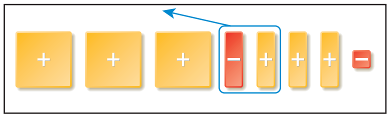
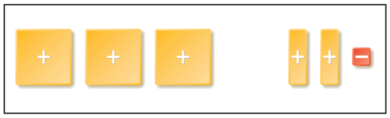
Activity 1

Use algebra tiles to find $(2x^2 - x) + (x^2 + 3x - 1)$.

MODEL		ALGEBRA
	*Use tiles to represent all terms from both expressions.*	$(2x^2 - x) + (x^2 + 3x - 1)$
	*Rearrange tiles so that like tiles are together. Like tiles are the same size and shape.*	$(2x^2 + x^2) + (-x + 3x) - 1$
	*Remove any zero pairs.*	$3x^2 - x + x + 2x - 1$
	*The remaining tiles represent the sum.*	$3x^2 + 2x - 1$

Try This

Use algebra tiles to find each sum.

1. $(-2x^2 + 1) + (-x^2)$

2. $(3x^2 + 2x + 5) + (x^2 - x - 4)$

3. $(x - 3) + (2x - 2)$

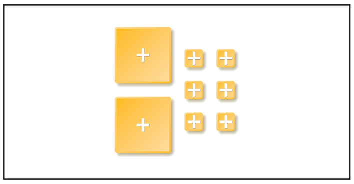
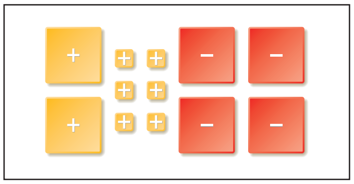
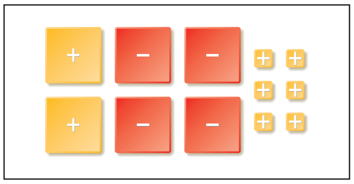
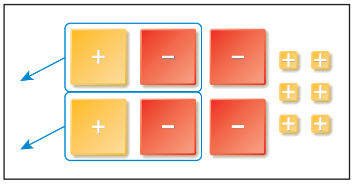
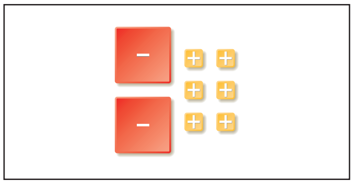
4. $(5x^2 - 3x - 6) + (x^2 + 3x + 6)$

5. $-5x^2 + (2x^2 + 5x)$

6. $(x^2 - x - 1) + (6x - 3)$

Activity 2

Use algebra tiles to find $(2x^2 + 6) - 4x^2$.

MODEL	ALGEBRA
 *Use tiles to represent the terms in the first expression.*	$2x^2 + 6$
To subtract $4x^2$, you would remove 4 yellow x^2-tiles, but there are not enough to do this. Remember that subtraction is the same as adding the opposite, so rewrite $(2x^2 + 6) - 4x^2$ as $(2x^2 + 6) + (-4x^2)$.	
 *Add 4 red x^2-tiles.*	$2x^2 + 6 + (-4x^2)$
 *Rearrange tiles so that like tiles are together.*	$2x^2 + (-4x^2) + 6$
 *Remove zero pairs.*	$2x^2 + (-2x^2) + (-2x^2) + 6$
 *The remaining tiles represent the difference.*	$-2x^2 + 6$

Try This

Use algebra tiles to find each difference.

7. $(6x^2 + 4x) - 3x^2$

8. $(2x^2 + x - 7) - 5x$

9. $(3x + 6) - 6$

10. $(8x + 5) - (-2x)$

11. $(x^2 + 2x) - (-4x^2 + x)$

12. $(3x^2 - 4) - (x^2 + 6x)$

13.  represents a zero pair. Use algebra tiles to model two other zero pairs.

14. When is it not necessary to "add the opposite" for polynomial subtraction using algebra tiles?

7-7 Adding and Subtracting Polynomials

Objective
Add and subtract polynomials.

Who uses this?
Business owners can add and subtract polynomials that model profit. (See Example 4.)

"This one pretty much sums it up."

Just as you can perform operations on numbers, you can perform operations on polynomials. To add or subtract polynomials, combine like terms.

Remember!
Like terms are constants or terms with the same variable(s) raised to the same power(s). To review combining like terms, see Lesson 1-7.

EXAMPLE 1 Adding and Subtracting Monomials

Add or subtract.

A $15m^3 + 6m^2 + 2m^3$

$15m^3 + 6m^2 + 2m^3$	*Identify like terms.*
$15m^3 + 2m^3 + 6m^2$	*Rearrange terms so that like terms are together.*
$17m^3 + 6m^2$	*Combine like terms.*

B $3x^2 + 5 - 7x^2 + 12$

$3x^2 + 5 - 7x^2 + 12$	*Identify like terms.*
$3x^2 - 7x^2 + 5 + 12$	*Rearrange terms so that like terms are together.*
$-4x^2 + 17$	*Combine like terms.*

C $0.9y^5 - 0.4y^5 + 0.5x^5 + y^5$

$0.9y^5 - 0.4y^5 + 0.5x^5 + y^5$	*Identify like terms.*
$0.9y^5 - 0.4y^5 + y^5 + 0.5x^5$	*Rearrange terms so that like terms are together.*
$1.5y^5 + 0.5x^5$	*Combine like terms.*

D $2x^2y - x^2y - x^2y$

$2x^2y - x^2y - x^2y$	*All terms are like terms.*
0	*Combine.*

CHECK IT OUT! **Add or subtract.**

1a. $2s^2 + 3s^2 + s$ **1b.** $4z^4 - 8 + 16z^4 + 2$

1c. $2x^8 + 7y^8 - x^8 - y^8$ **1d.** $9b^3c^2 + 5b^3c^2 - 13b^3c^2$

Polynomials can be added in either vertical or horizontal form.

In vertical form, align the like terms and add:

$$\begin{array}{r} 5x^2 + 4x + 1 \\ +\ 2x^2 + 5x + 2 \\ \hline 7x^2 + 9x + 3 \end{array}$$

In horizontal form, use the Associative and Commutative Properties to regroup and combine like terms:

$$(5x^2 + 4x + 1) + (2x^2 + 5x + 2)$$
$$= (5x^2 + 2x^2) + (4x + 5x) + (1 + 2)$$
$$= 7x^2 + 9x + 3$$

EXAMPLE 2

Adding Polynomials

Add.

A $(2x^2 - x) + (x^2 + 3x - 1)$

$(2x^2 - x) + (x^2 + 3x - 1)$ *Identify like terms.*

$(2x^2 + x^2) + (-x + 3x) + (-1)$ *Group like terms together.*

$3x^2 + 2x - 1$ *Combine like terms.*

B $(-2ab + b) + (2ab + a)$

$(-2ab + b) + (2ab + a)$ *Identify like terms.*

$(-2ab + 2ab) + b + a$ *Group like terms together.*

$0 + b + a$ *Combine like terms.*

$b + a$ *Simplify.*

C $(4b^5 + 8b) + (3b^5 + 6b - 7b^5 + b)$

$(4b^5 + 8b) + (3b^5 + 6b - 7b^5 + b)$ *Identify like terms.*

$(4b^5 + 8b) + (-4b^5 + 7b)$ *Combine like terms in the second polynomial.*

$$\begin{array}{r} 4b^5 + 8b \\ + \; -4b^5 + 7b \\ \hline 0 \quad + 15b \end{array}$$

Use the vertical method.

Combine like terms.

$15b$ *Simplify.*

D $(20.2y^2 + 6y + 5) + (1.7y^2 - 8)$

$(20.2y^2 + 6y + 5) + (1.7y^2 - 8)$ *Identify like terms.*

$$\begin{array}{r} 20.2y^2 + 6y + 5 \\ + \; 1.7y^2 + 0y - 8 \\ \hline 21.9y^2 + 6y - 3 \end{array}$$

Use the vertical method.

Write 0y as a placeholder in the second polynomial.

Combine like terms.

Writing Math

When you use the Associative and Commutative Properties to rearrange the terms, the sign in front of each term must stay with that term.

CHECK IT OUT! **2.** Add $(5a^3 + 3a^2 - 6a + 12a^2) + (7a^3 - 10a)$.

To subtract polynomials, remember that subtracting is the same as adding the opposite. To find the opposite of a polynomial, you must write the opposite of *each* term in the polynomial:

$$-(2x^3 - 3x + 7) = -2x^3 + 3x - 7$$

EXAMPLE 3

Subtracting Polynomials

Subtract.

A $(2x^2 + 6) - (4x^2)$

$(2x^2 + 6) + (-4x^2)$ *Rewrite subtraction as addition of the opposite.*

$(2x^2 + 6) + (-4x^2)$ *Identify like terms.*

$(2x^2 - 4x^2) + 6$ *Group like terms together.*

$-2x^2 + 6$ *Combine like terms.*

B $(a^4 - 2a) - (3a^4 - 3a + 1)$

$(a^4 - 2a) + (-3a^4 + 3a - 1)$ *Rewrite subtraction as addition of the opposite.*

$(a^4 - 2a) + (-3a^4 + 3a - 1)$ *Identify like terms.*

$(a^4 - 3a^4) + (-2a + 3a) - 1$ *Group like terms together.*

$-2a^4 + a - 1$ *Combine like terms.*

Subtract.

C $(3x^2 - 2x + 8) - (x^2 - 4)$

$(3x^2 - 2x + 8) + (-x^2 + 4)$ *Rewrite subtraction as addition of the opposite.*

$(3x^2 - 2x + 8) + (-x^2 + 4)$ *Identify like terms.*

$3x^2 - 2x + 8$ *Use the vertical method.*

$+ -x^2 + 0x + 4$ *Write 0x as a placeholder.*

$2x^2 - 2x + 12$ *Combine like terms.*

D $(11z^3 - 2z) - (z^3 - 5)$

$(11z^3 - 2z) + (-z^3 + 5)$ *Rewrite subtraction as addition of the opposite.*

$(11z^3 - 2z) + (-z^3 + 5)$ *Identify like terms.*

$11z^3 - 2z + 0$ *Use the vertical method.*

$+ -z^3 + 0z + 5$ *Write 0 and 0z as placeholders.*

$10z^3 - 2z + 5$ *Combine like terms.*

CHECK IT OUT! **3.** Subtract $(2x^2 - 3x^2 + 1) - (x^2 + x + 1)$.

EXAMPLE 4 Business Application

The profits of two different manufacturing plants can be modeled as shown, where x is the number of units produced at each plant.

Eastern:
$-0.03x^2 + 25x - 1500$

Southern:
$-0.02x^2 + 21x - 1700$

Write a polynomial that represents the difference of the profits at the eastern plant and the profits at the southern plant.

$(-0.03x^2 + 25x - 1500)$ *Eastern plant profits*

$-(-0.02x^2 + 21x - 1700)$ *Southern plant profits*

$-0.03x^2 + 25x - 1500$

$+(+0.02x^2 - 21x + 1700)$ *Write subtraction as addition of the opposite.*

$-0.01x^2 + 4x + 200$ *Combine like terms.*

CHECK IT OUT! **4.** Use the information above to write a polynomial that represents the total profits from both plants.

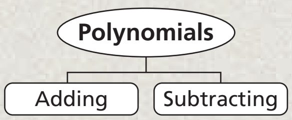
THINK AND DISCUSS

1. Identify the like terms in the following list: $-12x^2$, $-4.7y$, $\frac{1}{5}x^2y$, y, $3xy^2$, $-9x^2$, $5x^2y$, $-12x$

2. Describe how to find the opposite of $9t^2 - 5t + 8$.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example that shows how to perform the given operation.



7-7 Exercises

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Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

SEE EXAMPLE 1 p. 504

Add or subtract.

1. $7a^2 - 10a^2 + 9a$

2. $13x^2 + 9y^2 - 6x^2$

3. $0.07r^4 + 0.32r^3 + 0.19r^4$

4. $\frac{1}{4}p^3 + \frac{2}{3}p^3$

5. $5b^3c + b^3c - 3b^3c$

6. $-8m + 5 - 16 + 11m$

SEE EXAMPLE 2 p. 505

Add.

7. $(5n^3 + 3n + 6) + (18n^3 + 9)$

8. $(3.7q^2 - 8q + 3.7) + (4.3q^2 - 2.9q + 1.6)$

9. $(-3x + 12) + (9x^2 + 2x - 18)$

10. $(9x^4 + x^3) + (2x^4 + 6x^3 - 8x^4 + x^3)$

SEE EXAMPLE 3 p. 505

Subtract.

11. $(6c^4 + 8c + 6) - (2c^4)$

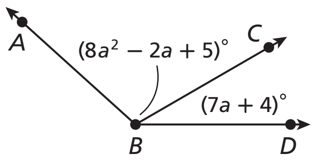
12. $(16y^2 - 8y + 9) - (6y^2 - 2y + 7y)$

13. $(2r + 5) - (5r - 6)$

14. $(-7k^2 + 3) - (2k^2 + 5k - 1)$

SEE EXAMPLE 4 p. 506

15. Geometry Write a polynomial that represents the measure of angle *ABD*.



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
16–24	1
25–28	2
29–32	3
33–34	4

Extra Practice
Skills Practice p. S17
Application Practice p. S34

Add or subtract.

16. $4k^3 + 6k^2 + 9k^3$

17. $5m + 12n^2 + 6n - 8m$

18. $2.5a^4 - 8.1b^4 - 3.6b^4$

19. $2d^5 + 1 - d^5$

20. $7xy - 4x^2y - 2xy$

21. $-6x^3 + 5x + 2x^3 + 4x^3$

22. $x^2 + x + 3x + 2x^2$

23. $3x^3 - 4 - x^3 - 1$

24. $3b^3 - 2b - 1 - b^3 - b$

Add.

25. $(2t^2 - 8t) + (8t^2 + 9t)$

26. $(-7x^2 - 2x + 3) + (4x^2 - 9x)$

27. $(x^5 - x) + (x^4 + x)$

28. $(-2z^3 + z + 2z^3 + z) + (3z^3 - 5z^2)$

Subtract.

29. $(t^3 + 8t^2) - (3t^3)$

30. $(3x^2 - x) - (x^2 + 3x - x)$

31. $(5m + 3) - (6m^3 - 2m^2)$

32. $(3s^2 + 4s) - (-10s^2 + 6s)$

33. Photography The measurements of a photo and its frame are shown in the diagram. Write a polynomial that represents the width of the photo.

34. Geometry The length of a rectangle is represented by $4a + 3b$, and its width is represented by $7a - 2b$. Write a polynomial for the perimeter of the rectangle.

Add or subtract.

35. $(2t - 7) + (-t + 2)$

36. $(4m^2 + 3m) + (-2m^2)$

37. $(4n - 2) - 2n$

38. $(-v - 7) - (-2v)$

39. $(4x^2 + 3x - 6) + (2x^2 - 4x + 5)$

40. $(2z^2 - 3z - 3) + (2z^2 - 7z - 1)$

41. $(5u^2 + 3u + 7) - (u^3 + 2u^2 + 1)$

42. $(-7h^2 - 4h + 7) - (7h^2 - 4h + 11)$

43. Geometry The length of a rectangle is represented by $2x + 3$, and its width is represented by $3x + 7$. The perimeter of the rectangle is 35 units. Find the value of x.

44. Write About It If the parentheses are removed from $(3m^2 - 5m) + (12m^2 + 7m - 10)$, is the new expression equivalent to the original? If the parentheses are removed from $(3m^2 - 5m) - (12m^2 + 7m - 10)$, is the new expression equivalent to the original? Explain.

45. ERROR ANALYSIS Two students found the sum of the polynomials $(-3n^4 + 6n^3 + 4n^2)$ and $(8n^4 - 3n^2 + 9n)$. Which is incorrect? Explain the error.

A

$$\begin{array}{r} -3n^4 + 6n^3 + 4n^2 + 0n \\ +\, 8n^4 + 0n^3 - 3n^2 + 9n \\ \hline 5n^4 + 6n^3 + n^2 + 9n \end{array}$$

B

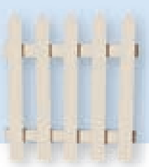
$$\begin{array}{r} -3n^4 + 6n^3 + 4n^2 \\ +\, 8n^4 - 3n^2 + 9n \\ \hline 5n^4 + 3n^3 + 13n^2 \end{array}$$

Copy and complete the table by finding the missing polynomials.

	Polynomial 1	Polynomial 2	Sum
46.	$x^2 - 6$	$3x^2 - 10x + 2$	■
47.	$12x + 5$	■	$15x + 11$
48.	■	$5x^4 + 8$	$6x^4 - 3x^2 - 1$
49.	$7x^3 - 6x - 3$	■	$7x^3 + 11$
50.	$2x^3 + 5x^2$	$7x^3 - 5x^2 + 1$	■
51.	■	$x + x^2 + 6$	$3x^2 + 2x + 1$

52. Critical Thinking Does the order in which you add polynomials affect the sum? Does the order in which you subtract polynomials affect the difference? Explain.

MULTI-STEP TEST PREP



53. This problem will prepare you for the Multi-Step Test Prep on page 528.

a. Ian plans to build a fenced dog pen. At first, he planned for the pen to be a square of length x feet on each side, but then he decided that a square may not be best. He added 4 feet to the length and subtracted 3 feet from the width. Draw a diagram to show the dimensions of the new pen.

b. Write a polynomial that represents the amount of fencing that Ian will need for the new dog pen.

c. How much fencing will Ian need if $x = 15$?

TEST PREP

54. What is the missing term?

$(-14y^2 + 9y^2 - 12y + 3) + (2y^2 + \blacksquare - 6y - 2) = (-3y^2 - 15y + 1)$

Ⓐ $-6y$ Ⓑ $-3y$ Ⓒ $3y$ Ⓓ $6y$

55. Which is NOT equivalent to $-5t^3 - t$?

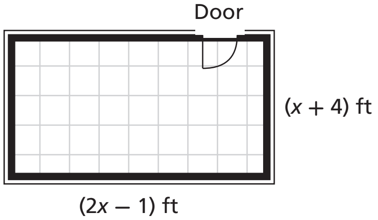
Ⓕ $-(5t^3 + t)$

Ⓗ $(t^3 + 6t) - (6t^3 + 7t)$

Ⓖ $(2t^3 - 4t) - (-7t - 3t)$

Ⓙ $(2t^3 - 3t^2 + t) - (7t^3 - 3t^2 + 2t)$

56. **Extended Response** Tammy plans to put a wallpaper border around the perimeter of her room. She will not put the border across the doorway, which is 3 feet wide.



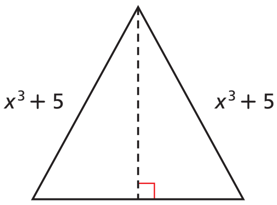
a. Write a polynomial that represents the number of feet of wallpaper border that Tammy will need.

b. A local store has 50 feet of the border that Tammy has chosen. What is the greatest whole-number value of *x* for which this amount would be enough for Tammy's room? Justify your answer.

c. Determine the dimensions of Tammy's room for the value of *x* that you found in part **b.**

CHALLENGE AND EXTEND

57. **Geometry** The legs of the isosceles triangle at right measure $(x^3 + 5)$ units. The perimeter of the triangle is $(2x^3 + 3x^2 + 8)$ units. Write a polynomial that represents the measure of the base of the triangle.



58. Write two polynomials whose sum is $4m^3 + 3m$.

59. Write two polynomials whose difference is $4m^3 + 3m$.

60. Write three polynomials whose sum is $4m^3 + 3m$.

61. Write two monomials whose sum is $4m^3 + 3m$.

62. Write three trinomials whose sum is $4m^3 + 3m$.

SPIRAL REVIEW

Solve each inequality and graph the solutions. *(Lesson 3-2)*

63. $d + 5 \geq -2$

64. $15 < m - 11$

65. $-6 + t < -6$

Write each equation in slope-intercept form. Then graph the line described by each equation. *(Lesson 5-7)*

66. $3x + y = 8$

67. $2y = \frac{1}{2}x + 6$

68. $y = 4(-x + 1)$

Simplify. *(Lesson 7-3)*

69. $b^4 \cdot b^7$

70. $cd^4 \cdot (c^{-5})^3$

71. $(-3z^6)^2$

72. $(j^3k^{-5})^3 \cdot (k^2)^4$

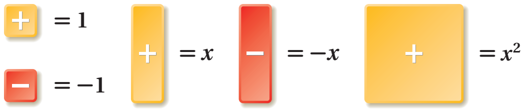
7-8 Algebra LAB

Use with Lesson 7-8

Model Polynomial Multiplication

You can use algebra tiles to multiply polynomials. Use the length and width of a rectangle to represent the factors. The area of the rectangle represents the product.

KEY

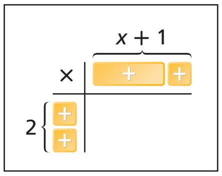
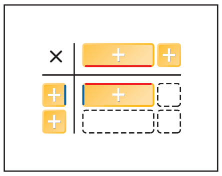
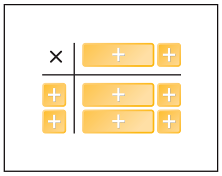


REMEMBER

- The product of two values with the same sign is positive.
- The product of two values with different signs is negative.

Activity 1

Use algebra tiles to find $2(x + 1)$.

MODEL		ALGEBRA
	*Place the first factor in a column along the left side of the grid. This will be the width of the rectangle.* *Place the second factor across the top of the grid. This will be the length of the rectangle.*	$2(x + 1)$
	*Fill in the grid with tiles that have the same width as the tiles in the left column and the same length as the tiles in the top row.*	
	*The area of the rectangle inside the grid represents the product.*	$x + x + 1 + 1$ $2x + 2$

The rectangle has an area of $2x + 2$, so $2(x + 1) = 2x + 2$. Notice that this is the same product you would get by using the Distributive Property to multiply $2(x + 1)$.

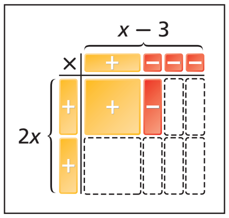
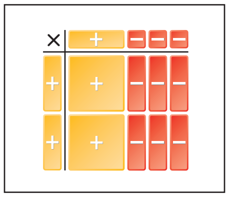
Try This

Use algebra tiles to find each product.

1. $3(x + 2)$ **2.** $2(2x + 1)$ **3.** $3(x + 1)$ **4.** $3(2x + 2)$

Activity 2

Use algebra tiles to find $2x(x - 3)$.

MODEL		ALGEBRA
	*Place tiles to form the length and width of a rectangle and fill in the rectangle. The product of two values with the same sign (same color) is positive (yellow). The product of two values with different signs (different colors) is negative (red).*	$2x(x - 3)$
	*The area of the rectangle inside the grid represents the product. The rectangle has an area of $2x^2 - 6x$, so $2x(x - 3) = 2x^2 - 6x$.*	$x^2 + x^2 - x - x - x - x - x - x$ $2x^2 - 6x$

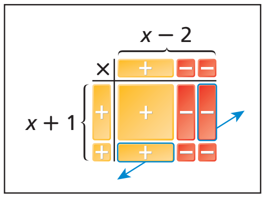
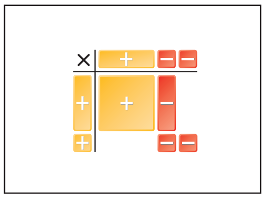
Try This

Use algebra tiles to find each product.

5. $3x(x - 2)$ **6.** $x(2x - 1)$ **7.** $x(x + 1)$ **8.** $(8x + 5)(-2x)$

Activity 3

Use algebra tiles to find $(x + 1)(x - 2)$.

MODEL		ALGEBRA
	*Place tiles for each factor to form the length and width of a rectangle. Fill in the grid and remove any zero pairs.*	$(x + 1)(x - 2)$ $x^2 - x - x + x - 1 - 1$
	*The area inside the grid represents the product. The remaining area is $x^2 - x - 2$, so $(x + 1)(x - 2) = x^2 - x - 2$.*	$x^2 - x - 1 - 1$ $x^2 - x - 2$

Try This

Use algebra tiles to find each product.

9. $(x + 2)(x - 3)$ **10.** $(x - 1)(x + 3)$ **11.** $(x - 2)(x - 3)$ **12.** $(x + 1)(x + 2)$

7-8 Multiplying Polynomials

Objective
Multiply polynomials.

Why learn this?
You can multiply polynomials to write expressions for areas, such as the area of a dulcimer. (See Example 5.)

To multiply monomials and polynomials, you will use some of the properties of exponents that you learned earlier in this chapter.

EXAMPLE 1 **Multiplying Monomials**

Multiply.

A $(5x^2)(4x^3)$

$(5x^2)(4x^3)$

$(5 \cdot 4)(x^2 \cdot x^3)$ *Group factors with like bases together.*

$20x^5$ *Multiply.*

B $(-3x^3y^2)(4xy^5)$

$(-3x^3y^2)(4xy^5)$

$(-3 \cdot 4)(x^3 \cdot x)(y^2 \cdot y^5)$ *Group factors with like bases together.*

$-12x^4y^7$ *Multiply.*

C $\left(\frac{1}{2}a^3b\right)(a^2c^2)(6b^2)$

$\left(\frac{1}{2}a^3b\right)(a^2c^2)(6b^2)$

$\left(\frac{1}{2} \cdot 6\right)(a^3 \cdot a^2)(b \cdot b^2)(c^2)$ *Group factors with like bases together.*

$3a^5b^3c^2$ *Multiply.*

Remember!
When multiplying powers with the same base, keep the base and add the exponents.
$x^2 \cdot x^3 = x^{2+3} = x^5$

CHECK IT OUT! **Multiply.**

1a. $(3x^3)(6x^2)$ **1b.** $(2r^2t)(5t^3)$ **1c.** $\left(\frac{1}{3}x^2y\right)(12x^3z^2)(y^4z^5)$

To multiply a polynomial by a monomial, use the Distributive Property.

EXAMPLE 2 **Multiplying a Polynomial by a Monomial**

Multiply.

A $5(2x^2 + x + 4)$

$5(2x^2 + x + 4)$

$(5)2x^2 + (5)x + (5)4$ *Distribute 5.*

$10x^2 + 5x + 20$ *Multiply.*

Multiply.

B $2x^2y(3x - y)$

$(2x^2y)(3x - y)$

$(2x^2y)3x + (2x^2y)(-y)$ — *Distribute $2x^2y$.*

$(2 \cdot 3)(x^2 \cdot x)y + 2(-1)(x^2)(y \cdot y)$ — *Group like bases together.*

$6x^3y - 2x^2y^2$ — *Multiply.*

C $4a(a^2b + 2b^2)$

$4a(a^2b + 2b^2)$

$(4a)a^2b + (4a)2b^2$ — *Distribute 4a.*

$(4)(a \cdot a^2)(b) + (4 \cdot 2)(a)(b^2)$ — *Group like bases together.*

$4a^3b + 8ab^2$ — *Multiply.*

CHECK IT OUT! **Multiply.**

2a. $2(4x^2 + x + 3)$ **2b.** $3ab(5a^2 + b)$ **2c.** $5r^2s^2(r - 3s)$

To multiply a binomial by a binomial, you can apply the Distributive Property more than once:

$(x + 3)(x + 2) = x(x + 2) + 3(x + 2)$ — *Distribute.*

$= x(x + 2) + 3(x + 2)$

$= x(x) + x(2) + 3(x) + 3(2)$ — *Distribute again.*

$= x^2 + 2x + 3x + 6$ — *Multiply.*

$= x^2 + 5x + 6$ — *Combine like terms.*

Another method for multiplying binomials is called the FOIL method.

1. Multiply the **F**irst terms. $(x+3)(x+2) \rightarrow x \cdot x = x^2$ (F)
2. Multiply the **O**uter terms. $(x+3)(x+2) \rightarrow x \cdot 2 = 2x$ (O)
3. Multiply the **I**nner terms. $(x+3)(x+2) \rightarrow 3 \cdot x = 3x$ (I)
4. Multiply the **L**ast terms. $(x+3)(x+2) \rightarrow 3 \cdot 2 = 6$ (L)

$(x+3)(x+2) = x^2 + 2x + 3x + 6 = x^2 + 5x + 6$

F O I L

EXAMPLE 3 Multiplying Binomials

Multiply.

A $(x+2)(x-5)$

$(x+2)(x-5)$

$x(x-5)+2(x-5)$ *Distribute.*

$x(x)+x(-5)+2(x)+2(-5)$ *Distribute again.*

$x^2-5x+2x-10$ *Multiply.*

$x^2-3x-10$ *Combine like terms.*

Helpful Hint

In the expression $(x+5)^2$, the base is $(x+5)$.

$(x+5)^2 =$
$(x+5)(x+5)$

B $(x+5)^2$

$(x+5)(x+5)$ *Write as a product of two binomials.*

$(x \cdot x)+(x \cdot 5)+(5 \cdot x)+(5 \cdot 5)$ *Use the FOIL method.*

$x^2+5x+5x+25$ *Multiply.*

$x^2+10x+25$ *Combine like terms.*

C $(3a^2-b)(a^2-2b)$

$3a^2(a^2)+3a^2(-2b)-b(a^2)-b(-2b)$ *Use the FOIL method.*

$3a^4-6a^2b-a^2b+2b^2$ *Multiply.*

$3a^4-7a^2b+2b^2$ *Combine like terms.*

CHECK IT OUT! **3a.** $(a+3)(a-4)$ **3b.** $(x-3)^2$ **3c.** $(2a-b^2)(a+4b^2)$

To multiply polynomials with more than two terms, you can use the Distributive Property several times. Multiply $(5x+3)$ by $(2x^2+10x-6)$:

$$(5x+3)(2x^2+10x-6) = 5x(2x^2+10x-6)+3(2x^2+10x-6)$$
$$= 5x(2x^2+10x-6)+3(2x^2+10x-6)$$
$$= 5x(2x^2)+5x(10x)+5x(-6)+3(2x^2)+3(10x)+3(-6)$$
$$= 10x^3+50x^2-30x+6x^2+30x-18$$
$$= 10x^3+56x^2-18$$

You can also use a rectangle model to multiply polynomials with more than two terms. This is similar to finding the area of a rectangle with length $(2x^2+10x-6)$ and width $(5x+3)$:

	$2x^2$	$+10x$	-6
$5x$	$10x^3$	$50x^2$	$-30x$
$+3$	$6x^2$	$30x$	-18

Write the product of the monomials in each row and column.

To find the product, add all of the terms inside the rectangle by combining like terms and simplifying if necessary.

$$10x^3+6x^2+50x^2+30x-30x-18$$
$$10x^3+56x^2-18$$

Another method that can be used to multiply polynomials with more than two terms is the vertical method. This is similar to methods used to multiply whole numbers.

$$\begin{array}{rrrrl}
 & 2x^2 & +\ 10x & -\ 6 & \\
\times & & 5x & +\ 3 & \\
\hline
 & 6x^2 & +\ 30x & -\ 18 & \text{Multiply each term in the top polynomial by 3.} \\
+\ 10x^3 & +\ 50x^2 & -\ 30x & & \text{Multiply each term in the top polynomial by 5x, and align like terms.} \\
\hline
10x^3 & +\ 56x^2 & +\ 0x & -\ 18 & \text{Combine like terms by adding vertically.} \\
10x^3 & +\ 56x^2 & & -\ 18 & \text{Simplify.}
\end{array}$$

EXAMPLE 4 Multiplying Polynomials

> **Helpful Hint**
>
> A polynomial with *m* terms multiplied by a polynomial with *n* terms has a product that, before simplifying, has *mn* terms. In Example 4A, there are 2 • 3, or 6, terms before simplifying.

Multiply.

A $(x + 2)(x^2 - 5x + 4)$

$(x + 2)(x^2 - 5x + 4)$

$x(x^2 - 5x + 4) + 2(x^2 - 5x + 4)$ *Distribute.*

$x(x^2) + x(-5x) + x(4) + 2(x^2) + 2(-5x) + 2(4)$ *Distribute again.*

$x^3 + 2x^2 - 5x^2 - 10x + 4x + 8$ *Simplify.*

$x^3 - 3x^2 - 6x + 8$ *Combine like terms.*

B $(3x - 4)(-2x^3 + 5x - 6)$

$(3x - 4)(-2x^3 + 5x - 6)$

$$\begin{array}{rrrrrl}
 & -2x^3 & +\ 0x^2 & +\ 5x & -\ 6 & \text{Add } 0x^2 \text{ as a placeholder.} \\
\times & & & 3x & -\ 4 & \\
\hline
 & 8x^3 & +\ 0x^2 & -\ 20x & +\ 24 & \text{Multiply each term in the top polynomial by } -4. \\
+\ -6x^4 & +\ 0x^3 & +\ 15x^2 & -\ 18x & & \text{Multiply each term in the top polynomial by 3x, and align like terms.} \\
\hline
-6x^4 & +\ 8x^3 & +\ 15x^2 & -\ 38x & +\ 24 & \text{Combine like terms by adding vertically.}
\end{array}$$

C $(x - 2)^3$

$[(x - 2)(x - 2)](x - 2)$ *Write as the product of three binomials.*

$[x \cdot x + x(-2) + (-2)x + (-2)(-2)](x - 2)$ *Use the FOIL method on the first two factors.*

$(x^2 - 2x - 2x + 4)(x - 2)$ *Multiply.*

$(x^2 - 4x + 4)(x - 2)$ *Combine like terms.*

$(x - 2)(x^2 - 4x + 4)$ *Use the Commutative Property of Multiplication.*

$x(x^2 - 4x + 4) + (-2)(x^2 - 4x + 4)$ *Distribute.*

$x(x^2) + x(-4x) + x(4) + (-2)(x^2) + (-2)(-4x) + (-2)(4)$ *Distribute again.*

$x^3 - 4x^2 + 4x - 2x^2 + 8x - 8$ *Simplify.*

$x^3 - 6x^2 + 12x - 8$ *Combine like terms.*

Multiply.

D $(2x+3)(x^2-6x+5)$

	x^2	$-6x$	$+5$
$2x$	$2x^3$	$-12x^2$	$10x$
$+3$	$3x^2$	$-18x$	15

Write the product of the monomials in each row and column.

$2x^3+3x^2-12x^2-18x+10x+15$ — *Add all terms inside the rectangle.*

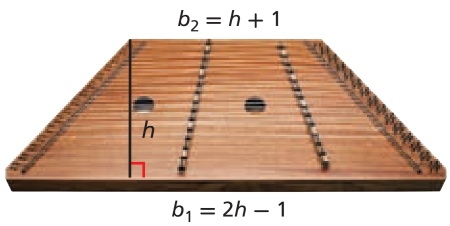
$2x^3-9x^2-8x+15$ — *Combine like terms.*

CHECK IT OUT! **Multiply.**

4a. $(x+3)(x^2-4x+6)$

4b. $(3x+2)(x^2-2x+5)$

EXAMPLE 5 *Music Application*



A dulcimer is a musical instrument that is sometimes shaped like a trapezoid.

A **Write a polynomial that represents the area of the dulcimer shown.**

$A=\frac{1}{2}h(b_1+b_2)$ — *Write the formula for area of a trapezoid.*

$=\frac{1}{2}h[(2h-1)+(h+1)]$ — *Substitute $2h-1$ for b_1 and $h+1$ for b_2.*

$=\frac{1}{2}h(3h)$ — *Combine like terms.*

$=\frac{3}{2}h^2$ — *Simplify.*

The area is represented by $\frac{3}{2}h^2$.

B **Find the area of the dulcimer when the height is 22 inches.**

$A=\frac{3}{2}h^2$ — *Use the polynomial from part **a**.*

$=\frac{3}{2}(22)^2$ — *Substitute 22 for h.*

$=\frac{3}{2}(484)=726$

The area is 726 square inches.

CHECK IT OUT!

5. The length of a rectangle is 4 meters shorter than its width.

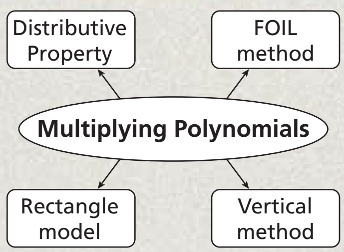
a. Write a polynomial that represents the area of the rectangle.

b. Find the area of the rectangle when the width is 6 meters.

THINK AND DISCUSS

1. Compare the vertical method for multiplying polynomials with the vertical method for multiplying whole numbers.

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, multiply two polynomials using the given method.



Know it! Note

7-8 Exercises

GUIDED PRACTICE

Multiply.

SEE EXAMPLE 1 p. 512

1. $(2x^2)(7x^4)$
2. $(-5mn^3)(4m^2n^2)$
3. $(6rs^2)(s^3t^2)\left(\frac{1}{2}r^4t^3\right)$
4. $\left(\frac{1}{3}a^5\right)(12a)$
5. $(-3x^4y^2)(-7x^3y)$
6. $(-2pq^3)(5p^2q^2)(-3q^4)$

SEE EXAMPLE 2 p. 512

7. $4(x^2 + 2x + 1)$
8. $3ab(2a^2 + 3b^3)$
9. $2a^3b(3a^2b + ab^2)$
10. $-3x(x^2 - 4x + 6)$
11. $5x^2y(2xy^3 - y)$
12. $5m^2n^3 \cdot mn^2(4m - n)$

SEE EXAMPLE 3 p. 514

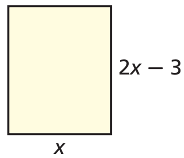
13. $(x + 1)(x - 2)$
14. $(x + 1)^2$
15. $(x - 2)^2$
16. $(y - 3)(y - 5)$
17. $(4a^3 - 2b)(a - 3b^2)$
18. $(m^2 - 2mn)(3mn + n^2)$

SEE EXAMPLE 4 p. 515

19. $(x + 5)(x^2 - 2x + 3)$
20. $(3x + 4)(x^2 - 5x + 2)$
21. $(2x - 4)(-3x^3 + 2x - 5)$
22. $(-4x + 6)(2x^3 - x^2 + 1)$
23. $(x - 5)(x^2 + x + 1)$
24. $(a + b)(a - b)(b - a)$

SEE EXAMPLE 5 p. 516

25. **Photography** The length of a rectangular photograph is 3 inches less than twice the width.
    a. Write a polynomial that represents the area of the photograph.
    b. Find the area of the photograph when the width is 4 inches.



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
26–34	1
35–43	2
44–52	3
53–61	4
62	5

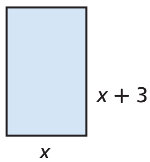
Extra Practice

Skills Practice p. S17
Application Practice p. S34

Multiply.

26. $(3x^2)(8x^5)$
27. $(-2r^3s^4)(6r^2s)$
28. $(15xy^2)\left(\frac{1}{3}x^2z^3\right)(y^3z^4)$
29. $(-2a^3)(-5a)$
30. $(6x^3y^2)(-2x^2y)$
31. $(-3a^2b)(-2b^3)(-a^3b^2)$
32. $(7x^2)(xy^5)(2x^3y^2)$
33. $(-4a^3bc^2)(a^3b^2c)(3ab^4c^5)$
34. $(12mn^2)(2m^2n)(mn)$
35. $9s(s + 6)$
36. $9(2x^2 - 5x)$
37. $3x(9x^2 - 4x)$
38. $3(2x^2 + 5x + 4)$
39. $5s^2t^3(2s - 3t^2)$
40. $x^2y^3 \cdot 5x^2y(6x + y^2)$
41. $-5x(2x^2 - 3x - 1)$
42. $-2a^2b^3(3ab^2 - a^2b)$
43. $-7x^3y \cdot x^2y^2(2x - y)$
44. $(x + 5)(x - 3)$
45. $(x + 4)^2$
46. $(m - 5)^2$
47. $(5x - 2)(x + 3)$
48. $(3x - 4)^2$
49. $(5x + 2)(2x - 1)$
50. $(x - 1)(x - 2)$
51. $(x - 8)(7x + 4)$
52. $(2x + 7)(3x + 7)$
53. $(x + 2)(x^2 - 3x + 5)$
54. $(2x + 5)(x^2 - 4x + 3)$
55. $(5x - 1)(-2x^3 + 4x - 3)$
56. $(x - 3)(x^2 - 5x + 6)$
57. $(2x^2 - 3)(4x^3 - x^2 + 7)$
58. $(x - 4)^3$
59. $(x - 2)(x^2 + 2x + 1)$
60. $(2x + 10)(4 - x + 6x^3)$
61. $(1 - x)^3$

62. **Geometry** The length of the rectangle at right is 3 feet longer than its width.
    a. Write a polynomial that represents the area of the rectangle.
    b. Find the area of the rectangle when the width is 5 feet.



63. A square tabletop has side lengths of $(4x - 6)$ units. Write a polynomial that represents the area of the tabletop.

MULTI-STEP TEST PREP

64. This problem will prepare you for the Multi-Step Test Prep on page 528.

a. Marie is creating a garden. She designs a rectangular garden with a length of $(x + 4)$ feet and a width of $(x + 1)$ feet. Draw a diagram of Marie's garden with the length and width labeled.

b. Write a polynomial that represents the area of Marie's garden.

c. What is the area when $x = 4$?

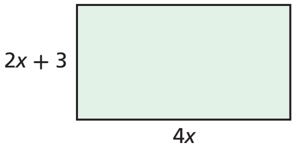
65. Copy and complete the table below.

	A	Degree of A	B	Degree of B	A • B	Degree of A • B
	$2x^2$	2	$3x^5$	5	$6x^7$	7
a.	$5x^3$		$2x^2 + 1$			
b.	$x^2 + 2$		$x^2 - x$			
c.	$x - 3$		$x^3 - 2x^2 + 1$			

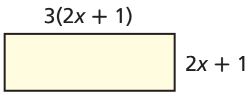
d. Use the results from the table to complete the following: The product of a polynomial of degree m and a polynomial of degree n has a degree of ▢.

Geometry **Write a polynomial that represents the area of each rectangle.**

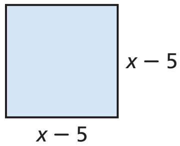
66.



67.

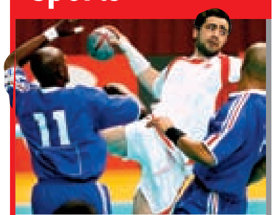


68.



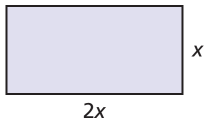
LINK

Sports



Team handball is a game with elements of soccer and basketball. It originated in Europe in the 1900s and was first played at the Olympics in 1936 with teams of 11 players. Today, a handball team consists of seven players—six court players and one goalie.

69. Sports The length of a regulation team handball court is twice its width.



a. Write a polynomial that represents the area of the court.

b. The width of a team handball court is 20 meters. Find the area of the court.

Multiply.

70. $(1.5a^3)(4a^6)$ **71.** $(2x + 5)(x - 6)$ **72.** $(3g - 1)(g + 5)$

73. $(4x - 2y)(2x - 3y)$ **74.** $(x + 3)(x - 3)$ **75.** $(1.5x - 3)(4x + 2)$

76. $(x - 10)(x + 4)$ **77.** $x^2(x + 3)$ **78.** $(x + 1)(x^2 + 2x)$

79. $(x - 4)(2x^2 + x - 6)$ **80.** $(a + b)(a - b)^2$ **81.** $(2p - 3q)^3$

82. Multi-Step A rectangular swimming pool is 25 feet long and 10 feet wide. It is surrounded by a fence that is x feet from each side of the pool.

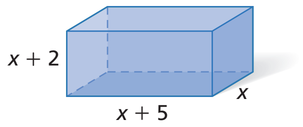
a. Draw a diagram of this situation.

b. Write expressions for the length and width of the fenced region. (*Hint:* How much longer is one side of the fenced region than the corresponding side of the pool?)

c. Write an expression for the area of the fenced region.

83. Write About It Explain why the FOIL method can be used to multiply only two binomials at a time.

84. **Geometry** Write a polynomial that represents the volume of the rectangular prism.



85. **Critical Thinking** Is there any value for x that would make the statement $(x+3)^3 = x^3 + 3^3$ true? Give an example to justify your answer.

86. **Estimation** The length of a rectangle is 1 foot more than its width. Write a polynomial that represents the area of the rectangle. Estimate the width of the rectangle if its area is 25 square feet.

TEST PREP

87. Which of the following products is equal to $a^2 - 5a - 6$?

(A) $(a-1)(a-5)$ (B) $(a-2)(a-3)$ (C) $(a+1)(a-6)$ (D) $(a+2)(a-3)$

88. Which of the following is equal to $2a(a^2-1)$?

(F) $2a^2 - 2a$ (G) $2a^3 - 1$ (H) $2a^3 - 2a$ (J) $2a^2 - 1$

89. What is the degree of the product of $3x^3y^2z$ and x^2yz?

(A) 5 (B) 6 (C) 7 (D) 10

CHALLENGE AND EXTEND

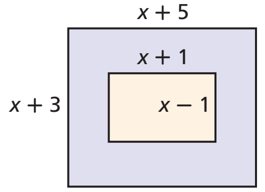
Simplify.

90. $6x^2 - 2(3x^2 - 2x + 4)$

91. $x^2 - 2x(x+3)$

92. $x(4x-2) + 3x(x+1)$

93. The diagram shows a sandbox and the frame that surrounds it.



a. Write a polynomial that represents the area of the sandbox.

b. Write a polynomial that represents the area of the frame that surrounds the sandbox.

94. **Geometry** The side length of a square is $(8 + 2x)$ units. The area of this square is the same as the perimeter of another square with a side length of $(x^2 + 48)$ units. Find the value of x.

95. Write a polynomial that represents the product of three consecutive integers. Let x represent the first integer.

96. Find m and n so that $x^m(x^n + x^{n-2}) = x^5 + x^3$.

97. Find a so that $2x^a(5x^{2a-3} + 2x^{2a+2}) = 10x^3 + 4x^8$

SPIRAL REVIEW

98. A stop sign is 2.5 meters tall and casts a shadow that is 3.5 meters long. At the same time, a flagpole casts a shadow that is 28 meters long. How tall is the flagpole? *(Lesson 2-8)*

Find the distance, to the nearest hundredth, between each pair of points. *(Lesson 5-5)*

99. (2, 3) and (4, 6)

100. (−1, 4) and (0, 8)

101. (−3, 7) and (−6, −2)

Graph the solutions of each linear inequality. *(Lesson 6-5)*

102. $y \le x - 2$

103. $4x - 2y < 10$

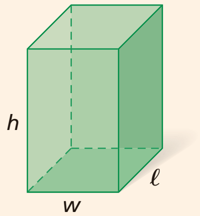
104. $-y \ge -3x + 1$

Connecting Algebra to **Geometry**

Volume and Surface Area

The volume V of a three-dimensional figure is the amount of space it occupies. The surface area S is the total area of the two-dimensional surfaces that make up the figure.

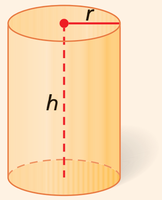
Rectangular Prism



$V = \ell wh$

$S = 2(\ell w + \ell h + wh)$

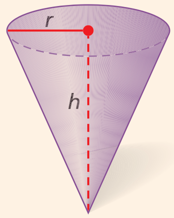
Cylinder



$V = \pi r^2 h$

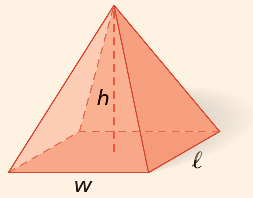
$S = 2\pi r^2 + 2\pi rh$

Cone



$V = \frac{1}{3}\pi r^2 h$

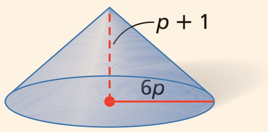
Pyramid



$V = \frac{1}{3}\ell wh$

Example

Write and simplify a polynomial expression for the volume of the cone. Leave the symbol π in your answer.



$V = \frac{1}{3}\pi r^2 h$ — *Choose the correct formula.*

$= \frac{1}{3}\pi(6p)^2(p + 1)$ — *Substitute 6p for r and p + 1 for h.*

$= \frac{1}{3}\pi(36p^2)(p + 1)$ — *Use the Power of a Product Property.*

$= \frac{1}{3}(36)\pi[p^2(p + 1)]$ — *Use the Associative and Commutative Properties of Multiplication.*

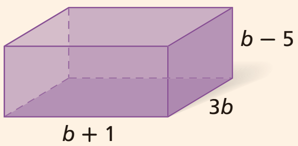
$= 12\pi p^2(p + 1)$ — *Distribute $12\pi p^2$.*

$= 12\pi p^3 + 12\pi p^2$

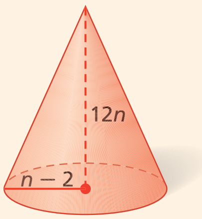
Try This

Write and simplify a polynomial expression for the volume of each figure.

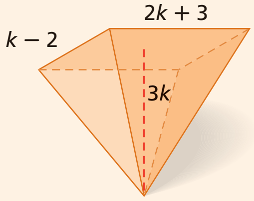
1.



2.

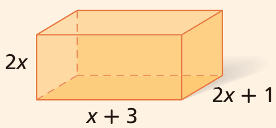


3.

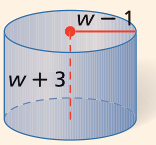


Write and simplify a polynomial expression for the surface area of each figure.

4.



5.



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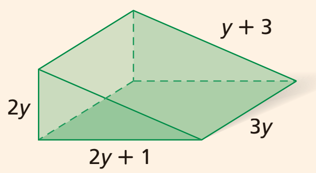


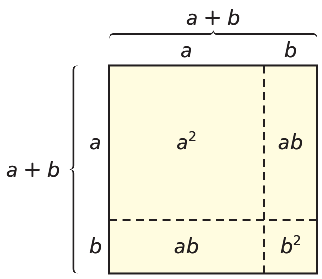
7-9 Special Products of Binomials

Objective
Find special products of binomials.

Vocabulary
perfect-square trinomial
difference of two squares

Why learn this?
You can use special products to find areas, such as the area of a deck around a pond. (See Example 4.)

Imagine a square with sides of length $(a + b)$:



The area of this square is $(a + b)(a + b)$, or $(a + b)^2$. The area of this square can also be found by adding the areas of the smaller squares and rectangles inside. The sum of the areas inside is $a^2 + ab + ab + b^2$.

This means that $(a + b)^2 = a^2 + 2ab + b^2$.

You can use the FOIL method to verify this:

$$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

(F, L, I, O)

A trinomial of the form $a^2 + 2ab + b^2$ is called a *perfect-square trinomial*. A **perfect-square trinomial** is a trinomial that is the result of squaring a binomial.

EXAMPLE 1 Finding Products in the Form $(a + b)^2$

Multiply.

A $(x + 4)^2$

$(a + b)^2 = a^2 + 2ab + b^2$ *Use the rule for $(a + b)^2$.*

$(x + 4)^2 = x^2 + 2(x)(4) + 4^2$ *Identify a and b: $a = x$ and $b = 4$.*

$= x^2 + 8x + 16$ *Simplify.*

B $(3x + 2y)^2$

$(a + b)^2 = a^2 + 2ab + b^2$ *Use the rule for $(a + b)^2$.*

$(3x + 2y)^2 = (3x)^2 + 2(3x)(2y) + (2y)^2$ *Identify a and b: $a = 3x$ and $b = 2y$.*

$= 9x^2 + 12xy + 4y^2$ *Simplify.*

Multiply.

C $(4+s^2)^2$

$(a+b)^2 = a^2 + 2ab + b^2$ — *Use the rule for $(a+b)^2$.*

$(4+s^2)^2 = (4)^2 + 2(4)(s^2) + (s^2)^2$ — *Identify a and b: $a = 4$ and $b = s^2$.*

$= 16 + 8s^2 + s^4$ — *Simplify.*

D $(-m+3)^2$

$(a+b)^2 = a^2 + 2ab + b^2$ — *Use the rule for $(a+b)^2$.*

$(-m+3)^2 = (-m)^2 + 2(-m)(3) + 3^2$ — *Identify a and b: $a = -m$ and $b = 3$.*

$= m^2 - 6m + 9$ — *Simplify.*

CHECK IT OUT! **Multiply.**

1a. $(x+6)^2$ **1b.** $(5a+b)^2$ **1c.** $(1+c^3)^2$

You can use the FOIL method to find products in the form $(a-b)^2$:

$$(a-b)^2 = (a-b)(a-b) = a^2 - ab - ab + b^2$$

$$= a^2 - 2ab + b^2$$

A trinomial of the form $a^2 - 2ab + b^2$ is also a perfect-square trinomial because it is the result of squaring the binomial $(a-b)$.

EXAMPLE 2 Finding Products in the Form $(a-b)^2$

Multiply.

A $(x-5)^2$

$(a-b)^2 = a^2 - 2ab + b^2$ — *Use the rule for $(a-b)^2$.*

$(x-5)^2 = x^2 - 2(x)(5) + 5^2$ — *Identify a and b: $a = x$ and $b = 5$.*

$= x^2 - 10x + 25$ — *Simplify.*

B $(6a-1)^2$

$(a-b)^2 = a^2 - 2ab + b^2$ — *Use the rule for $(a-b)^2$.*

$(6a-1)^2 = (6a)^2 - 2(6a)(1) + (1)^2$ — *Identify a and b: $a = 6a$ and $b = 1$.*

$= 36a^2 - 12a + 1$ — *Simplify.*

C $(4c-3d)^2$

$(a-b)^2 = a^2 - 2ab + b^2$ — *Use the rule for $(a-b)^2$.*

$(4c-3d)^2 = (4c)^2 - 2(4c)(3d) + (3d)^2$ — *Identify a and b: $a = 4c$ and $b = 3d$.*

$= 16c^2 - 24cd + 9d^2$ — *Simplify.*

D $(3-x^2)^2$

$(a-b)^2 = (a)^2 - 2ab + b^2$ — *Use the rule for $(a-b)^2$.*

$(3-x^2)^2 = (3)^2 - 2(3)(x^2) + (x^2)^2$ — *Identify a and b: $a = 3$ and $b = x^2$.*

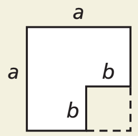
$= 9 - 6x^2 + x^4$ — *Simplify.*

CHECK IT OUT! **Multiply.**

2a. $(x-7)^2$ **2b.** $(3b-2c)^2$ **2c.** $(a^2-4)^2$

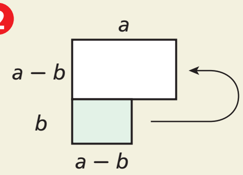
You can use an area model to see that $(a + b)(a - b) = a^2 - b^2$.

1



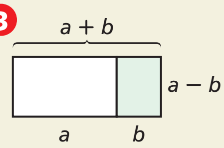
Begin with a square with area a^2. Remove a square with area b^2. The area of the new figure is $a^2 - b^2$.

2



Then remove the smaller rectangle on the bottom. Turn it and slide it up next to the top rectangle.

3



The new arrangement is a rectangle with length $a + b$ and width $a - b$. Its area is $(a + b)(a - b)$.

So $(a + b)(a - b) = a^2 - b^2$. A binomial of the form $a^2 - b^2$ is called a **difference of two squares**.

EXAMPLE 3 Finding Products in the Form $(a + b)(a - b)$

Multiply.

A $(x + 6)(x - 6)$

$(a + b)(a - b) = a^2 - b^2$ *Use the rule for $(a + b)(a - b)$.*

$(x + 6)(x - 6) = x^2 - 6^2$ *Identify a and b: $a = x$ and $b = 6$.*

$= x^2 - 36$ *Simplify.*

B $(x^2 + 2y)(x^2 - 2y)$

$(a + b)(a - b) = a^2 - b^2$ *Use the rule for $(a + b)(a - b)$.*

$(x^2 + 2y)(x^2 - 2y) = (x^2)^2 - (2y)^2$ *Identify a and b: $a = x^2$ and $b = 2y$.*

$= x^4 - 4y^2$ *Simplify.*

C $(7 + n)(7 - n)$

$(a + b)(a - b) = a^2 - b^2$ *Use the rule for $(a + b)(a - b)$.*

$(7 + n)(7 - n) = 7^2 - n^2$ *Identify a and b: $a = 7$ and $b = n$.*

$= 49 - n^2$ *Simplify.*

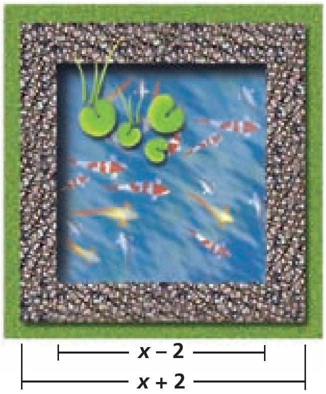
CHECK IT OUT! **Multiply.**

3a. $(x + 8)(x - 8)$ **3b.** $(3 + 2y^2)(3 - 2y^2)$ **3c.** $(9 + r)(9 - r)$

EXAMPLE 4 *Problem-Solving Application*

PROBLEM SOLVING

A square koi pond is surrounded by a gravel path. Write an expression that represents the area of the path.



1 Understand the Problem

The **answer** will be an expression that represents the area of the path.

List the important information:

- The pond is a square with a side length of $x - 2$.
- The path has a side length of $x + 2$.

2 Make a Plan

The area of the pond is $(x - 2)^2$. The total area of the path plus the pond is $(x + 2)^2$. You can subtract the area of the pond from the total area to find the area of the path.

3 Solve

Step 1 Find the total area.

$$(x + 2)^2 = x^2 + 2(x)(2) + 2^2$$ *Use the rule for $(a + b)^2$: $a = x$ and $b = 2$.*

$$= x^2 + 4x + 4$$

Step 2 Find the area of the pond.

$$(x - 2)^2 = x^2 - 2(x)(2) + 2^2$$ *Use the rule for $(a - b)^2$: $a = x$ and $b = 2$.*

$$= x^2 - 4x + 4$$

Step 3 Find the area of the path.

Remember!

To subtract a polynomial, add the opposite of *each* term.

area of path = total area − area of pond

$$a = x^2 + 4x + 4 - (x^2 - 4x + 4)$$

$$= x^2 + 4x + 4 - x^2 + 4x - 4$$ *Identify like terms.*

$$= (x^2 - x^2) + (4x + 4x) + (4 - 4)$$ *Group like terms together.*

$$= 8x$$

The area of the path is $8x$. *Combine like terms.*

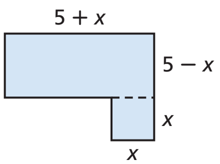
4 Look Back

Suppose that $x = 10$. Then one side of the path is 12, and the total area is 12^2, or 144. Also, if $x = 10$, one side of the pond is 8, and the area of the pond is 8^2, or 64. This means the area of the path is $144 - 64 = 80$.

According to the solution above, the area of the path is $8x$. If $x = 10$, then $8x = 8(10) = 80$. ✓

CHECK IT OUT!

4. Write an expression that represents the area of the swimming pool at right.



Know it! Note

Special Products of Binomials

Perfect-Square Trinomials

$$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$$

$$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$$

Difference of Two Squares

$$(a + b)(a - b) = a^2 - b^2$$

THINK AND DISCUSS

1. Use the FOIL method to verify that $(a + b)(a - b) = a^2 - b^2$.

2. When a binomial is squared, the middle term of the resulting trinomial is twice the ______?______ of the first and last terms.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. Complete the special product rules and give an example of each.

Special Products of Binomials		
Perfect-Square Trinomials		Difference of Two Squares
$(a + b)^2 = ?$	$(a - b)^2 = ?$	$(a + b)(a - b) = ?$

7-9 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MA11 7-9
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

1. Vocabulary In your own words, describe a *perfect-square trinomial*.

Multiply.

SEE EXAMPLE 1 p. 521

2. $(x + 7)^2$ **3.** $(2 + x)^2$ **4.** $(x + 1)^2$

5. $(2x + 6)^2$ **6.** $(5x + 9)^2$ **7.** $(2a + 7b)^2$

SEE EXAMPLE 2 p. 522

8. $(x - 6)^2$ **9.** $(x - 2)^2$ **10.** $(2x - 1)^2$

11. $(8 - x)^2$ **12.** $(6p - q)^2$ **13.** $(7a - 2b)^2$

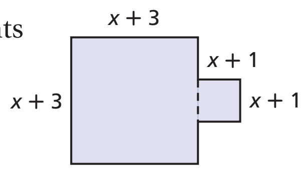
SEE EXAMPLE 3 p. 523

14. $(x + 5)(x - 5)$ **15.** $(x + 6)(x - 6)$ **16.** $(5x + 1)(5x - 1)$

17. $(2x^2 + 3)(2x^2 - 3)$ **18.** $(9 - x^3)(9 + x^3)$ **19.** $(2x - 5y)(2x + 5y)$

SEE EXAMPLE 4 p. 523

20. Geometry Write a polynomial that represents the area of the figure.



PRACTICE AND PROBLEM SOLVING

Independent Practice	
For Exercises	See Example
21–26	1
27–32	2
33–38	3
39	4

Extra Practice
Skills Practice p. S17
Application Practice p. S34

Multiply.

21. $(x + 3)^2$ **22.** $(4 + z)^2$ **23.** $(x^2 + y^2)^2$

24. $(p + 2q^3)^2$ **25.** $(2 + 3x)^2$ **26.** $(r^2 + 5t)^2$

27. $(s^2 - 7)^2$ **28.** $(2c - d^3)^2$ **29.** $(a - 8)^2$

30. $(5 - w)^2$ **31.** $(3x - 4)^2$ **32.** $(1 - x^2)^2$

33. $(a - 10)(a + 10)$ **34.** $(y + 4)(y - 4)$ **35.** $(7x + 3)(7x - 3)$

36. $(x^2 - 2)(x^2 + 2)$ **37.** $(5a^2 + 9)(5a^2 - 9)$ **38.** $(x^3 + y^2)(x^3 - y^2)$

39. **Entertainment** Write a polynomial that represents the area of the circular puzzle. Remember that the formula for area of a circle is $A = \pi r^2$, where r is the radius of the circle. Leave the symbol π in your answer.

40. **Multi-Step** A square has sides that are $(x - 1)$ units long and a rectangle has a length of x units and a width of $(x - 2)$ units.

a. What are the possible values of x? Explain.

b. Which has the greater area, the square or the rectangle?

c. What is the difference in the areas?

Multiply.

41. $(x + y)^2$
42. $(x - y)^2$
43. $(x^2 + 4)(x^2 - 4)$
44. $(x^2 + 4)^2$
45. $(x^2 - 4)^2$
46. $(1 - x)^2$
47. $(1 + x)^2$
48. $(1 - x)(1 + x)$
49. $(x^3 - a^3)(x^3 - a^3)$
50. $(5 + n)(5 + n)$
51. $(6a - 5b)(6a + 5b)$
52. $(r - 4t^4)(r - 4t^4)$

Copy and complete the tables to verify the special products of binomials.

	a	b	$(a - b)^2$	$a^2 - 2ab + b^2$
	1	4	$(1 - 4)^2 = 9$	$1^2 - 2(1)(4) + 4^2 = 9$
53.	2	4	■	■
54.	3	2	■	■

	a	b	$(a + b)^2$	$a^2 + 2ab + b^2$
55.	1	4	■	■
56.	2	5	■	■
57.	3	0	■	■

	a	b	$(a + b)(a - b)$	$a^2 - b^2$
58.	1	4	■	■
59.	2	3	■	■
60.	3	2	■	■

Math History

Beginning about 3000 B.C.E., the Babylonians lived in what is now Iraq and Turkey. Around 575 B.C.E., they built the Ishtar Gate to serve as one of eight main entrances into the city of Babylon. The image above is a relief sculpture from a restoration of the Ishtar Gate.

61. **Math History** The Babylonians used tables of squares and the formula $ab = \frac{(a + b)^2 - (a - b)^2}{4}$ to multiply two numbers. Use this formula to find the product $35 \cdot 24$.

62. **Critical Thinking** Find a value of c that makes $16x^2 - 24x + c$ a perfect-square trinomial.

63. **ERROR ANALYSIS** Explain the error below. What is the correct product?
$(a - b)^2 = a^2 - b^2$

MULTI-STEP TEST PREP

64. This problem will prepare you for the Multi-Step Test Prep on page 528.

a. Michael is fencing part of his yard. He started with a square of length x on each side. He then added 3 feet to the length and subtracted 3 feet from the width. Make a sketch to show the fenced area with the length and width labeled.

b. Write a polynomial that represents the area of the fenced region.

c. Michael bought a total of 48 feet of fencing. What is the area of his fenced region?

65. Critical Thinking The polynomial $ax^2 - 49$ is a difference of two squares. Find all possible values of a between 1 and 100 inclusive.

66. Write About It When is the product of two binomials also a binomial? Explain and give an example.

TEST PREP

67. What is $(5x - 6y)(5x - 6y)$?

Ⓐ $25x^2 - 22xy + 36y^2$ Ⓒ $25x^2 + 22xy + 36y^2$

Ⓑ $25x^2 - 60xy + 36y^2$ Ⓓ $25x^2 + 60xy + 36y^2$

68. Which product is represented by the model?

$25x^2$	$10x$
$10x$	4

Ⓕ $(2x + 5)(2x + 5)$ Ⓗ $(5x + 2)(5x - 2)$

Ⓖ $(5x - 2)(5x - 2)$ Ⓙ $(5x + 2)(5x + 2)$

69. If $a + b = 12$ and $a^2 - b^2 = 96$ what is the value of a?

Ⓐ 2 Ⓑ 4 Ⓒ 8 Ⓓ 10

70. If $rs = 15$ and $(r + s)^2 = 64$, what is the value of $r^2 + s^2$?

Ⓕ 25 Ⓖ 30 Ⓗ 34 Ⓙ 49

CHALLENGE AND EXTEND

71. Multiply $(x + 4)(x + 4)(x - 4)$.

72. Multiply $(x + 4)(x - 4)(x - 4)$.

73. If $x^2 + bx + c$ is a perfect-square trinomial, what is the relationship between b and c?

74. You can multiply two numbers by rewriting the numbers as the difference of two squares. For example:

$$36 \cdot 24 = (30 + 6)(30 - 6) = 30^2 - 6^2 = 900 - 36 = 864$$

Use this method to multiply $27 \cdot 19$. Explain how you rewrote the numbers.

SPIRAL REVIEW

75. The square paper that Yuki is using to make an origami frog has an area of 165 cm^2. Find the side length of the paper to the nearest centimeter. *(Lesson 1-5)*

Use intercepts to graph the line described by each equation. *(Lesson 5-2)*

76. $2x + 3y = 6$

77. $y = -3x + 9$

78. $\frac{1}{2}x + y = 4$

Add or subtract. *(Lesson 7-7)*

79. $3x^2 + 8x - 2x + 9x^2$

80. $(8m^4 + 2n - 3m^3 + 6) + (9m^3 + 5 - 4m^4)$

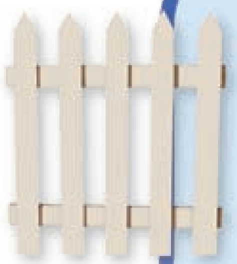
81. $(2p^3 + p) - (5p^3 + 9p)$

82. $(12t - 3t^2 + 10) - (-5t^2 - 7 - 4t)$

CHAPTER 7

MULTI-STEP TEST PREP

SECTION 7B



Polynomials

Don't Fence Me In James has 500 feet of fencing to enclose a rectangular region on his farm for some sheep.

1. Make a sketch of three possible regions that James could enclose and give the corresponding areas.

2. If the length of the region is x, find an expression for the width.

3. Use your answer to Problem 2 to write an equation for the area of the region.

4. Graph your equation from Problem 3 on your calculator. Sketch the graph.

5. James wants his fenced region to have the largest area possible using 500 feet of fencing. Find this area using the graph or a table of values.

6. What are the length and width of the region with the area from Problem 5? Describe this region.

READY TO GO ON?

Quiz for Lessons 7-6 Through 7-9

7-6 Polynomials

Write each polynomial in standard form and give the leading coefficient.

1. $4r^2 + 2r^6 - 3r$ **2.** $y^2 + 7 - 8y^3 + 2y$ **3.** $-12t^3 - 4t + t^4$

4. $n + 3 + 3n^2$ **5.** $2 + 3x^3$ **6.** $-3a^2 + 16 + a^7 + a$

Classify each polynomial according to its degree and number of terms.

7. $2x^3 + 5x - 4$ **8.** $5b^2$ **9.** $6p^2 + 3p - p^4 + 2p^3$

10. $x^2 + 12 - x$ **11.** $-2x^3 - 5 + x - 2x^7$ **12.** $5 - 6b^2 + b - 4b^4$

13. Business The function $C(x) = x^3 - 15x + 14$ gives the cost to manufacture x units of a product. What is the cost to manufacture 900 units?

7-7 Adding and Subtracting Polynomials

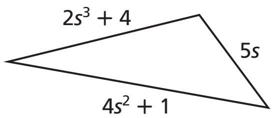
Add or subtract.

14. $(10m^3 + 4m^2) + (7m^2 + 3m)$ **15.** $(3t^2 - 2t) + (9t^2 + 4t - 6)$

16. $(12d^6 - 3d^2) + (2d^4 + 1)$ **17.** $(6y^3 + 4y^2) - (2y^2 + 3y)$

18. $(7n^2 - 3n) - (5n^2 + 5n)$ **19.** $(b^2 - 10) - (-5b^3 + 4b)$

20. Geometry The measures of the sides of a triangle are shown as polynomials. Write a simplified polynomial to represent the perimeter of the triangle.



7-8 Multiplying Polynomials

Multiply.

21. $2h^3 \cdot 5h^5$ **22.** $(s^8t^4)(-6st^3)$ **23.** $2ab(5a^3 + 3a^2b)$

24. $(3k + 5)^2$ **25.** $(2x^3 + 3y)(4x^2 + y)$ **26.** $(p^2 + 3p)(9p^2 - 6p - 5)$

27. Geometry Write a simplified polynomial expression for the area of a parallelogram whose base is $(x + 7)$ units and whose height is $(x - 3)$ units.

7-9 Special Products of Binomials

Multiply.

28. $(d + 9)^2$ **29.** $(3 + 2t)^2$ **30.** $(2x + 5y)^2$ **31.** $(m - 4)^2$

32. $(a - b)^2$ **33.** $(3w - 1)^2$ **34.** $(c + 2)(c - 2)$ **35.** $(5r + 6)(5r - 6)$

36. Sports A child's basketball has a radius of $(x - 5)$ inches. Write a polynomial that represents the surface area of the basketball. (The formula for the surface area of a sphere is $S = 4\pi r^2$, where r represents the radius of the sphere.) Leave the symbol π in your answer.

CHAPTER 7

Study Guide: Review

Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. A(n) ___?___ polynomial is a polynomial of degree 3.
2. When a polynomial is written with the terms in order from highest to lowest degree, it is in ___?___.
3. A(n) ___?___ is a number, a variable, or a product of numbers and variables with whole-number exponents.
4. A(n) ___?___ is a polynomial with three terms.
5. ___?___ is a method of writing numbers that are very large or very small.

7-1 Integer Exponents *(pp. 460–465)*

EXAMPLES

Simplify.

- -2^{-4}

  $-2^{-4} = -\frac{1}{2^4} = -\frac{1}{2 \cdot 2 \cdot 2 \cdot 2} = -\frac{1}{16}$

- 3^0

  $3^0 = 1$ *Any nonzero number raised to the zero power is 1.*

- **Evaluate r^3s^{-4} for $r = -3$ and $s = 2$.**

  r^3s^{-4}

  $(-3)^3(2)^{-4} = \frac{(-3)(-3)(-3)}{2 \cdot 2 \cdot 2 \cdot 2} = -\frac{27}{16}$

- **Simplify $\frac{a^{-3}b^4}{c^{-2}}$.**

  $\frac{a^{-3}b^4}{c^{-2}} = \frac{b^4c^2}{a^3}$

EXERCISES

6. The diameter of a certain bearing is 2^{-5} in. Simplify this expression.

Simplify.

7. $(3.6)^0$
8. $(-1)^{-4}$
9. 5^{-3}
10. 10^{-4}

Evaluate each expression for the given value(s) of the variable(s).

11. b^{-4} for $b = 2$
12. $\left(\frac{2}{5}b\right)^{-4}$ for $b = 10$
13. $-2p^3q^{-3}$ for $p = 3$ and $q = -2$

Simplify.

14. m^{-2}
15. bc^0
16. $-\frac{1}{2}x^{-2}y^{-4}$
17. $\frac{2b^6}{c^{-4}}$
18. $\frac{3a^2c^{-2}}{4b^0}$
19. $\frac{q^{-1}r^{-2}}{s^{-3}}$

7-2 Powers of 10 and Scientific Notation *(pp. 466–471)*

EXAMPLES

- **Write 1,000,000 as a power of 10.**

1,000,000 — *The decimal point is 6 places to the right of 1.*

$1{,}000{,}000 = 10^6$

- **Find the value of 386.21×10^5.**

386.2 1 0 0 0 — *Move the decimal point 5 places to the right.*

38,621,000

- **Write 0.000000041 in scientific notation.**

0.0 0 0 0 0 0 0 4 1 — *Move the decimal point 8 places to the right to get a number between 1 and 10.*

4.1×10^{-8}

EXERCISES

Find the value of each power of 10.

20. 10^7 **21.** 10^{-5}

Write each number as a power of 10.

22. 100 **23.** 0.00000000001

Find the value of each expression.

24. 3.25×10^5 **25.** 0.18×10^4

26. 17×10^{-2} **27.** 299×10^{-6}

28. Order the list of numbers from least to greatest. 6.3×10^{-3}, 1.2×10^4, 5.8×10^{-7}, 2.2×10^2

29. In 2003, the average daily value of shares traded on the New York Stock Exchange was about $\$3.85 \times 10^{10}$. Write this amount in standard form.

7-3 Multiplication Properties of Exponents *(pp. 474–480)*

EXAMPLES

Simplify.

- $5^3 \cdot 5^{-2}$

$5^3 \cdot 5^{-2}$ — *The powers have the same base.*

$5^{3+(-2)}$ — *Add the exponents.*

5^1

5

- $a^4 \cdot b^{-3} \cdot b \cdot a^{-2}$

$a^4 \cdot b^{-3} \cdot b \cdot a^{-2}$ — *Use properties to group factors.*

$(a^4 \cdot a^{-2}) \cdot (b^{-3} \cdot b)$

$a^2 \cdot b^{-2}$ — *Add the exponents of powers with the same base.*

$\frac{a^2}{b^2}$ — *Write with a positive exponent.*

- $(a^{-3}b^2)^{-2}$

$(a^{-3})^{-2} \cdot (b^2)^{-2}$ — *Power of a Product Property*

$a^6 \cdot b^{-4}$ — *Power of a Power Property*

$\frac{a^6}{b^4}$ — *Write with a positive exponent.*

EXERCISES

Simplify.

30. $5^3 \cdot 5^6$ **31.** $2^6 \cdot 3 \cdot 2^{-3} \cdot 3^3$

32. $b^2 \cdot b^8$ **33.** $r^4 \cdot r$

34. $(x^3)^4$ **35.** $(s^3)^0$

36. $(2^3)^{-1}$ **37.** $(5^2)^{-2}$

38. $(4b^3)^{-2}$ **39.** $(g^3h^2)^4$

40. $(-x^2y)^2$ **41.** $-(x^2y)^2$

42. $(x^2y^3)(xy^3)^4$ **43.** $(j^2k^3)(j^4k^6)$

44. $(5^3 \cdot 5^{-2})^{-1}$ **45.** $(mn^3)^5(mn^5)^3$

46. $(4 \times 10^8)(2 \times 10^3)$ **47.** $(3 \times 10^2)(3 \times 10^5)$

48. $(5 \times 10^3)(2 \times 10^6)$ **49.** $(7 \times 10^5)(4 \times 10^9)$

50. $(3 \times 10^{-4})(2 \times 10^5)$ **51.** $(3 \times 10^{-8})(6 \times 10^{-1})$

52. In 2003, Wyoming's population was about 5.0×10^5. California's population was about 7.1×10 times as large as Wyoming's. What was the approximate population of California? Write your answer in scientific notation.

7-4 Division Properties of Exponents *(pp. 481–487)*

EXAMPLES

- Simplify $\frac{x^9}{x^2}$.

$\frac{x^9}{x^2} = x^{9-2} = x^7$ *Subtract the exponents.*

EXERCISES

Simplify.

53. $\frac{2^8}{2^2}$ **54.** $\frac{m^6}{m}$ **55.** $\frac{2^6 \cdot 4 \cdot 7^3}{2^5 \cdot 4^4 \cdot 7^2}$

56. $\frac{24b^6}{4b^5}$ **57.** $\frac{t^4v^5}{tv}$ **58.** $\left(\frac{1}{2}\right)^{-4}$

Simplify each quotient and write the answer in scientific notation.

59. $(2.5 \times 10^8) \div (0.5 \times 10^7)$

60. $(2 \times 10^{10}) \div (8 \times 10^2)$

7-5 Rational Exponents *(pp. 488–493)*

EXAMPLES

- Simplify $\sqrt[3]{r^6s^{12}}$.

$\sqrt[3]{r^6s^{12}} = (r^6s^{12})^{\frac{1}{3}}$ *Definition of $b^{\frac{1}{n}}$*

$= (r^6)^{\frac{1}{3}} \cdot (s^{12})^{\frac{1}{3}}$ *Power of a Product Property*

$= \left(r^{6 \cdot \frac{1}{3}}\right) \cdot \left(s^{12 \cdot \frac{1}{3}}\right)$ *Power of a Power Property*

$= (r^2) \cdot (s^4)$ *Simplify exponents.*

$= r^2s^4$

EXERCISES

Simplify each expression.

61. $81^{\frac{1}{2}}$ **62.** $343^{\frac{1}{3}}$

63. $64^{\frac{2}{3}}$ **64.** $(2^6)^{\frac{1}{2}}$

Simplify each expression. All variables represent nonnegative numbers.

65. $\sqrt[5]{z^{10}}$ **66.** $\sqrt[3]{125x^6}$

67. $\sqrt{x^8y^6}$ **68.** $\sqrt[3]{m^6n^{12}}$

7-6 Polynomials *(pp. 496–501)*

EXAMPLES

- Find the degree of the polynomial $3x^2 + 8x^5$.

$3x^2 + 8x^5$ *$8x^5$ has the highest degree.*

The degree is 5.

- Classify the polynomial $y^3 - 2y$ according to its degree and number of terms.

Degree: 3
Terms: 2

The polynomial $y^3 - 2y$ is a cubic binomial.

EXERCISES

Find the degree of each polynomial.

69. 5 **70.** $8st^3$

71. $3z^6$ **72.** $6h$

Write each polynomial in standard form. Then give the leading coefficient.

73. $2n - 4 + 3n^2$ **74.** $2a - a^4 - a^6 + 3a^3$

Classify each polynomial according to its degree and number of terms.

75. $2s - 6$ **76.** $-8p^5$

77. $-m^4 - m^2 - 1$ **78.** 2

7-7 Adding and Subtracting Polynomials *(pp. 504–509)*

EXAMPLES

Add.

- $(h^3 - 2h) + (3h^2 + 4h) - 2h^3$

  $(h^3 - 2h) + (3h^2 + 4h) - 2h^3$

  $(h^3 - 2h^3) + (3h^2) + (4h - 2h)$

  $-h^3 + 3h^2 + 2h$

Subtract.

- $(n^3 + 5 - 6n^2) - (3n^2 - 7)$

  $(n^3 + 5 - 6n^2) + (-3n^2 + 7)$

  $(n^3 + 5 - 6n^2) + (-3n^2 + 7)$

  $n^3 + (-6n^2 - 3n^2) + (5 + 7)$

  $n^3 - 9n^2 + 12$

EXERCISES

Add or subtract.

79. $3t + 5 - 7t - 2$

80. $4x^5 - 6x^6 + 2x^5 - 7x^5$

81. $-h^3 - 2h^2 + 4h^3 - h^2 + 5$

82. $(3m - 7) + (2m^2 - 8m + 6)$

83. $(12 + 6p) - (p - p^2 + 4)$

84. $(3z - 9z^2 + 2) + (2z^2 - 4z + 8)$

85. $(10g - g^2 + 3) - (-4g^2 + 8g - 1)$

92. $(-5x^3 + 2x^2 - x + 5) - (-5x^3 + 3x^2 - 5x - 3)$

7-8 Multiplying Polynomials *(pp. 512–519)*

EXAMPLES

Multiply.

- $(2x - 4)(3x + 5)$

  $2x(3x) + 2x(5) - 4(3x) - 4(5)$

  $6x^2 + 10x - 12x - 20$

  $6x^2 - 2x - 20$

- $(b - 2)(b^2 + 4b - 5)$

  $b(b^2) + b(4b) - b(5) - 2(b^2) - 2(4b) - 2(-5)$

  $b^3 + 4b^2 - 5b - 2b^2 - 8b + 10$

  $b^3 + 2b^2 - 13b + 10$

EXERCISES

Multiply.

87. $(2r)(4r)$

88. $(3a^5)(2ab)$

89. $(-3xy)(-6x^2y)$

90. $(3s^3t^2)(2st^4)\left(\frac{1}{2}s^2t^8\right)$

91. $2(x^2 - 4x + 6)$

92. $-3ab(ab - 2a^2b + 5a)$

93. $(a + 3)(a - 6)$

94. $(b - 9)(b + 3)$

95. $(x - 10)(x - 2)$

96. $(t - 1)(t + 1)$

97. $(2q + 6)(4q + 5)$

98. $(5g - 8)(4g - 1)$

7-9 Special Products of Binomials *(pp. 521–527)*

EXAMPLES

Multiply.

- $(2h - 6)^2$

  $(2h - 6)^2 = (2h)^2 + 2(2h)(-6) + (-6)^2$

  $4h^2 - 24h + 36$

- $(4x - 3)(4x + 3)$

  $(4x - 3)(4x + 3) = (4x)^2 - 3^2$

  $16x^2 - 9$

EXERCISES

Multiply.

99. $(p - 4)^2$

100. $(x + 12)^2$

101. $(m + 6)^2$

102. $(3c + 7)^2$

103. $(2r - 1)^2$

104. $(3a - b)^2$

105. $(2n - 5)^2$

106. $(h - 13)^2$

107. $(x - 1)(x + 1)$

108. $(z + 15)(z - 15)$

109. $(c^2 - d)(c^2 + d)$

110. $(3k^2 + 7)(3k^2 - 7)$

CHAPTER 7

CHAPTER TEST

Evaluate each expression for the given value(s) of the variable(s).

1. $\left(\frac{1}{3}b\right)^{-2}$ for $b = 12$
2. $\left(14 - a^0b^2\right)^{-3}$ for $a = -2$ and $b = 4$

Simplify.

3. $2r^{-3}$
4. $-3f^0g^{-1}$
5. m^2n^{-3}
6. $\frac{1}{2}s^{-5}t^3$

Write each number as a power of 10.

7. 0.0000001
8. 10,000,000,000,000
9. 1

Find the value of each expression.

10. 1.25×10^{-5}
11. $10^8 \times 10^{-11}$
12. 325×10^{-2}

13. **Technology** In 2002, there were approximately 544,000,000 Internet users worldwide. Write this number in scientific notation.

Simplify.

14. $\left(f^4\right)^3$
15. $\left(4b^2\right)^0$
16. $\left(a^3b^6\right)^6$
17. $-\left(x^3\right)^5 \cdot \left(x^2\right)^6$

Simplify each quotient and write the answer in scientific notation.

18. $\left(3.6 \times 10^9\right) \div \left(6 \times 10^4\right)$
19. $\left(3 \times 10^{12}\right) \div \left(9.6 \times 10^{16}\right)$

Simplify.

20. $\frac{y^4}{y}$
21. $\frac{d^2f^5}{\left(d^3\right)^2f^{-4}}$
22. $\frac{2^5 \cdot 3^3 \cdot 5^4}{2^8 \cdot 3^2 \cdot 5^4}$
23. $\left(\frac{4s}{3t}\right)^{-2} \cdot \left(\frac{2s}{6t}\right)^2$

24. **Geometry** The surface area of a cone is approximated by the polynomial $3.14r^2 + 3.14r\ell$, where r is the radius and ℓ is the slant height. Find the approximate surface area of a cone when $\ell = 5$ cm and $r = 3$ cm.

Simplify each expression. All variables represent nonnegative numbers.

25. $\left(\frac{27}{125}\right)^{\frac{1}{3}}$
26. $\sqrt[3]{43^3}$
27. $\sqrt{25y^8}$
28. $\sqrt[5]{3^5t^{10}}$

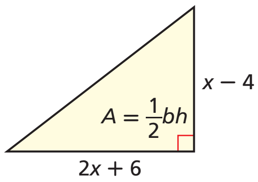
Add or subtract.

29. $3a - 4b + 2a$
30. $\left(2b^2 - 4b^3\right) - \left(6b^3 + 8b^2\right)$
31. $-9g^2 + 3g - 4g^3 - 2g + 3g^2 - 4$

Multiply.

32. $-5\left(r^2s - 6\right)$
33. $(2t - 7)(t + 4)$
34. $(4g - 1)\left(4g^2 - 5g - 3\right)$
35. $(m + 6)^2$
36. $(3t - 7)(3t + 7)$
37. $\left(3x^2 - 7\right)^2$

38. **Carpentry** Carpenters use a tool called a *speed square* to help them mark right angles. A speed square is a right triangle.
    a. Write a polynomial that represents the area of the speed square shown.
    b. Find the area when $x = 4.5$ in.



COLLEGE ENTRANCE EXAM PRACTICE

FOCUS ON SAT

When you receive your SAT scores, you will find a percentile for each score. The percentile tells you what percent of students scored lower than you on the same test. Your percentile at the national and state levels may differ because of the different groups being compared.

You may want to time yourself as you take this practice test. It should take you about 7 minutes to complete.

HOT TIP! You may use some types of calculators on the math section of the SAT. For about 40% of the test items, a graphing calculator is recommended. Bring a calculator that you are comfortable using. You won't have time to figure out how a new calculator works.

---

1. If $(x + 1)(x + 4) - (x - 1)(x - 2) = 0$, what is the value of x?

(A) -1

(B) $-\frac{1}{4}$

(C) 0

(D) $\frac{1}{4}$

(E) 1

---

2. Which of the following is equal to 4^5?

I. $3^5 \times 1^5$

II. 2^{10}

III. $4^0 \times 4^5$

(A) I only

(B) II only

(C) I and II only

(D) II and III only

(E) I, II, and III

---

3. If $x^{-4} = 81$, then $x =$

(A) -3

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) 3

(E) 9

---

4. What is the value of $2x^3 - 4x^2 + 3x + 1$ when $x = -2$?

(A) -37

(B) -25

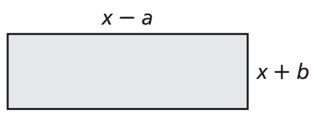
(C) -5

(D) 7

(E) 27

---

5. What is the area of a rectangle with a length of $x - a$ and a width of $x + b$?



(A) $x^2 - a^2$

(B) $x^2 + b^2$

(C) $x^2 - abx + ab$

(D) $x^2 - ax - bx - ab$

(E) $x^2 + bx - ax - ab$

---

6. For integers greater than 0, define the following operations.

$$a \square b = 2a^2 + 3b$$

$$a \triangle b = 5a^2 - 2b$$

What is $(a \square b) + (a \triangle b)$?

(A) $7a^2 + b$

(B) $-3a^2 + 5b$

(C) $7a^2 - b$

(D) $3a^2 - 5b$

(E) $-3a^2 - b$

Any Question Type: Use a Diagram

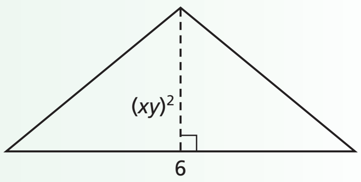
When a test item includes a diagram, use it to help solve the problem. Gather as much information from the drawing as possible. However, keep in mind that diagrams are not always drawn to scale and can be misleading.

EXAMPLE 1

Multiple Choice **What is the height of the triangle when $x = 4$ and $y = 1$?**

Ⓐ 2 Ⓒ 8

Ⓑ 4 Ⓓ 16



In the diagram, the height appears to be less than 6, so you might eliminate choices C and D. However, doing the math shows that the height is actually greater than 6. Do not rely solely on visual information. Always use the numbers given in the problem.

The height of the triangle is $(xy)^2$.

When $x = 4$ and $y = 1$, $(xy)^2 = (4 \cdot 1)^2 = (4)^2 = 16$.

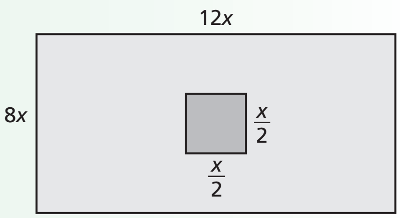
Choice D is the correct answer.

If a test item does not have a diagram, draw a quick sketch of the problem situation. Label your diagram with the data given in the problem.

EXAMPLE 2

Short Response **A square placemat is lying in the middle of a rectangular table. The side length of the placemat is $\left(\frac{x}{2}\right)$. The length of the table is $12x$, and the width is $8x$. Write a polynomial to represent the area of the placemat. Then write a polynomial to represent the area of the table that surrounds the placemat.**

Use the information in the problem to draw and label a diagram. Then write the polynomials.



area of placemat $= s^2 = \left(\frac{x}{2}\right)^2 = \left(\frac{x}{2}\right)\left(\frac{x}{2}\right) = \frac{x^2}{4}$

area of table $= \ell w = (12x)(8x) = 96x^2$

area of table $-$ area of placemat $= 96x^2 - \frac{x^2}{4} = \frac{384x^2 - x^2}{4} = \frac{383x^2}{4}$

The area of the placemat is $\frac{x^2}{4}$.

The area of the table that surrounds the placemat is $\frac{383x^2}{4}$.

HOT TIP! If a given diagram does not reflect the problem, draw a sketch that is more accurate. If a test item does not have a diagram, use the given information to sketch your own. Try to make your sketch as accurate as possible.

Read each test item and answer the questions that follow.

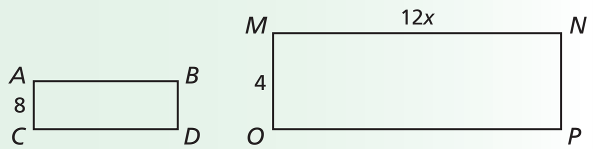
Item A

Short Response The width of a rectangle is 1.5 feet more than 4 times its length. Write a polynomial expression for the area of the rectangle. What is the area when the length is 16.75 feet?

1. What is the unknown measure in this problem?
2. How will drawing a diagram help you solve the problem?
3. Draw and label a sketch of the situation.

Item B

Multiple Choice Rectangle *ABDC* is similar to rectangle *MNPO*. If the width of rectangle *ABDC* is 8, what is its length?



Ⓐ 2

Ⓑ $2x$

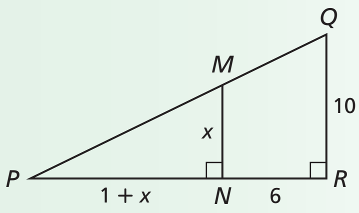
Ⓒ $24x$

Ⓓ 24

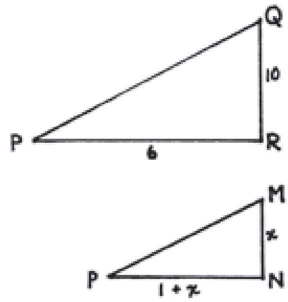
4. Look at the dimensions in the diagram. Do you think that the length of rectangle *ABDC* is greater or less than the length of rectangle *MNPO*?
5. Do you think the drawings reflect the information in the problem accurately? Why or why not?
6. Draw your own sketch to match the information in the problem.

Item C

Short Response Write a polynomial expression for the area of triangle *QRP*. Write a polynomial expression for the area of triangle *MNP*. Then use these expressions to write a polynomial expression for the area of *QRNM*.

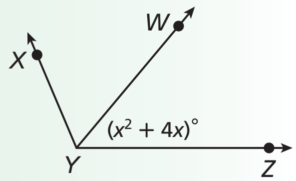


7. Describe how redrawing the figure can help you better understand the information in the problem.
8. After reading this test item, a student redrew the figure as shown below. Is this a correct interpretation of the original figure? Explain.



Item D

Multiple Choice The measure of angle *XYZ* is $(x^2 + 10x + 15)^\circ$. What is the measure of angle *XYW*?



Ⓕ $(6x + 15)^\circ$

Ⓖ $(2x^2 + 14x + 15)^\circ$

Ⓗ $(14x + 15)^\circ$

Ⓙ $(6x^2 + 15)^\circ$

9. What information does the diagram provide that the problem does not?
10. Will the measure of angle *XYW* be less than or greater than the measure of angle *XYZ*? Explain.

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CUMULATIVE ASSESSMENT, CHAPTERS 1–7

Multiple Choice

1. A negative number is raised to a power. The result is a negative number. What do you know about the power?

Ⓐ It is an even number.

Ⓑ It is an odd number.

Ⓒ It is zero.

Ⓓ It is a whole number.

2. Which expression represents the phrase *eight less than the product of a number and two*?

Ⓕ $2 - 8x$

Ⓖ $8 - 2x$

Ⓗ $2x - 8$

Ⓙ $\frac{x}{2} - 8$

3. An Internet service provider charges a \$20 set-up fee plus \$12 per month. A competitor charges \$15 per month. Which equation can you use to find x, the number of months when the total charge will be the same for both companies?

Ⓐ $15 = 20 + 12x$

Ⓑ $20 + 12x = 15x$

Ⓒ $20x + 12 = 15x$

Ⓓ $20 = 15x + 12x$

4. Which is a solution of the inequality $7 - 3(x - 3) > 2(x + 3)$?

Ⓕ 0

Ⓖ 2

Ⓗ 5

Ⓙ 12

5. One dose of Ted's medication contains 0.625 milligram, or $\frac{5}{8}$ milligram, of a drug. Which expression is equivalent to 0.625?

Ⓐ $5(4)^{-2}$

Ⓑ $5(2)^{-4}$

Ⓒ $5(-2)^{3}$

Ⓓ $5(2)^{-3}$

6. A restaurant claims to have served 352×10^6 hamburgers. What is this number in scientific notation?

Ⓕ 3.52×10^6

Ⓖ 3.52×10^8

Ⓗ 3.52×10^4

Ⓙ 352×10^6

7. Janet is ordering game cartridges from an online retailer. The retailer's prices, including shipping and handling, are given in the table below.

Game Cartridges	Total Cost (\$)
1	54.95
2	104.95
3	154.95
4	204.95

Which equation best describes the relationship between the total cost c and the number of game cartridges g?

Ⓐ $c = 54.95g$

Ⓑ $c = 51g + 0.95$

Ⓒ $c = 50g + 4.95$

Ⓓ $c = 51.65g$

8. Which equation describes a line parallel to $y = 5 - 2x$?

Ⓕ $y = -2x + 8$

Ⓖ $y = 2x - 5$

Ⓗ $y = 5 + \frac{1}{2}x$

Ⓙ $y = 5 - \frac{1}{2}x$

9. A square has sides of length $x - 4$. A rectangle has a length of $x + 2$ and a width of $2x - 1$. What is the total combined area of the square and the rectangle?

Ⓐ $10x - 14$

Ⓑ $4x - 3$

Ⓒ $3x^2 - 5x + 14$

Ⓓ $3x^2 + 3x - 18$

HOT TIP! Test writers develop multiple-choice test options with distracters. Distracters are incorrect options that are based on common student errors. Be cautious! Even if the answer you calculated is one of the options, it may not be the correct answer. Always check your work carefully.

10. Jennifer has a pocketful of change, all in nickels and quarters. There are 11 coins with a total value of $1.15. Which system of equations can you use to find the number of each type of coin?

(F) $\begin{cases} n + q = 11 \\ n + q = 1.15 \end{cases}$

(G) $\begin{cases} n + q = 11 \\ 5n + 25q = 1.15 \end{cases}$

(H) $\begin{cases} 5n + 25q = 11 \\ n + q = 1.15 \end{cases}$

(J) $\begin{cases} n + q = 11 \\ 0.05n + 0.25q = 1.15 \end{cases}$

11. Which of the following is a true statement?

(A) $\left[(a^m)^n\right]^p = a^{m+n+p}$

(B) $\left[(a^m)^n\right]^p = a^{mn+p}$

(C) $\left[(a^m)^n\right]^p = a^{mnp}$

(D) $\left[(a^m)^n\right]^p = (a^{m+n})^p$

12. In 1867, the United States purchased the Alaska Territory from Russia for $\$7.2 \times 10^6$. The total area was about 6×10^5 square miles. What was the price per square mile?

(F) About $0.12 per square mile

(G) About $1.20 per square mile

(H) About $12.00 per square mile

(J) About $120.00 per square mile

Gridded Response

13. Evaluate the expression $3b^{-2}c^0$ for $b = 2$ and $c = -3$.

14. What is the slope of the line described by $-3y = -6x - 12$?

15. The quotient $(5.6 \times 10^8) \div (8 \times 10^3)$ is written in scientific notation as (7×10^n). What is the value of n?

16. The volume of a plastic cylinder is 64 cubic centimeters. A glass cylinder has the same height and a radius that is half that of the plastic cylinder. What is the volume in cubic centimeters of the glass cylinder?

Short Response

17. A sweater that normally sells for $35 was marked down 20% and placed on the sale rack. Later, the sweater was marked down an additional 30% and placed on the clearance rack.

a. Find the price of the sweater while on the sale rack. Show your work.

b. Find the price of the sweater while on the clearance rack. Show your work.

18. A set of positive integers (a, b, c) is called a *Pythagorean triple* if $a^2 + b^2 = c^2$.

a. Find a^2, b^2, and c^2 when $a = 2x$, $b = x^2 - 1$, and $c = x^2 + 1$. Show your work.

b. Is $(2x, x^2 - 1, x^2 + 1)$ a Pythagorean triple? Explain your reasoning.

19. Ron is making an ice sculpture. The block of ice is in the shape of a rectangular prism with a length of $(x + 2)$ inches, a width of $(x - 2)$ inches, and a height of $2x$ inches.

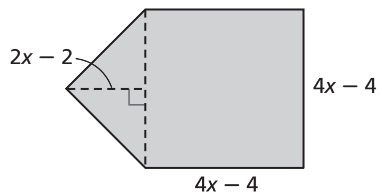
a. Write and simplify a polynomial expression for the volume of the block of ice. Show your work.

b. The final volume of the ice sculpture is $(x^3 + 4x^2 - 10x + 1)$ cubic inches. Write an expression for the volume of ice that Ron carved away. Show your work.

20. Simplify the expression $(3 \cdot a^2 \cdot b^{-4} \cdot a \cdot b^{-3})^{-3}$ using two different methods. Show that the results are the same.

Extended Response

21. Look at the pentagon below.



a. Write and simplify an expression that represents the area of the pentagon. Show your work or explain your answer.

b. Show one method of checking that your expression in part **a** is correct.

c. The triangular part of the pentagon can be rearranged to form a square. Write the area of this square as the square of a binomial.

d. Expand the product that you wrote in part **c.** What type of polynomial is this?

e. Is the square of a binomial ever a binomial? Explain your reasoning.