CHAPTER

8

Factoring Polynomials

Chapter Focus

- Factor polynomials.
- Apply factoring techniques to solve problems involving area and volume.

High Fliers

You can use polynomials to model area. When given the area of a kite as a polynomial, you can factor to find the kite's dimensions.

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ARE YOU READY?

✓ Vocabulary

Match each term on the left with a definition on the right.

1. binomial
2. composite number
3. factor
4. multiple
5. prime number

A. a whole number greater than 1 that has more than two positive factors

B. a polynomial with two terms

C. the product of any number and a whole number

D. a number that is written as the product of its prime factors

E. a whole number greater than 1 that has exactly two positive factors, itself and 1

F. a number that is multiplied by another number to get a product

✓ Multiples

Write the first four multiples of each number.

6. 3   **7.** 4   **8.** 8   **9.** 15

✓ Factors

Tell whether the second number is a factor of the first number.

10. 20, 5   **11.** 50, 6   **12.** 120, 8   **13.** 245, 7

✓ Prime and Composite Numbers

Tell whether each number is prime or composite. If the number is composite, write it as the product of two numbers.

14. 2   **15.** 7   **16.** 10   **17.** 38

18. 115   **19.** 147   **20.** 151   **21.** 93

✓ Multiply Monomials and Polynomials

Multiply.

22. $2(x + 5)$ **23.** $3h(h + 1)$ **24.** $xy(x^2 - xy^3)$ **25.** $6m(m^2 - 4m - 1)$

✓ Multiply Binomials

Find each product.

26. $(x + 3)(x + 8)$ **27.** $(b - 7)(b + 1)$

28. $(2p - 5)(p - 1)$ **29.** $(3n + 4)(2n + 3)$

Study Guide: Preview

Where You've Been

Previously, you
- used properties of exponents to evaluate and simplify expressions.
- added and subtracted polynomials by combining like terms.
- multiplied polynomials.

In This Chapter

You will study
- greatest common factors.
- how to factor polynomials.
- how to factor special products.
- how to choose a factoring method.

Where You're Going

You can use the skills in this chapter
- in Geometry to solve area problems.
- in Physics to solve quadratic equations.
- in the real world to calculate dimensions in landscaping, construction, or design work.

Key Vocabulary/Vocabulario

greatest common factor	máximo común divisor
prime factorization	factorización prima

Vocabulary Connections

To become familiar with the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word *factor* refers to a number or polynomial that is multiplied by another number or polynomial to form a product. What do you think the word *factor* means when it is used as a verb (action word)?

2. List some words that end with the suffixes *-ize* or *-ization*. What does the ending *-ization* seem to mean? What do you think *factorization* means?

3. The words *prime, primer, primary*, and *primitive* all come from the same root word. What are the meanings of these words? How can their meanings help you understand what a *prime factor* is?

4. What is a prime number? How might the **prime factorization** of a number differ from another factorization?

5. What does the word *common* mean? How can you use this meaning to understand the term **greatest common factor**?

Reading and Writing Math

Reading Strategy: Read a Lesson for Understanding

To help you learn new concepts, you should read each lesson with a purpose. As you read a lesson, make notes. Include the main ideas of the lesson and any questions you have. In class, listen for explanations of the vocabulary, clarification of the examples, and answers to your questions.

Reading Tips

Objectives
Evaluate and multiply by powers of 10.

Convert between standard notation and scientific notation.

> The objectives tell you the main idea of the lesson.

If a power of 10 has a negative integer exponent, does that make the number negative?
How do I enter numbers written in scientific notation into my calculator?

> Write down questions you have as you read the lesson.

EXAMPLE 1 Evaluating Powers of 10

Find the value of each power of 10.

A 10^{-3}

Start with 1 and move the decimal point three places to the left.

0. 0 0 1

0.001

> Work through the examples and write down any questions you have.

CHECK IT OUT!

> Practice what you've learned in the Check It Out sections.

Try This

Read Lesson 8-1 prior to your next class. Then answer the questions below.

1. What are the lesson objectives?

2. What vocabulary, formulas, and symbols are new?

3. Which examples, if any, are unclear?

4. What questions do you have about the lesson?

Factors and Greatest Common Factors

Who uses this?
Web site designers who sell electronic greeting cards can use the greatest common factor of numbers to design their Web sites. (See Example 4.)

The numbers that are multiplied to find a product are called *factors* of that product. A number is divisible by its factors.

You can use the factors of a number to write the number as a product. The number 12 can be factored several ways.

Remember!

A prime number is a whole number that has exactly two positive factors, itself and 1. The number 1 is not prime because it only has one positive factor.

Factorizations of 12

$1 \cdot 12$ $2 \cdot 6$ $3 \cdot 4$ $1 \cdot 4 \cdot 3$ $2 \cdot 2 \cdot 3$

The order of the factors does not change the product, but there is only one example above that cannot be factored further. The circled factorization is the **prime factorization** because all the factors are prime numbers. The prime factors can be written in any order, and, except for changes in the order, there is only one way to write the prime factorization of a number.

E X A M P L E 1 **Writing Prime Factorizations**

Write the prime factorization of 60.

Method 1 Factor tree
Choose any two factors of 60 to begin. Keep finding factors until each branch ends in a prime factor.

60
2 • 30
10 • 3
2 • 5

$60 = 2 \cdot 2 \cdot 5 \cdot 3$
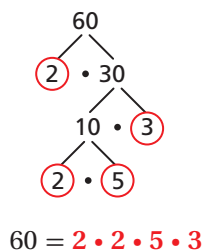
Method 2 Ladder diagram
Choose a prime factor of 60 to begin. Keep dividing by prime factors until the quotient is 1.

2	60
3	30
2	10
5	5
	1

$60 = 2 \cdot 3 \cdot 2 \cdot 5$
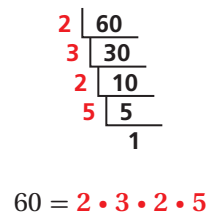
The prime factorization of 60 is $2 \cdot 2 \cdot 3 \cdot 5$ or $2^2 \cdot 3 \cdot 5$.

CHECK IT OUT! Write the prime factorization of each number.
1a. 40   **1b.** 33   **1c.** 49   **1d.** 19

Factors that are shared by two or more whole numbers are called *common factors*. The greatest of these common factors is called the **greatest common factor**, or GCF.

Factors of 12: **1**, **2**, 3, **4**, 6, 12

Factors of 32: **1**, **2**, **4**, 8, 16, 32

Common factors: **1**, **2**, ④

The greatest of the common factors is 4.

E X A M P L E **2** **Finding the GCF of Numbers**

Find the GCF of each pair of numbers.

A **24 and 60**

Method 1 List the factors.

factors of 24: **1**, **2**, **3**, **4**, **6**, 8, ⑫, 24   *List all the factors.*

factors of 60: **1**, **2**, **3**, **4**, 5, **6**, 10, ⑫, 15, 20, 30, 60   *Circle the GCF.*

The GCF of 24 and 60 is 12.

B **18 and 27**

Method 2 Use prime factorization.

$18 = 2 \cdot 3 \cdot 3$ *Write the prime factorization of each number.*

$27 = \quad 3 \cdot 3 \cdot 3$ *Align the common factors.*

$3 \cdot 3 = 9$

The GCF of 18 and 27 is 9.

CHECK IT OUT! **Find the GCF of each pair of numbers.**

2a. 12 and 16   **2b.** 15 and 25

You can also find the GCF of monomials that include variables. To find the GCF of monomials, write the prime factorization of each coefficient and write all powers of variables as products. Then find the product of the common factors.

E X A M P L E **3** **Finding the GCF of Monomials**

Find the GCF of each pair of monomials.

A $3x^3$ **and** $6x^2$

$3x^3 = \quad 3 \cdot x \cdot x \cdot x$ *Write the prime factorization of each*

$6x^2 = 2 \cdot 3 \cdot x \cdot x$ *coefficient and write powers as products.*

          Align the common factors.

$3 \cdot x \cdot x = 3x^2$ *Find the product of the common factors.*

The GCF of $3x^3$ and $6x^2$ is $3x^2$.

B $4x^2$ **and** $5y^3$

$4x^2 = 2 \cdot 2 \cdot \quad x \cdot x$ *Write the prime factorization of each*

$5y^3 = \quad 5 \cdot \quad y \cdot y \cdot y$ *coefficient and write powers as products.*

          Align the common factors.

          There are no common factors other than 1.

The GCF of $4x^2$ and $5y^3$ is 1.

Helpful Hint

If two terms contain the same variable raised to different powers, the GCF will contain that variable raised to the lower power.

CHECK IT OUT! **Find the GCF of each pair of monomials.**

3a. $18g^2$ and $27g^3$ **3b.** $16a^6$ and $9b$ **3c.** $8x$ and $7v^2$

EXAMPLE 4 *Technology Application*

Garrison is creating a Web page that offers electronic greeting cards. He has 24 special occasion designs and 42 birthday designs. The cards will be displayed with the same number of designs in each row. Special occasion and birthday designs will not appear in the same row. How many rows will there be if Garrison puts the greatest possible number of designs in each row?

The 24 special occasion designs and 42 birthday designs must be divided into groups of equal size. The number of designs in each row must be a common factor of 24 and 42.

factors of 24: **1, 2, 3,** 4, **6,** 8, 12, 24

factors of 42: **1, 2, 3, 6,** 7, 14, 21, 42

Find the common factors of 24 and 42.

The GCF of 24 and 42 is 6.

The greatest possible number of designs in each row is 6. Find the number of rows of each group of designs when there are 6 designs in each row.

$$\frac{24 \text{ special occasion designs}}{6 \text{ designs per row}} = 4 \text{ rows}$$

$$\frac{42 \text{ birthday designs}}{6 \text{ designs per row}} = 7 \text{ rows}$$

When the greatest possible number of designs is in each row, there are 11 rows in total.

CHECK IT OUT!

4. Adrianne is shopping for a CD storage unit. She has 36 CDs by pop music artists and 48 CDs by country music artists. She wants to put the same number of CDs on each shelf without putting pop music and country music CDs on the same shelf. If Adrianne puts the greatest possible number of CDs on each shelf, how many shelves does her storage unit need?

THINK AND DISCUSS

1. Describe two ways you can find the prime factorization of a number.

2. GET ORGANIZED Copy and complete the graphic organizer. Show how to write the prime factorization of $100x^2$ by filling in each box.

Know it! Note

8-1 **Exercises**

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Homework Help Online
KEYWORD: MA7 8-1
Parent Resources Online
KEYWORD: MA7 Parent
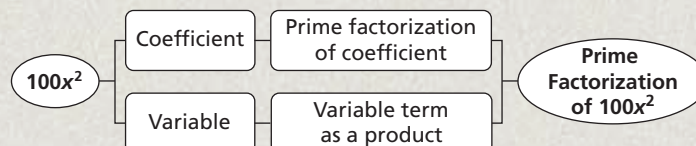
GUIDED PRACTICE

1. Vocabulary Define the term *greatest common factor* in your own words.

SEE EXAMPLE 1
p. 544

Write the prime factorization of each number.

2. 20　　　　　　**3.** 36　　　　　　**4.** 27　　　　　　**5.** 54

6. 96　　　　　　**7.** 7　　　　　　　**8.** 100　　　　　**9.** 75

SEE EXAMPLE 2
p. 545

Find the GCF of each pair of numbers.

10. 12 and 60　　　　**11.** 14 and 49　　　　**12.** 55 and 121

SEE EXAMPLE 3
p. 545

Find the GCF of each pair of monomials.

13. $6x^2$ and $5x^2$　　　**14.** $15y^3$ and $-20y$　　　**15.** $13q^4$ and $2p^2$

SEE EXAMPLE 4
p. 546

16. Samantha is making beaded necklaces using 54 glass beads and 18 clay beads. She wants each necklace to have the same number of beads, but each necklace will have only one type of bead. If she puts the greatest possible number of beads on each necklace, how many necklaces can she make?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
17–24	1
25–27	2
28–30	3
31	4

Extra Practice
Skills Practice p. S18
Application Practice p. S35

Write the prime factorization of each number.

17. 18　　　　**18.** 64　　　　**19.** 12　　　　**20.** 150

21. 17　　　　**22.** 226　　　**23.** 49　　　　**24.** 63

Find the GCF of each pair of numbers.

25. 36 and 63　　　**26.** 14 and 15　　　**27.** 30 and 40

Find the GCF of each pair of monomials.

28. $8a^2$ and 11　　　**29.** $9s$ and $63s^3$　　　**30.** $-64n^4$ and $24n^2$

31. José is making fruit-filled tart shells for a party. He has 72 raspberries and 108 blueberries. The tarts will each have the same number of berries. Raspberries and blueberries will not be in the same tart. If he puts the greatest possible number of fruits in each tart, how many tarts can he make?

Find the GCF of each pair of products.

32. $3 \cdot 5 \cdot t$ and $2 \cdot 2 \cdot 5 \cdot t \cdot t$　　　　**33.** $-1 \cdot 2 \cdot 2 \cdot x \cdot x$ and $2 \cdot 2 \cdot 7 \cdot x \cdot x \cdot x$

34. $2 \cdot 2 \cdot 2 \cdot 11 \cdot x \cdot x \cdot x$ and $3 \cdot 11$　　　**35.** $2 \cdot 5 \cdot n \cdot n \cdot n$ and $-1 \cdot 2 \cdot 3 \cdot n$

36. Write About It The number 2 is even and is prime. Explain why all other prime numbers are odd numbers.

37. Critical Thinking The GCF of two numbers is 1. Explain whether this means the two numbers must be prime.

38. Multi-Step Angelo is making a rectangular floor for a clubhouse with an area of 84 square feet. The length of each side of the floor is a whole number of feet.
  a. What are the possible lengths and widths for Angelo's clubhouse floor?
  b. What is the minimum perimeter for the clubhouse floor?
  c. What is the maximum perimeter for the clubhouse floor?

39. Music The Cavaliers and the Blue Devils are two of the marching bands that are members of Drum Corps International (DCI). DCI bands are made up of percussionists, brass players, and color guard members who use flags and other props.
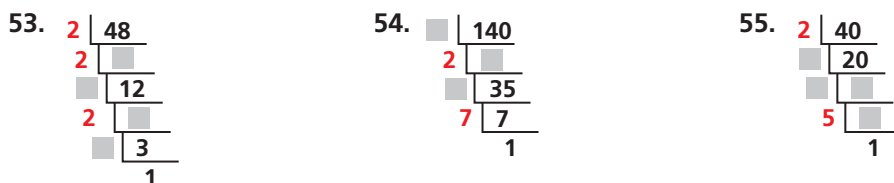
In 2004, there were 35 color guard members in the Cavaliers and 40 in the Blue Devils. The two color guards will march in rows with the same number of people in each row without mixing the guards together. If the greatest possible number of people are in each row, how many rows will there be?

For each set of numbers, determine which two numbers have a GCF greater than 1, and find that GCF.

40. 11, 12, 14

41. 8, 20, 63

42. 16, 21, 27

43. 32, 63, 105

44. 25, 35, 54

45. 35, 54, 72

46. Number Sense The prime factorization of 24 is $2^3 \cdot 3$. Without using a diagram, write the prime factorization of 48. Explain your reasoning.

Fill in each diagram. Then write the prime factorization of the number.

47.
```
        72
    2 · ▢
      ● · 18
        2 · ▢
          ● · 3
```

48.
```
        81
    3 · ▢
      ● · ▢
        3 · 3
```

49.
```
        210
    2 · ▢
      ● · 21
        3 · ●
```

50.
```
  ▢ | 56
  2 | 28
  2 | ▢
  ▢ | 7
      1
```

51.
```
  ▢ | 108
  ▢ | 54
  3 | ▢
  ▢ | 9
  3 | 3
      1
```

52.
```
  2 | 136
  ▢ | 68
  2 | ▢
  ▢ | 17
      1
```

53.
```
  2 | 48
  2 | ▢
  ▢ | 12
  2 | ▢
  ▢ | 3
      1
```

54.
```
  ▢ | 140
  2 | ▢
  ▢ | 35
  7 | 7
      1
```

55.
```
  2 | 40
  ▢ | 20
  ▢ | ▢
  5 | ▢
      1
```
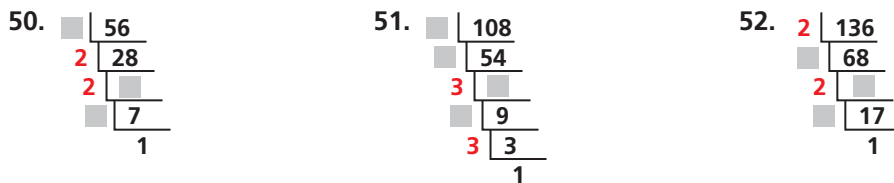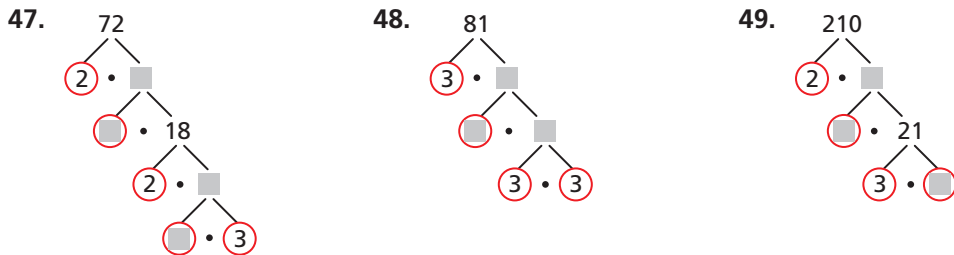
56. This problem will prepare you for the Multi-Step Test Prep on page 576.

The equation for the motion of an object with constant acceleration is $d = vt + \frac{1}{2}at^2$ where d is distance traveled in feet, v is starting velocity in ft/s, a is acceleration in ft/s^2, and t is time in seconds.

a. A toy car begins with a velocity of 2 ft/s and accelerates at 2 ft/s^2. Write an expression for the distance the toy car travels after t seconds.

b. What is the GCF of the terms in your expression from part **a**?

57. Which set of numbers has a GCF greater than 6?

Ⓐ 18, 24, 36   Ⓑ 30, 35, 40   Ⓒ 11, 29, 37   Ⓓ 16, 24, 48

58. The slope of a line is the GCF of 48 and 12. The y-intercept is the GCF of the slope and 8. Which equation describes the line?

Ⓕ $y = 12x + 4$ Ⓖ $y = 6x + 2$ Ⓗ $y = 4x + 4$ Ⓙ $y = 3x + 1$

59. **Extended Response** Patricia is making a dog pen in her back yard. The pen will be rectangular and have an area of 24 square feet. Draw and label a diagram that shows all possible whole-number dimensions for the pen. Find the perimeter of each rectangle you drew. Which dimensions should Patricia use in order to spend the least amount of money on fencing materials? Explain your reasoning.

CHALLENGE AND EXTEND

Find the GCF of each set.

60. $4n^3$, $16n^2$, $8n$

61. $27y^3$, $18y^2$, $81y$

62. 100, $25s^5$, $50s$

63. $2p^4r$, $8p^3r^2$, $16p^2r^3$

64. $2x^3y$, $8x^2y^2$, $17xy^3$

65. $8a^4b^3$, $4a^3b^3$, $12a^2b^3$

66. **Geometry** The area of a triangle is 10 in^2. What are the possible whole-number dimensions for the base and height of the triangle?

67. **Number Sense** The GCF of three different numbers is 7. The sum of the three numbers is 105. What are the three numbers?

68. **Critical Thinking** Find three different *composite* numbers whose GCF is 1. (*Hint:* A composite number has factors other than 1 and itself.)

SPIRAL REVIEW

Use dimensional analysis to convert each measure. *(Lesson 2-7)*

69. 11,232 s to h

70. 9 gal to pt

71. 18 in./day to mi/yr

Find each value. Round to the nearest tenth if necessary. *(Lesson 2-9)*

72. 40% of 60

73. 250% of 16

74. What percent of 80 is 20?

75. Write a simplified polynomial expression for the perimeter of the triangle. *(Lesson 7-7)*

$x^2 + 3x$

$6x - 2$

$2x^2 + 5x - 1$
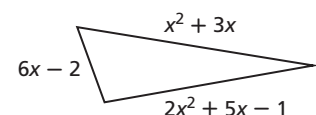
Model Factoring

You can use algebra tiles to write a polynomial as the product of its factors. This process is called *factoring*. Factoring is the reverse of multiplying.

Use with Lesson 8-2

KEY

$\boxed{+} = 1$

$\boxed{-} = -1$

$\boxed{+} = x$

$\boxed{-} = -x$

$\boxed{+} = x^2$
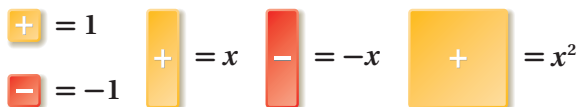
Activity

Use algebra tiles to factor $4x + 8$.

MODEL		ALGEBRA
	Model $4x + 8$.	$4x + 8$
$x + 2$ *4*	*Arrange the tiles into a rectangle. The total area represents $4x + 8$. The length and width represent the factors. The rectangle has a width of $x + 2$ and a length of 4.*	$4x + 8 = 4(x + 2)$

Use algebra tiles to factor $x^2 - 2x$.

MODEL		ALGEBRA
	Model $x^2 - 2x$.	$x^2 - 2x$
$x - 2$ *x*	*Arrange the tiles into a rectangle. The total area represents $x^2 - 2x$. The length and width represent the factors. The rectangle has a width of $x - 2$ and a length of x.*	$x^2 - 2x = x(x - 2)$

Try This

Use algebra tiles to factor each polynomial.

1. $3x + 9$ **2.** $2x + 8$ **3.** $4x - 12$ **4.** $3x - 12$

5. $2x^2 + 2x$ **6.** $x^2 + 4x$ **7.** $x^2 - 3x$ **8.** $2x^2 - 4x$

8-2 Factoring by GCF

Objective
Factor polynomials by using the greatest common factor.

Why learn this?
You can determine the dimensions of a solar panel by factoring an expression representing the panel's area. (See Example 2.)

Recall that the Distributive Property states that $ab + ac = a(b + c)$. The Distributive Property allows you to factor out the GCF of the terms in a polynomial to write a factored form of the polynomial.

A polynomial is in its factored form when it is written as a product of monomials and polynomials that cannot be factored further. The expression $2(3x - 4x)$ is not fully factored because the terms in the parentheses have a common factor of x.

EXAMPLE 1 **Factoring by Using the GCF**

Factor each polynomial. Check your answer.

Writing Math

Aligning common factors can help you find the greatest common factor of two or more terms.

A $4x^2 - 3x$

$$4x^2 = 2 \cdot 2 \cdot \boxed{x} \cdot x$$
$$3x = 3 \cdot \boxed{x}$$
Find the GCF.

$$x$$
The GCF of $4x^2$ and $3x$ is x.

$$4x(x) - 3(x)$$
Write terms as products using the GCF as a factor.

$$x(4x - 3)$$
Use the Distributive Property to factor out the GCF.

Check $x(4x - 3)$
Multiply to check your answer.
$$4x^2 - 3x \checkmark$$
The product is the original polynomial.

B $10y^3 + 20y^2 - 5y$

$$10y^3 = 2 \cdot \boxed{5} \cdot \boxed{y} \cdot y \cdot y$$
$$20y^2 = 2 \cdot 2 \cdot \boxed{5} \cdot \boxed{y} \cdot y$$
$$5y = \boxed{5} \cdot \boxed{y}$$
Find the GCF.

$$5 \cdot y = 5y$$
The GCF of $10y^3$, $20y^2$, and $5y$ is $5y$.

$$2y^2(5y) + 4y(5y) - 1(5y)$$
Write terms as products using the GCF as a factor.

$$5y(2y^2 + 4y - 1)$$
Use the Distributive Property to factor out the GCF.

Check $5y(2y^2 + 4y - 1)$
Multiply to check your answer.
$$10y^3 + 20y^2 - 5y \checkmark$$
The product is the original polynomial.

Factor each polynomial. Check your answer.

C $-12x - 8x^2$

$\quad -1(12x + 8x^2)$ *Both coefficients are negative. Factor out −1.*

$\qquad\qquad 12x = \boxed{2}\cdot\boxed{2}\cdot 3 \cdot \boxed{x}$ *Find the GCF.*
$\qquad\qquad 8x^2 = \boxed{2}\cdot\boxed{2}\cdot 2 \cdot \boxed{x}\cdot x$

$\qquad\qquad\quad 2\cdot 2\cdot\quad x = 4x$ *The GCF of 12x and 8x² is 4x.*

$\quad -1[3(4x) + 2x(4x)]$ *Write each term as a product using the GCF.*
$\quad -1[4x(3 + 2x)]$ *Use the Distributive Property to factor out*
$\quad -1(4x)(3 + 2x)$ *the GCF.*
$\quad -4x(3 + 2x)$

Check

$\quad -4x(3 + 2x) = -12x - 8x^2$ ✓ *Multiply to check your answer.*

D $5x^2 + 7$

$\qquad 5x^2 = 5\quad\cdot x \cdot x$ *Find the GCF.*
$\qquad\quad 7 = \quad 7$

$\qquad 5x^2 + 7$ *There are no common factors other than 1.*

The polynomial cannot be factored.

> **Caution!**
>
> When you factor out −1 as the first step, be sure to include it in all the other steps as well.

CHECK IT OUT! **Factor each polynomial. Check your answer.**

1a. $5b + 9b^3$ **1b.** $9d^2 - 8^2$

1c. $-18y^3 - 7y^2$ **1d.** $8x^4 + 4x^3 - 2x^2$

To write expressions for the length and width of a rectangle whose area is expressed as a polynomial, you need to write the polynomial as a product. You can write a polynomial as a product by factoring it.

EXAMPLE 2 *Science Application*

Mandy's calculator is powered by solar energy. The area of the solar panel is $(7x^2 + x)$ cm². Factor this polynomial to find possible expressions for the dimensions of the solar panel.

$\quad A = 7x^2 + x$ *The GCF of 7x² and x is x.*

$\qquad = 7x(x) + 1(x)$ *Write each term as a product using the GCF as a factor.*

$\qquad = x(7x + 1)$ *Use the Distributive Property to factor out the GCF.*

Possible expressions for the dimensions of the solar panel are x cm and $(7x + 1)$ cm.

CHECK IT OUT! **2. What if...?** The area of the solar panel on another calculator is $(2x^2 + 4x)$ cm². Factor this polynomial to find possible expressions for the dimensions of the solar panel.

Sometimes the GCF of terms is a binomial. This GCF is called a *common binomial factor*. You factor out a common binomial factor the same way you factor out a monomial factor.

EXAMPLE 3 | **Factoring Out a Common Binomial Factor**

Factor each expression.

A $7(x - 3) - 2x(x - 3)$

$7(x - 3) - 2x(x - 3)$ *(x − 3) is a common binomial factor.*

$(x - 3)(7 - 2x)$ *Factor out (x − 3).*

B $-t(t^2 + 4) + (t^2 + 4)$

$-t(t^2 + 4) + (t^2 + 4)$ *(t² + 4) is a common binomial factor.*

$-t(t^2 + 4) + 1(t^2 + 4)$ *(t² + 4) = 1(t² + 4)*

$(t^2 + 4)(-t + 1)$ *Factor out (t² + 4).*

C $9x(x + 4) - 5(4 + x)$

$9x(x + 4) - 5(4 + x)$ *(x + 4) = (4 + x), so (x + 4) is a common binomial factor.*

$9x(x + 4) - 5(x + 4)$

$(x + 4)(9x - 5)$ *Factor out (x + 4).*

D $-3x^2(x + 2) + 4(x - 7)$

$-3x^2(x + 2) + 4(x - 7)$ *There are no common factors.*

The expression cannot be factored.

CHECK IT OUT!

Factor each expression.

3a. $4s(s + 6) - 5(s + 6)$ **3b.** $7x(2x + 3) + (2x + 3)$

3c. $3x(y + 4) - 2y(x + 4)$ **3d.** $5x(5x - 2) - 2(5x - 2)$

You may be able to factor a polynomial by grouping. When a polynomial has four terms, you can make two groups and factor out the GCF from each group.

EXAMPLE 4 | **Factoring by Grouping**

Factor each polynomial by grouping. Check your answer.

A $12a^3 - 9a^2 + 20a - 15$

$(12a^3 - 9a^2) + (20a - 15)$ *Group terms that have a common number or variable as a factor.*

$3a^2(4a - 3) + 5(4a - 3)$ *Factor out the GCF of each group.*

$3a^2(4a - 3) + 5(4a - 3)$ *(4a − 3) is a common factor.*

$(4a - 3)(3a^2 + 5)$ *Factor out (4a − 3).*

Check $(4a - 3)(3a^2 + 5)$ *Multiply to check your solution.*

$4a(3a^2) + 4a(5) - 3(3a^2) - 3(5)$

$12a^3 + 20a - 9a^2 - 15$

$12a^3 - 9a^2 + 20a - 15 \checkmark$ *The product is the original polynomial.*

Factor each polynomial by grouping. Check your answer.

B $9x^3 + 18x^2 + x + 2$

$(9x^3 + 18x^2) + (x + 2)$ *Group terms.*

$\mathbf{9x^2}(x + 2) + \mathbf{1}(x + 2)$ *Factor out the GCF of each group.*

$9x^2\mathbf{(x + 2)} + 1\mathbf{(x + 2)}$ *(x + 2) is a common factor.*

$\mathbf{(x + 2)}(9x^2 + 1)$ *Factor out (x + 2).*

Check $(x + 2)(9x^2 + 1)$ *Multiply to check your solution.*

$x(9x^2) + x(1) + 2(9x^2) + 2(1)$

$9x^3 + x + 18x^2 + 2$

$9x^3 + 18x^2 + x + 2$ ✓   *The product is the original polynomial.*

CHECK IT OUT! Factor each polynomial by grouping. Check your answer.

4a. $6b^3 + 8b^2 + 9b + 12$ **4b.** $4r^3 + 24r + r^2 + 6$

Recognizing opposite binomials can help you factor polynomials. The binomials $(5 - x)$ and $(x - 5)$ are opposites. Notice $(5 - x)$ can be written as $-1(x - 5)$.

$\mathbf{-1}(x - 5) = \mathbf{(-1)}(x) + \mathbf{(-1)}(-5)$ *Distributive Property*

$= -x + 5$ *Simplify.*

$= 5 - x$ *Commutative Property of Addition*

So, $(5 - x) = -1(x - 5)$.

EXAMPLE 5   **Factoring with Opposites**

Factor $3x^3 - 15x^2 + 10 - 2x$ by grouping.

$3x^3 - 15x^2 + 10 - 2x$

$(3x^3 - 15x^2) + (10 - 2x)$ *Group terms.*

$3x^2(x - 5) + 2(5 - x)$ *Factor out the GCF of each group.*

$3x^2\mathbf{(x - 5)} + 2\mathbf{(-1)}\mathbf{(x - 5)}$ *Write (5 − x) as −1(x − 5).*

$3x^2\mathbf{(x - 5)} - 2\mathbf{(x - 5)}$ *Simplify. (x − 5) is a common factor.*

$\mathbf{(x - 5)}(3x^2 - 2)$ *Factor out (x − 5).*

CHECK IT OUT! Factor each polynomial by grouping. Check your answer.

5a. $15x^2 - 10x^3 + 8x - 12$ **5b.** $8y - 8 - x + xy$

Know it! Note

THINK AND DISCUSS

1. Explain how finding the GCF of monomials helps you factor a polynomial.

2. GET ORGANIZED Copy and complete the graphic organizer.

```
                    Factoring by GCF

1. Find the __?__     2. Write each term    3. Use the __?__    4. Check by
   common factor.        as a __?__ using       to factor out       __?__.
                         the GCF.               the GCF.
```

8-2 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MA7 8-2
Parent Resources Online
KEYWORD: MA7 Parent
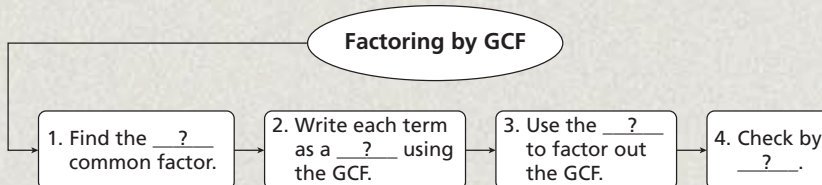
GUIDED PRACTICE

SEE EXAMPLE 1
p. 551

Factor each polynomial. Check your answer.

1. $15a - 5a^2$

2. $10g^3 - 3g$

3. $-35x + 42$

4. $-4x^2 - 6x$

5. $12h^4 + 8h^2 - 6h$

6. $3x^2 - 9x + 3$

7. $9m^2 + m$

8. $14n^3 + 7n + 7n^2$

9. $36f + 18f^2 + 3$

10. $-15b^2 + 7b$

SEE EXAMPLE 2
p. 552

11. Physics A model rocket is fired vertically into the air at 320 ft/s. The expression $-16t^2 + 320t$ gives the rocket's height after t seconds. Factor this expression.

SEE EXAMPLE 3
p. 553

Factor each expression.

12. $5(m - 2) - m(m - 2)$

13. $2b(b + 3) + 5(b + 3)$

14. $4(x - 3) - x(y + 2)$

Factor each polynomial by grouping. Check your answer.

SEE EXAMPLE 4
p. 553

15. $x^3 + 4x^2 + 2x + 8$

16. $6x^3 + 4x^2 + 3x + 2$

17. $4b^3 - 6b^2 + 10b - 15$

18. $2m^3 + 4m^2 + 6m + 12$

19. $7r^3 - 35r^2 + 6r - 30$

20. $10a^3 + 4a^2 + 5a + 2$

SEE EXAMPLE 5
p. 554

21. $2r^2 - 6r + 12 - 4r$

22. $6b^2 - 3b + 4 - 8b$

23. $14q^2 - 21q + 6 - 4q$

24. $3r - r^2 + 2r - 6$

25. $2m^3 - 6m^2 + 9 - 3m$

26. $6a^3 - 9a^2 - 12 + 8a$

PRACTICE AND PROBLEM SOLVING

Extra Practice
Skills Practice p. S18
Application Practice p. S35

Factor each polynomial. Check your answer.

27. $9y^2 + 45y$

28. $36d^3 + 24$

29. $-14x^4 + 5x^2$

30. $-15f - 10f^2$

31. $-4d^4 + d^3 - 3d^2$

32. $14x^3 + 63x^2 - 7x$

33. $21c^2 + 14c$

34. $33d^3 + 22d + 11$

35. $-5g^3 - 15g^2$

36. Finance After t years, the amount of money in a savings account that earns simple interest is $P + Prt$, where P is the starting amount and r is the yearly interest rate. Factor this expression.

Factor each expression.

37. $6a(a - 2) - 5b(b + 4)$

38. $-4x(x + 2) + 9(x + 2)$

39. $6y(y - 7) + (y - 7)$

40. $a(x - 3) + 2b(x - 3)$

41. $-3(2 + b) + 4b(b + 2)$

42. $5(3x - 2) + x(3x - 2)$

Factor each polynomial by grouping. Check your answer.

43. $2a^3 - 8a^2 + 3a - 12$

44. $x^3 + 3x^2 + 5x + 15$

45. $6x^3 + 18x^2 + x + 3$

46. $7x^3 + 2x^2 + 28x + 8$

47. $n^3 - 2n^2 + 5n - 10$

48. $10b^3 - 16b^2 + 25b - 40$

49. $2m^3 - 2m^2 + 3 - 3m$

50. $2d^3 - d^2 - 3 + 6d$

51. $6f^3 - 8f^2 + 20 - 15f$

52. $5k^2 - k^3 + 3k - 15$

53. $b^3 - 2b - 8 + 4b^2$

54. $20 - 15x - 6x^2 + 8x$

Fill in the missing part of each factorization.

55. $16v + 12v^2 = 4v(4 + \blacksquare)$

56. $15x - 25x^2 = 5x(3 - \blacksquare)$

57. $-16k^3 - 24k^2 = -8k^2(\blacksquare + 3)$

58. $-x - 10 = -1(\blacksquare + 10)$

Copy and complete the table.

	Polynomial	Number of Terms	Name	Completely Factored Form
	$3y + 3x + 9$	3	trinomial	$3(y + x + 3)$
59.	$x^2 + 5x$	$\blacksquare$	$\blacksquare$	$\blacksquare$
60.	$28c^2 - 49c$	$\blacksquare$	$\blacksquare$	$\blacksquare$
61.	$a^4 + a^3 + a^2$	$\blacksquare$	$\blacksquare$	$\blacksquare$
62.	$36 + 99r - 40r^2 - 110r^3$	$\blacksquare$	$\blacksquare$	$\blacksquare$

63. Personal Finance The final amount of money earned by a certificate of deposit (CD) after n years is $P(1 + r)^n$, where P is the original amount contributed and r is the interest rate as a decimal.

Year	Amount of CD
2004	$100.00
2005	$200.00
2006	$400.00

Justin's aunt purchased three CDs with the same interest rate to help him pay for college. The table shows the amount of the CD she purchased each year. In 2007, she paid $800.00 directly to the college.

a. Let $x = 1 + r$. Write expressions in terms of x for the value of the CDs purchased in 2004, 2005, and 2006 when Justin started college in 2007.

b. Write a polynomial in terms of x to represent the total value of the CDs purchased in 2004, 2005, and 2006 plus the amount paid to the college in 2007.

c. Factor the polynomial in part **b** by grouping. Evaluate the factored form of the polynomial when the interest rate is 9%. (*Hint*: Remember that $x = 1 +$ the interest rate expressed as a decimal.)

64. Write About It Describe how to find the area of the figure shown. Show each step and write your answer in factored form.

$2x + 6$ $x + 8$ $2x$
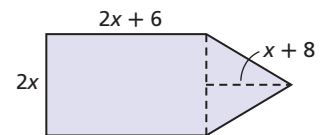
65. Critical Thinking Show two methods of factoring the expression $3a - 3b - 4a + 4b$.

66. Geometry The area of a triangle is $\frac{1}{2}(x^3 - 2x + 2x^2 - 4)$. The height h is $x + 2$. Write an expression for the base b of the triangle. $\left(Hint: \text{Area of a triangle} = \frac{1}{2}bh\right)$

67. Write About It Explain how you know when two binomials are opposites.

MULTI-STEP TEST PREP

68. This problem will prepare you for the Multi-Step Test Prep on page 576.

a. The Multiplication Property of Zero states that the product of any number and 0 is 0. What must be true about either a or b to make $ab = 0$?

b. A toy car's distance in feet from the starting point is given by the equation $d = t(3 - t)$. Explain why $t(3 - t) = 0$ means that either $t = 0$ or $3 - t = 0$.

c. When $d = 0$, the car is at the starting point. Use the fact that $t = 0$ or $3 - t = 0$ when $d = 0$ to find the two times when the car is at the starting point.

Fill in each blank with a property or definition that justifies the step.

69. $7x^3 + 2x + 21x^2 + 6 = 7x^3 + 21x^2 + 2x + 6$ **a.** _____?_____

$= (7x^3 + 21x^2) + (2x + 6)$ **b.** _____?_____

$= 7x^2(x + 3) + 2(x + 3)$ **c.** _____?_____

$= (x + 3)(7x^2 + 2)$ **d.** _____?_____

70. ///**ERROR ANALYSIS**/// Which factorization of $3n^3 - n^2$ is incorrect? Explain.

A

| $3n^3 - n^2$ |
| $n^2(3n) - n^2(0)$ |
| $n^2(3n - 0)$ |

B

| $3n^3 - n^2$ |
| $n^2(3n) - n^2(1)$ |
| $n^2(3n - 1)$ |

TEST PREP

71. Which is the complete factorization of $24x^3 - 12x^2$?

Ⓐ $6(4x^3 - 2x^2)$ Ⓑ $12(2x^3 - x^2)$ Ⓒ $12x(2x^2 - x)$ Ⓓ $12x^2(2x - 1)$

72. Which is NOT a factor of $18x^2 + 36x$?

Ⓕ 1   Ⓖ $4x$ Ⓗ $x + 2$ Ⓙ $18x$

73. The area of a rectangle is represented by the polynomial $x^2 + 3x - 6x - 18$. Which of the following could represent the length and width of the rectangle?

Ⓐ Length: $x + 3$; width: $x + 6$ Ⓒ Length: $x + 3$; width: $x - 6$

Ⓑ Length: $x - 3$; width: $x - 6$ Ⓓ Length: $x - 3$; width: $x + 6$

CHALLENGE AND EXTEND

Factor each polynomial.

74. $6ab^2 - 24a^2$ **75.** $-72a^2b^2 - 45ab$ **76.** $-18a^2b^2 + 21ab$

77. $ab + bc + ad + cd$ **78.** $4y^2 + 8ay - y - 2a$ **79.** $x^3 - 4x^2 + 3x - 12$

80. Geometry The area between two concentric circles is called an *annulus*. The formula for area of an annulus is $A = \pi R^2 - \pi r^2$, where R is the radius of the larger circle and r is the radius of the smaller circle.

a. Factor the GCF from the formula for area of an annulus.

b. Use your answer from part **a** to find the area of an annulus with $R = 12$ cm and $r = 5$ cm.
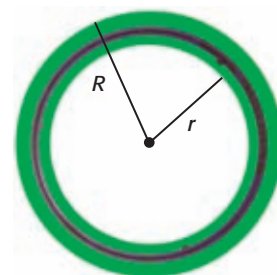
SPIRAL REVIEW

Solve each equation. *(Lesson 2-4)*

81. $4(x + 1) = 3(2x - 6)$ **82.** $-20 + 8n = n + 29$ **83.** $8\left(n + \frac{3}{4}\right) = 10n - 4$

84. The coordinates of the vertices of a quadrilateral are $A(-2, 5)$, $B(6, 5)$, $C(4, -3)$, and $D(-4, -3)$. Use slope to show that $ABCD$ is a parallelogram. *(Lesson 5-9)*

85. Graph the data in the table and show the rates of change. *(Lesson 5-3)*

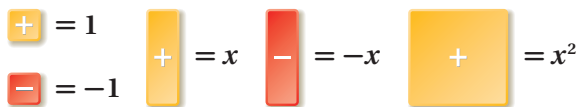| Time (yr)	1998	1999	2002	2004	2005
Profit (million $)	0.6	0.8	1.3	1.9	2.4

Algebra LAB

Model Factorization of Trinomials

You can use algebra tiles to write a trinomial as a product of two binomials. This is called *factoring a trinomial.*

Use with Lesson 8-3

KEY

$\boxed{+} = 1$

$\boxed{-} = -1$

$\boxed{+} = x$ $\boxed{-} = -x$ $\boxed{+} = x^2$

Activity 1

Use algebra tiles to factor $x^2 + 7x + 6$.

MODEL		ALGEBRA
	Model $x^2 + 7x + 6$.	$x^2 + 7x + 6$
	Try to arrange all of the tiles in a rectangle. Start by placing the x^2-tile in the upper left corner. Arrange the unit tiles in a rectangle so that the top left corner of this rectangle touches the bottom right corner of the x^2-tile. Arrange the x-tiles so that all the tiles together make one large rectangle. *This arrangement does not work because two x-tiles are left over.*	$x^2 + 7x + 6 \neq (x + 2)(x + 3)$
	Rearrange the unit tiles to form another rectangle.	
	Fill in the empty spaces with x-tiles. All 7 x-tiles fit. This is the correct arrangement. The total area represents the trinomial. The length and width represent the factors.	$x^2 + 7x + 6 = (x + 1)(x + 6)$

The rectangle has width $x + 1$ and length $x + 6$. So $x^2 + 7x + 6 = (x + 1)(x + 6)$.

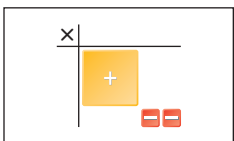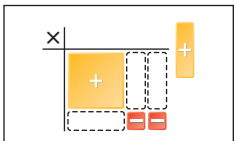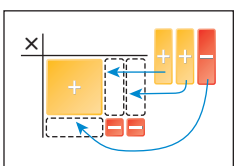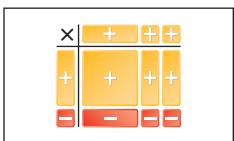Use algebra tiles to factor each trinomial.

1. $x^2 + 2x + 1$ **2.** $x^2 + 3x + 2$ **3.** $x^2 + 6x + 5$ **4.** $x^2 + 6x + 9$

5. $x^2 + 5x + 4$ **6.** $x^2 + 6x + 8$ **7.** $x^2 + 5x + 6$ **8.** $x^2 + 8x + 12$

Activity 2

Use algebra tiles to factor $x^2 + x - 2$.

MODEL	ALGEBRA
Model $x^2 + x - 2$.	$x^2 + x - 2$
Start by placing the x^2-tile in the upper left corner. *Arrange the unit tiles in a rectangle so that the top left corner of this rectangle touches the bottom right corner of the x^2-tile.* *To make a rectangle, you need to fill in the empty spaces, but there aren't enough x-tiles to fill in the empty spaces.*	
Add a zero pair. Arrange the x-tiles to complete the rectangle. *Remember that the product of two positive values is positive and the product of a positive and a negative value is negative.*	
The total area represents the trinomial. The length and width represent the factors.	$x^2 + x - 2 = (x - 1)(x + 2)$

The rectangle has width $x - 1$ and length $x + 2$. So, $x^2 + x - 2 = (x - 1)(x + 2)$.

Try This

9. Why can you add one red $-x$-tile and one yellow x-tile?

Use algebra tiles to factor each trinomial.

10. $x^2 - x - 2$ **11.** $x^2 - 2x - 3$ **12.** $x^2 - 5x + 4$ **13.** $x^2 - 7x + 10$

14. $x^2 - 2x + 1$ **15.** $x^2 - 6x + 5$ **16.** $x^2 + 5x - 6$ **17.** $x^2 + 3x - 4$

18. $x^2 - x - 6$ **19.** $x^2 + 3x - 10$ **20.** $x^2 - 2x - 8$ **21.** $x^2 + x - 12$

Factoring $x^2 + bx + c$

Objective
Factor quadratic
trinomials of the form
$x^2 + bx + c$.

Why learn this?
Factoring polynomials will help you find the dimensions of rectangular shapes, such as a fountain. (See Exercise 71.)

In Chapter 7, you learned how to multiply two binomials using the Distributive Property or the FOIL method. In this lesson, you will learn how to factor a trinomial into two binomials.

Notice that when you multiply $(x + 2)(x + 5)$, the constant term in the trinomial is the product of the constants in the binomials.

$$(x + 2)(x + 5) = x^2 + 7x + 10$$

You can use this fact to factor a trinomial into its binomial factors. Look for two numbers that are factors of the constant term in the trinomial. Write two binomials with those numbers, and then multiply to see if you are correct.

EXAMPLE 1 **Factoring Trinomials by Guess and Check**

Factor $x^2 + 19x + 60$ by guess and check.

$(\blacksquare + \blacksquare)(\blacksquare + \blacksquare)$ *Write two sets of parentheses.*

$(x + \blacksquare)(x + \blacksquare)$ *The first term is x^2, so the variable terms have a coefficient of 1.*

Remember!

When you multiply two binomials, multiply:

First terms
Outer terms
Inner terms
Last terms

The constant term in the trinomial is 60.

$(x + 1)(x + 60) = x^2 + 61x + 60$ ✗   *Try factors of 60 for the constant terms in the binomials.*
$(x + 2)(x + 30) = x^2 + 32x + 60$ ✗
$(x + 3)(x + 20) = x^2 + 23x + 60$ ✗
$(x + 4)(x + 15) = x^2 + 19x + 60$ ✓

The factors of $x^2 + 19x + 60$ are $(x + 4)$ and $(x + 15)$.

$x^2 + 19x + 60 = (x + 4)(x + 15)$

CHECK IT OUT! Factor each trinomial by guess and check.
1a. $x^2 + 10x + 24$ **1b.** $x^2 + 7x + 12$

The guess and check method is usually not the most efficient method of factoring a trinomial. Look at the product of $(x + 3)$ and $(x + 4)$.

$$(x+3)(x+4) = x^2 + 7x + 12$$

The coefficient of the middle term is the sum of 3 and 4. The third term is the product of 3 and 4.

Know it!
Note

Factoring $x^2 + bx + c$

WORDS	EXAMPLE
To factor a quadratic trinomial of the form $x^2 + bx + c$, find two factors of c whose sum is b. If no such integers exist, the trinomial is not factorable.	To factor $x^2 + 9x + 18$, look for factors of 18 whose sum is 9.

Factors of 18	Sum	
1 and 18	19	✗
2 and 9	11	✗
3 and 6	9	✓ $x^2 + 9x + 18 = (x + 3)(x + 6)$

When c is positive, its factors have the same sign. The sign of b tells you whether the factors are positive or negative. When b is positive, the factors are positive, and when b is negative, the factors are negative.

E X A M P L E 2 **Factoring $x^2 + bx + c$ When c Is Positive**

Factor each trinomial. Check your answer.

A $x^2 + 6x + 8$

$(x + \boxed{})(x + \boxed{})$ *$b = 6$ and $c = 8$; look for factors of 8 whose sum is 6.*

Factors of 8	Sum	
1 and 8	9	✗
2 and 4	6	✓ *The factors needed are 2 and 4.*

$(x + 2)(x + 4)$

Check $(x + 2)(x + 4) = x^2 + 4x + 2x + 8$ *Use the FOIL method.*

$= x^2 + 6x + 8$ ✓   *The product is the original polynomial.*

B $x^2 + 5x + 6$

$(x + \boxed{})(x + \boxed{})$ *$b = 5$ and $c = 6$; look for factors of 6 whose sum is 5.*

Factors of 6	Sum	
1 and 6	7	✗
2 and 3	5	✓ *The factors needed are 2 and 3.*

$(x + 2)(x + 3)$

Use the FOIL method.

Check $(x + 2)(x + 3) = x^2 + 3x + 2x + 6$

The product is the original polynomial.

$= x^2 + 5x + 6$ ✓

Factor each trinomial. Check your answer.

C $x^2 - 10x + 16$

$(x + \boxed{})(x + \boxed{})$ *b = −10 and c = 16; look for factors of 16 whose sum is −10.*

Factors of 16	Sum	
−1 and −16	−17	✗
−2 and −8	**−10**	✓

The factors needed are −2 and −8.

$(x - 2)(x - 8)$

Check $(x - 2)(x - 8) = x^2 - 8x - 2x + 16$ *Use the FOIL method.*

$\qquad\qquad\qquad\qquad\quad = x^2 - 10x + 16$ ✓ *The product is the original polynomial.*

CHECK IT OUT! Factor each trinomial. Check your answer.

2a. $x^2 + 8x + 12$ $\qquad\qquad$ **2b.** $x^2 - 5x + 6$

2c. $x^2 + 13x + 42$ $\qquad\quad$ **2d.** $x^2 - 13x + 40$

When c is negative, its factors have opposite signs. The sign of b tells you which factor is positive and which is negative. The factor with the greater absolute value has the same sign as b.

EXAMPLE 3 **Factoring $x^2 + bx + c$ When c Is Negative**

Factor each trinomial.

A $x^2 + 7x - 18$

$(x + \boxed{})(x + \boxed{})$ *b = 7 and c = −18; look for factors of −18 whose sum is 7. The factor with the greater absolute value is positive.*

Factors of −18	Sum	
−1 and 18	17	✗
−2 and 9	**7**	✓

The factors needed are −2 and 9.

$(x - 2)(x + 9)$

B $x^2 - 5x - 24$

$(x + \boxed{})(x + \boxed{})$ *b = −5 and c = −24; look for factors of −24 whose sum is −5. The factor with the greater absolute value is negative.*

Factors of −24	Sum	
1 and −24	−23	✗
2 and −12	−10	✗
3 and −8	**−5**	✓

The factors needed are 3 and −8.

$(x + 3)(x - 8)$

Helpful Hint

If you have trouble remembering the rules for which factor is positive and which is negative, you can try all the factor pairs and check their sums.

CHECK IT OUT! Factor each trinomial. Check your answer.

3a. $x^2 + 2x - 15$ $\qquad$ **3b.** $x^2 - 6x + 8$ $\qquad$ **3c.** $x^2 - 8x - 20$

A polynomial and the factored form of the polynomial are equivalent expressions. When you evaluate these two expressions for the same value of the variable, the results are the same.

EXAMPLE 4 **Evaluating Polynomials**

Factor $n^2 + 11n + 24$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3,$ and 4.

$n^2 + 11n + 24$

$(n + \blacksquare)(n + \blacksquare)$ *b = 11 and c = 24; look for factors of 24 whose sum is 11.*

Factors of 24	Sum	
1 and 24	25	✗
2 and 12	14	✗
3 and **8**	**11**	✓

The factors needed are 3 and 8.

$(n + 3)(n + 8)$

Evaluate the original polynomial and the factored form for $n = 0, 1, 2, 3,$ and 4.

n	$n^2 + 11n + 24$
0	$0^2 + 11(0) + 24 = 24$
1	$1^2 + 11(1) + 24 = 36$
2	$2^2 + 11(2) + 24 = 50$
3	$3^2 + 11(3) + 24 = 66$
4	$4^2 + 11(4) + 24 = 84$

n	$(n + 3)(n + 8)$
0	$(0 + 3)(0 + 8) = 24$
1	$(1 + 3)(1 + 8) = 36$
2	$(2 + 3)(2 + 8) = 50$
3	$(3 + 3)(3 + 8) = 66$
4	$(4 + 3)(4 + 8) = 84$

The original polynomial and the factored form have the same value for the given values of n.

CHECK IT OUT! **4.** Factor $n^2 - 7n + 10$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3,$ and 4.

THINK AND DISCUSS

1. Explain in your own words how to factor $x^2 + 9x + 14$. Show how to check your answer.

2. Explain how you can determine the signs of the factors of c when factoring a trinomial of the form $x^2 + bx + c$.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of a trinomial with the given properties and factor it.
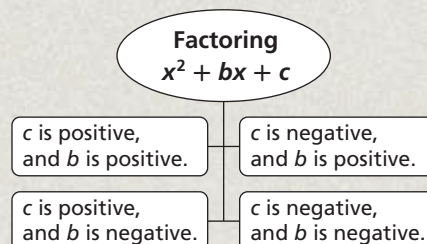
Factoring $x^2 + bx + c$

c is positive, and b is positive.	c is negative, and b is positive.
c is positive, and b is negative.	c is negative, and b is negative.

GUIDED PRACTICE

SEE EXAMPLE 1
p. 560

Factor each trinomial by guess and check.

1. $x^2 + 13x + 36$ **2.** $x^2 + 11x + 24$ **3.** $x^2 + 14x + 40$

SEE EXAMPLE 2
p. 561

Factor each trinomial. Check your answer.

4. $x^2 + 4x + 3$ **5.** $x^2 + 10x + 16$ **6.** $x^2 + 15x + 44$

7. $x^2 - 7x + 6$ **8.** $x^2 - 9x + 14$ **9.** $x^2 - 11x + 24$

SEE EXAMPLE 3
p. 562

10. $x^2 - 6x - 7$ **11.** $x^2 + 6x - 27$ **12.** $x^2 + x - 30$

13. $x^2 - x - 2$ **14.** $x^2 - 3x - 18$ **15.** $x^2 - 4x - 45$

SEE EXAMPLE 4
p. 563

16. Factor $n^2 + 6n - 7$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3,$ and 4.

PRACTICE AND PROBLEM SOLVING

Factor each trinomial by guess and check.

17. $x^2 + 13x + 30$ **18.** $x^2 + 11x + 28$ **19.** $x^2 + 16x + 48$

Factor each trinomial. Check your answer.

20. $x^2 + 12x + 11$ **21.** $x^2 + 16x + 28$ **22.** $x^2 + 15x + 36$

23. $x^2 - 6x + 5$ **24.** $x^2 - 9x + 18$ **25.** $x^2 - 12x + 32$

26. $x^2 + x - 12$ **27.** $x^2 + 4x - 21$ **28.** $x^2 + 9x - 36$

29. $x^2 - 12x - 13$ **30.** $x^2 - 10x - 24$ **31.** $x^2 - 2x - 35$

32. Factor $n^2 - 12n - 45$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3,$ and 4.

Match each trinomial with its correct factorization.

33. $x^2 + 3x - 10$ **A.** $(x - 2)(x - 5)$

34. $x^2 - 7x + 10$ **B.** $(x + 1)(x + 10)$

35. $x^2 - 9x - 10$ **C.** $(x - 2)(x + 5)$

36. $x^2 + 11x + 10$ **D.** $(x + 1)(x - 10)$

37. Write About It Compare multiplying binomials with factoring polynomials into binomial factors.

Factor each trinomial. Check your answer.

38. $x^2 + x - 20$ **39.** $x^2 - 11x + 18$ **40.** $x^2 - 4x - 21$

41. $x^2 + 10x + 9$ **42.** $x^2 - 12x + 32$ **43.** $x^2 + 13x + 42$

44. $x^2 - 7x + 12$ **45.** $x^2 + 11x + 18$ **46.** $x^2 - 6x - 27$
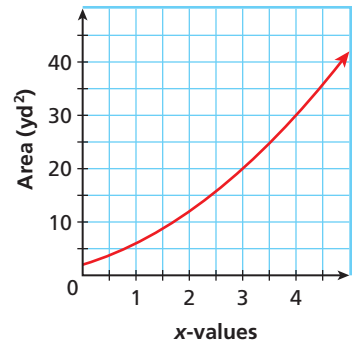
47. $x^2 + 5x - 24$ **48.** $x^2 - 10x + 21$ **49.** $x^2 + 4x - 45$

50. Factor $n^2 + 11n + 28$. Show that the original polynomial and the factored form have the same value for $n = 0, 1, 2, 3,$ and 4.

51. Estimation The graph shows the areas of rectangles with dimensions $(x + 1)$ yards and $(x + 2)$ yards. Estimate the value of x for a rectangle with area 9 square yards.

52. Geometry The area of a rectangle in square feet can be represented by $x^2 + 8x + 12$. The length is $(x + 6)$ ft. What is the width of the rectangle?

53. Remodeling A homeowner wants to enlarge a rectangular closet that has an area of $(x^2 + 3x + 2)$ ft². The length is $(x + 2)$ ft. After construction, the area will be $(x^2 + 8x + 15)$ ft² with a length of $(x + 3)$ ft.

  a. Find the dimensions of the closet before construction.

  b. Find the dimensions of the closet after construction.

  c. By how many feet will the length and width increase after construction?

Art Write the polynomial modeled and then factor.

54.

55.

56.

Copy and complete the table.

$x^2 + bx + c$	Sign of c	Binomial Factors	Signs of Numbers in Binomials
$x^2 + 4x + 3$	Positive	$(x + 1)(x + 3)$	Both positive
57. $x^2 - 4x + 3$	▢	$(x\ ▢\ 1)(x\ ▢\ 3)$	▢
58. $x^2 + 2x - 3$	▢	$(x\ ▢\ 1)(x\ ▢\ 3)$	▢
59. $x^2 - 2x - 3$	▢	$(x\ ▢\ 1)(x\ ▢\ 3)$	▢

60. Geometry A rectangle has area $x^2 + 6x + 8$. The length is $x + 4$. Find the width of the rectangle. Could the rectangle be a square? Explain why or why not.

MULTI-STEP TEST PREP

61. This problem will prepare you for the Multi-Step Test Prep on page 576.

The equation for the motion of an object with constant acceleration is $d = vt + \frac{1}{2}at^2$ where d is distance traveled in feet, v is starting velocity in feet per second, a is acceleration in feet per second squared, and t is time in seconds.

  a. Janna has two toy race cars on a track. One starts with a velocity of 0 ft/s and accelerates at 2 ft/s². Write an equation for the distance the car travels in time t.

  b. The second car travels at a constant speed of 4 ft/s. Write an equation for the distance the second car travels in time t. (*Hint:* When speed is constant, the acceleration is 0 ft/s².)

  c. By setting the equations equal to each other you can determine when the cars have traveled the same distance: $t^2 = 4t$. This can be written as $t^2 - 4t = 0$. Factor the left side of the equation.

Art

The Dutch painter Theo van Doesburg (1883–1931) is most famous for his paintings composed of lines and rectangles, such as the one shown above.

62. Construction The length of a rectangular porch is $(x + 7)$ ft. The area of the porch is $(x^2 + 9x + 14)$ ft^2. Find the width of the porch.
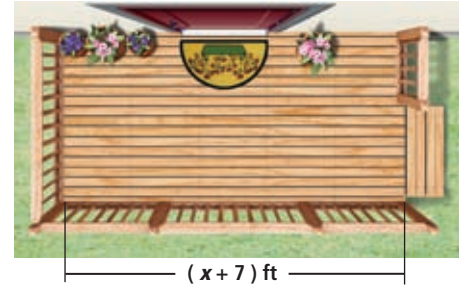
Tell whether each statement is true or false. If false, explain.

63. The third term in a factorable trinomial is equal to the product of the constants in its binomial factors.

64. The constants in the binomial factors of $x^2 + x - 2$ are both negative.

65. The correct factorization of $x^2 - 3x - 4$ is $(x + 4)(x - 1)$.

66. All trinomials of the form $x^2 + bx + c$ can be factored.

Fill in the missing part of each factorization.

67. $x^2 - 6x + 8 = (x - 2)(x - \blacksquare)$
68. $x^2 - 2x - 8 = (x + 2)(x - \blacksquare)$
69. $x^2 + 2x - 8 = (x - 2)(x + \blacksquare)$
70. $x^2 + 6x + 8 = (x + 2)(x + \blacksquare)$

71. Construction The area of a rectangular fountain is $(x^2 + 12x + 20)$ ft^2. The width is $(x + 2)$ ft.
  a. Find the length of the fountain.
  b. A 2-foot walkway is built around the fountain. Find the dimensions of the outside border of the walkway.
  c. Find the total area covered by the fountain and walkway.

72. Critical Thinking Find all possible values of b so that $x^2 + bx + 6$ can be factored into binomial factors.

TEST PREP
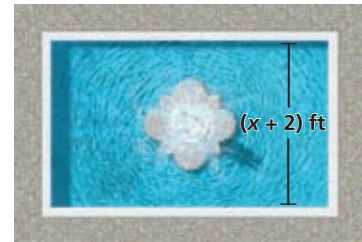
73. Which is the correct factorization of $x^2 - 10x - 24$?
  Ⓐ $(x - 4)(x - 6)$ Ⓒ $(x - 2)(x + 12)$
  Ⓑ $(x + 4)(x - 6)$ Ⓓ $(x + 2)(x - 12)$

74. Which value of b would make $x^2 + bx - 20$ factorable?
  Ⓕ 9 Ⓖ 12 Ⓗ 19 Ⓙ 21

75. Which value of b would NOT make $x^2 + bx - 36$ factorable?
  Ⓐ 5 Ⓑ 9 Ⓒ 15 Ⓓ 16

76. Short Response What are the factors of $x^2 + 2x - 24$? Show and explain each step of factoring the polynomial.

CHALLENGE AND EXTEND

Factor each expression.

77. $x^4 + 18x^2 + 81$ **78.** $y^4 - 5y^2 - 24$ **79.** $d^4 + 22d^2 + 21$

80. $(u + v)^2 + 2(u + v) - 3$ **81.** $(de)^2 - (de) - 20$ **82.** $(m - n)^2 - 4(m - n) - 45$

83. Find all possible values of b such that, when $x^2 + bx + 28$ is factored, both constants in the binomials are positive.

84. Find all possible values of b such that, when $x^2 + bx + 32$ is factored, both constants in the binomials are negative.

85. The area of Beth's rectangular garden is $\left(x^2 + 13x + 42\right)$ ft². The width is $(x + 6)$ ft.

Item	Cost
Fertilizer	0.28 ($/ft²)
Fencing	2.00 ($/ft)

  a. What is the length of the garden?

  b. Find the perimeter in terms of x.

  c. Find the cost to fence the garden when x is 5.

  d. Find the cost of fertilizer when x is 5.

  e. Find the total cost to fence and fertilize Beth's garden when x is 5.

SPIRAL REVIEW

86. Choose the situation that best describes the graph. *(Lesson 4-1)*

  A. An object increases speed, stops, and then moves in reverse.

  B. An object starts at rest, increases speed steadily, maintains constant speed, and then comes to an immediate stop.

  C. An object increases speed quickly, then increases speed slowly, and then comes to an immediate stop.
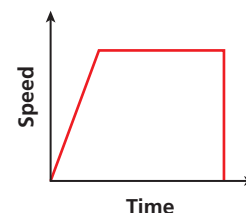
Simplify. *(Lesson 7-3)*

87. $x^3 x^2$

88. $m^8 n^3 m^{-12}$

89. $\left(t^4\right)^3$

90. $\left(-2xy^3\right)^5$

Factor each polynomial by grouping. *(Lesson 8-2)*

91. $x^3 + 2x^2 + 5x + 10$

92. $2n^3 - 8n^2 - 3n + 12$

93. $2p^4 - 4p^3 + 7p - 14$

94. $x^3 - 4x^2 + x - 4$

Career Path

Jessica Rubino
Environmental Sciences major

Q: What math classes did you take in high school?

A: Algebra 1, Algebra 2, and Geometry

Q: What college math classes have you taken?

A: I took several computer modeling and programming classes as well as Statistics and Probability.

Q: How is math used in some of your projects?

A: Computer applications help me analyze data collected from a local waste disposal site. I used my mathematical knowledge to make recommendations on how to preserve surrounding water supplies.

Q: What plans do you have for the future?

A: I enjoy my studies in the area of water pollution. I would also like to research more efficient uses of natural energy resources.

Factoring $ax^2 + bx + c$

Objective
Factor quadratic trinomials of the form $ax^2 + bx + c$.

Why learn this?
The height of a football that has been kicked can be modeled by a factored polynomial. (See Exercise 69.)

In the previous lesson you factored trinomials of the form $x^2 + bx + c$. Now you will factor trinomials of the form $ax^2 + bx + c$, where $a \neq 0$ or 1.

When you multiply $(3x + 2)(2x + 5)$, the coefficient of the x^2-term is the product of the coefficients of the x-terms. Also, the constant term in the trinomial is the product of the constants in the binomials.

$$(3x + 2)(2x + 5) = 6x^2 + 19x + 10$$

To factor a trinomial like $ax^2 + bx + c$ into its binomial factors, write two sets of parentheses: $(\blacksquare x + \blacksquare)(\blacksquare x + \blacksquare)$.

Write two numbers that are factors of a next to the x's and two numbers that are factors of c in the other blanks. Then multiply to see if the product is the original trinomial. If there are not two such integers, the trinomial is unfactorable.

EXAMPLE 1 **Factoring $ax^2 + bx + c$ by Guess and Check**

Factor $4x^2 + 16x + 15$ by guess and check.

$(\blacksquare + \blacksquare)(\blacksquare + \blacksquare)$ *Write two sets of parentheses.*

$(\blacksquare x + \blacksquare)(\blacksquare x + \blacksquare)$ *The first term is $4x^2$, so at least one variable term has a coefficient other than 1.*

The coefficient of the x^2-term is **4**. The constant term in the trinomial is **15**.

$(1x + 15)(4x + 1) = 4x^2 + 61x + 15$ ✗ *Try factors of 4 for the coefficients and factors of 15 for the constant terms.*

$(1x + 5)(4x + 3) = 4x^2 + 23x + 15$ ✗

$(1x + 3)(4x + 5) = 4x^2 + 17x + 15$ ✗

$(1x + 1)(4x + 15) = 4x^2 + 19x + 15$ ✗

$(2x + 15)(2x + 1) = 4x^2 + 32x + 15$ ✗

$(2x + 5)(2x + 3) = 4x^2 + 16x + 15$ ✓

The factors of $4x^2 + 16x + 15$ are $(2x + 5)$ and $(2x + 3)$.

$4x^2 + 16x + 15 = (2x + 5)(2x + 3)$

CHECK IT OUT!

Factor each trinomial by guess and check.

1a. $6x^2 + 11x + 3$ **1b.** $3x^2 - 2x - 8$

So, to factor $ax^2 + bx + c$, check the factors of a and the factors of c in the binomials. The sum of the products of the outer and inner terms should be b.

Product = a — Product = c

$$(\square x + \square)(\square x + \square) = ax^2 + bx + c$$

Sum of outer and inner products = b

Since you need to check all the factors of a and all the factors of c, it may be helpful to make a table. Then check the products of the outer and inner terms to see if the sum is b. You can multiply the binomials to check your answer.

EXAMPLE 2 **Factoring $ax^2 + bx + c$ When c Is Positive**

Factor each trinomial. Check your answer.

A $2x^2 + 11x + 12$

$(\square x + \square)(\square x + \square)$ $a = 2$ and $c = 12$; Outer + Inner = 11

Factors of 2	Factors of 12	Outer + Inner	
1 and 2	1 and 12	$1(12) + 2(1) = 14$	✗
1 and 2	12 and 1	$1(1) + 2(12) = 25$	✗
1 and 2	2 and 6	$1(6) + 2(2) = 10$	✗
1 and 2	6 and 2	$1(2) + 2(6) = 14$	✗
1 and 2	3 and 4	$1(4) + 2(3) = 10$	✗
1 and 2	4 and 3	$1(3) + 2(4) = 11$	✓

$$(x + 4)(2x + 3)$$

Check $(x + 4)(2x + 3) = 2x^2 + 3x + 8x + 12$ *Use the FOIL method.*
$$= 2x^2 + 11x + 12 \checkmark$$

B $5x^2 - 14x + 8$

$(\square x + \square)(\square x + \square)$ $a = 5$ and $c = 8$; Outer + Inner = -14

Factors of 5	Factors of 8	Outer + Inner	
1 and 5	-1 and -8	$1(-8) + 5(-1) = -13$	✗
1 and 5	-8 and -1	$1(-1) + 5(-8) = -41$	✗
1 and 5	-2 and -4	$1(-4) + 5(-2) = -14$	✓

$$(x - 2)(5x - 4)$$

Check $(x - 2)(5x - 4) = 5x^2 - 4x - 10x + 8$ *Use the FOIL method.*
$$= 5x^2 - 14x + 8 \checkmark$$

Remember!

When b is negative and c is positive, the factors of c are both negative.

CHECK IT OUT! Factor each trinomial. Check your answer.

2a. $6x^2 + 17x + 5$ **2b.** $9x^2 - 15x + 4$ **2c.** $3x^2 + 13x + 12$

When c is negative, one factor of c will be positive and the other factor will be negative. Only some of the factors are shown in the examples, but you may need to check all of the possibilities.

EXAMPLE 3

Factoring $ax^2 + bx + c$ When c Is Negative

Factor each trinomial. Check your answer.

A $4y^2 + 7y - 2$

$(\boxed{}y + \boxed{})(\boxed{}y + \boxed{})$ *$a = 4$ and $c = -2$; Outer + Inner = 7*

Factors of 4	Factors of -2	Outer + Inner	
1 and 4	1 and -2	$1(-2) + 4(1) = 2$	✗
1 and 4	-1 and 2	$1(2) + 4(-1) = -2$	✗
1 and 4	2 and -1	$1(-1) + 4(2) = 7$	✓

$(y + 2)(4y - 1)$

Check $(y + 2)(4y - 1) = 4y^2 - y + 8y - 2$ *Use the FOIL method.*

                 $= 4y^2 + 7y - 2$ ✓

B $4x^2 + 19x - 5$

$(\boxed{}x + \boxed{})(\boxed{}x + \boxed{})$ *$a = 4$ and $c = -5$; Outer + Inner = 19*

Factors of 4	Factors of -5	Outer + Inner	
1 and 4	1 and -5	$1(-5) + 4(1) = -1$	✗
1 and 4	-1 and 5	$1(5) + 4(-1) = 1$	✗
1 and 4	5 and -1	$1(-1) + 4(5) = 19$	✓

$(x + 5)(4x - 1)$

Check $(x + 5)(4x - 1) = 4x^2 - x + 20x - 5$ *Use the FOIL method.*

                 $= 4x^2 + 19x - 5$ ✓

C $2x^2 - 7x - 15$

$(\boxed{}x + \boxed{})(\boxed{}x + \boxed{})$ *$a = 2$ and $c = -15$; Outer + Inner = -7*

Factors of 2	Factors of -15	Outer + Inner	
1 and 2	1 and -15	$1(-15) + 2(1) = -13$	✗
1 and 2	-1 and 15	$1(15) + 2(-1) = 13$	✗
1 and 2	3 and -5	$1(-5) + 2(3) = 1$	✗
1 and 2	-3 and 5	$1(5) + 2(-3) = -1$	✗
1 and 2	5 and -3	$1(-3) + 2(5) = 7$	✗
1 and 2	-5 and 3	$1(3) + 2(-5) = -7$	✓

$(x - 5)(2x + 3)$

Check $(x - 5)(2x + 3) = 2x^2 + 3x - 10x - 15$ *Use the FOIL method.*

                 $= 2x^2 - 7x - 15$ ✓

CHECK IT OUT! Factor each trinomial. Check your answer.

3a. $6x^2 + 7x - 3$ **3b.** $4n^2 - n - 3$

Factoring $ax^2 + bx + c$

Reggie Wilson
Franklin High School

When a, b, and c are positive, I like to use a box to help me factor. I look for factors of ac that add to b. Then I arrange the terms in a box and factor.

To factor $6x^2 + 7x + 2$, first I find the factors I need.

$ac = 2(6) = 12 \quad b = 7$

Factors of 12	Sum
1 and 12	13
2 and 6	8
3 and 4	7

Then I rewrite the trinomial as $6x^2 + 3x + 4x + 2$.

Now I arrange $6x^2 + 3x + 4x + 2$ in a box and factor out the common factors from each row and column.

$6x^2$	$3x$	→ **3x**
$4x$	2	→ **2**
2x	**1**	

The factors are $(2x + 1)$ and $(3x + 2)$.

When the leading coefficient is negative, factor out -1 from each term before using other factoring methods.

EXAMPLE 4 Factoring $ax^2 + bx + c$ When a Is Negative

Factor $-2x^2 - 15x - 7$.

$-1(2x^2 + 15x + 7)$ *Factor out −1.*

$-1(\square x + \square)(\square x + \square)$ *$a = 2$ and $c = 7$; Outer + Inner = 15*

Factors of 2	Factors of 7	Outer + Inner	
1 and 2	1 and 7	$1(7) + 2(1) = 9$	✗
1 and 2	7 and 1	$1(1) + 2(7) = 15$	✓

$(x + 7)(2x + 1)$

$-1(x + 7)(2x + 1)$

Caution!
When you factor out −1 in an early step, you must carry it through the rest of the steps.

CHECK IT OUT! Factor each trinomial. Check your answer.

4a. $-6x^2 - 17x - 12$ **4b.** $-3x^2 - 17x - 10$

THINK AND DISCUSS

1. Let a, b, and c be positive. If $ax^2 + bx + c$ is the product of two binomials, what do you know about the signs of the numbers in the binomials?

2. GET ORGANIZED Copy and complete the graphic organizer. Write each of the following trinomials in the appropriate box and factor each one.

$3x^2 + 10x - 8$ $3x^2 + 10x + 8$
$3x^2 - 10x + 8$ $3x^2 - 10x - 8$

Know it! Note

Factoring $ax^2 + bx + c$	
$c > 0$	
$b > 0$	$b < 0$
$c < 0$	
$b < 0$	$b > 0$

8-4

Exercises

go.hrw.com
Homework Help Online
KEYWORD: MA7 8-4
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

SEE EXAMPLE **1**
p. 568

Factor each trinomial by guess and check.

1. $2x^2 + 9x + 10$ **2.** $5x^2 + 31x + 6$ **3.** $5x^2 + 7x - 6$

4. $6x^2 + 37x + 6$ **5.** $3x^2 - 14x - 24$ **6.** $6x^2 + x - 2$

Factor each trinomial. Check your answer.

SEE EXAMPLE **2**
p. 569

7. $5x^2 + 11x + 2$ **8.** $2x^2 + 11x + 5$ **9.** $4x^2 - 9x + 5$

10. $2y^2 - 11y + 14$ **11.** $5x^2 + 9x + 4$ **12.** $3x^2 + 7x + 2$

SEE EXAMPLE **3**
p. 570

13. $4a^2 + 8a - 5$ **14.** $15x^2 + 4x - 3$ **15.** $2x^2 + x - 6$

16. $6n^2 - 11n - 10$ **17.** $10x^2 - 9x - 1$ **18.** $7x^2 - 3x - 10$

SEE EXAMPLE **4**
p. 571

19. $-2x^2 + 5x + 12$ **20.** $-4n^2 - 16n + 9$ **21.** $-5x^2 + 7x + 6$

22. $-6x^2 + 13x - 2$ **23.** $-4x^2 - 8x + 5$ **24.** $-5x^2 + x + 18$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
25–33	1
34–42	2
43–48	3
49–51	4

Extra Practice
Skills Practice p. S19
Application Practice p. S35

Factor each trinomial by guess and check.

25. $9x^2 + 9x + 2$ **26.** $2x^2 + 7x + 5$ **27.** $3n^2 + 8n + 4$

28. $10d^2 + 17d + 7$ **29.** $4c^2 - 17c + 15$ **30.** $6x^2 + 14x + 4$

31. $8x^2 + 22x + 5$ **32.** $6x^2 - 13x + 6$ **33.** $5x^2 + 9x - 18$

Factor each trinomial. Check your answer.

34. $6x^2 + 23x + 7$ **35.** $10n^2 - 17n + 7$ **36.** $3x^2 + 11x + 6$

37. $7x^2 + 15x + 2$ **38.** $3n^2 + 4n + 1$ **39.** $3x^2 - 19x + 20$

40. $6x^2 + 11x + 4$ **41.** $4x^2 - 31x + 21$ **42.** $10x^2 + 31x + 15$

43. $12y^2 + 17y - 5$ **44.** $3x^2 + 10x - 8$ **45.** $4x^2 + 4x - 3$

46. $2n^2 - 7n - 4$ **47.** $3x^2 - 4x - 15$ **48.** $3n^2 - n - 4$

49. $-4x^2 - 4x + 15$ **50.** $-3x^2 + 16x - 16$ **51.** $-3x^2 - x + 2$

Geometry For Exercises 52–54, write the polynomial modeled and then factor.

52.

$12x^2$	$24x$
$3x$	6

53.

$2x^2$	$-x$
$-4x$	2

54.

$5x^2$	$-4x$
$35x$	-28

Factor each trinomial, if possible.

55. $9n^2 + 17n + 8$ **56.** $2x^2 - 7x - 4$ **57.** $4x^2 - 12x + 5$

58. $5x^2 - 4x + 12$ **59.** $3x^2 + 14x + 16$ **60.** $-3x^2 - 11x + 4$

61. $6x^2 - x - 12$ **62.** $10a^2 + 11a + 3$ **63.** $4x^2 - 12x + 9$

64. Geometry The area of a rectangle is $(6x^2 + 11x + 5)$ cm^2. The width is $(x + 1)$ cm. What is the length of the rectangle?

(x + 1) cm

65. Write About It Write a paragraph describing how to factor $6x^2 + 13x + 6$. Show each step you would take and explain your steps.

Complete each factorization.

66.

$8x^2 + 18x - 5$

$8x^2 + 20x - 2x - 5$

$(8x^2 + 20x) - (2x + 5)$

$\blacksquare(\blacksquare + \blacksquare) - \blacksquare(2x + 5)$

$(\blacksquare - \blacksquare)(2x + 5)$

67.

$4x^2 + 9x + 2$

$4x^2 + 8x + x + 2$

$(4x^2 + 8x) + (x + 2)$

$\blacksquare(\blacksquare + \blacksquare) + \blacksquare(x + 2)$

$(\blacksquare + \blacksquare)(x + 2)$

68. Gardening The length of Rebecca's rectangular garden was two times the width w. Rebecca increased the length and width of the garden so that the area of the new garden is $(2w^2 + 7w + 6)$ square yards. By how much did Rebecca increase the length and the width?

|← 2w ──────|── ? ─→|

W

?
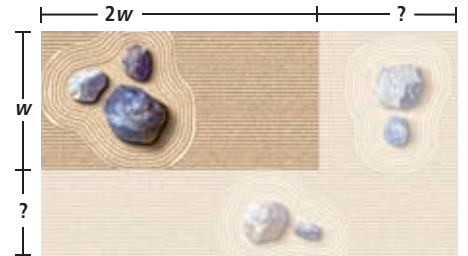
69. Physics The height of a football that has been thrown or kicked can be described by the expression $-16t^2 + vt + h$ where t is the time in seconds, v is the initial upward velocity, and h is the initial height in feet.

a. Write an expression for the height of a football at time t when the initial upward velocity is 20 feet per second and the initial height is 6 feet.

b. Factor your expression from part **a**.

c. Find the height of the football after 1 second.

70. ///ERROR ANALYSIS/// A student attempted to factor $2x^2 + 11x + 12$ as shown. Find and explain the error.

$2x^2 + 11x + 12$

Factors of 12	Sum	
1 and 12	13	✓
2 and 6	8	✗
3 and 4	7	✗

$(2x + 1)(x + 12)$

MULTI-STEP TEST PREP

71. This problem will prepare you for the Multi-Step Test Prep on page 576.

The equation $d = 2t^2$ gives the distance from the start point of a toy boat that starts at rest and accelerates at 4 cm/s^2. The equation $d = 10t - 8$ gives the distance from the start point of a second boat that starts at rest 8 cm behind the first boat and travels at a constant rate of 10 cm/s.

a. By setting the equations equal to each other, you can determine when the cars are the same distance from the start point: $2t^2 = 10t - 8$. Use properties of algebra to collect all terms on the left side of the equation, leaving 0 on the right side.

b. Factor the expression on the left side of the equation.

c. The boats are the same distance from the start point at $t = 1$ and $t = 4$. Explain how the factors you found in part **b** were used to find these two times.

Match each trinomial with its correct factorization.

72. $6x^2 - 29x - 5$ **A.** $(x + 5)(6x + 1)$

73. $6x^2 - 31x + 5$ **B.** $(x - 5)(6x - 1)$

74. $6x^2 + 31x + 5$ **C.** $(x + 5)(6x - 1)$

75. $6x^2 + 29x - 5$ **D.** $(x - 5)(6x + 1)$

76. **Critical Thinking** The quadratic trinomial $ax^2 + bx + c$ has $a > 0$ and can be factored into the product of two binomials.

   a. Explain what you know about the signs of the constants in the factors if $c > 0$.

   b. Explain what you know about the signs of the constants in the factors if $c < 0$.

TEST PREP

77. What value of b would make $3x^2 + bx - 8$ factorable?

   Ⓐ 3   Ⓑ 10   Ⓒ 11   Ⓓ 25

78. Which product of binomials is represented by the model?

   Ⓕ $(x + 4)(3x + 5)$ Ⓗ $(x + 3)(5x + 4)$

   Ⓖ $(x + 4)(5x + 3)$ Ⓙ $(x + 5)(3x + 4)$

$5x^2$	$4x$
$15x$	12

79. Which binomial is a factor of $24x^2 - 49x + 2$?

   Ⓐ $x - 2$ Ⓑ $x - 1$ Ⓒ $x + 1$ Ⓓ $x + 2$

80. Which value of c would make $2x^2 + x + c$ NOT factorable?

   Ⓕ -15 Ⓖ -9 Ⓗ -6 Ⓙ -1

CHALLENGE AND EXTEND

Factor each trinomial. Check your answer.

81. $1 + 4x + 4x^2$ **82.** $1 - 14x + 49x^2$ **83.** $1 + 18x + 81x^2$

84. $25 + 30x + 9x^2$ **85.** $4 + 20x + 25x^2$ **86.** $4 - 12x + 9x^2$

87. Find all possible values of b such that $3x^2 + bx + 2$ can be factored.

88. Find all possible values of b such that $3x^2 + bx - 2$ can be factored.

89. Find all possible values of b such that $5x^2 + bx + 1$ can be factored.

SPIRAL REVIEW

90. Archie makes \$12 per hour. The function $f(x) = 12x$ gives the amount of money that Archie makes in x hours. Graph this function and give its domain and range. *(Lesson 5-1)*

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions. *(Lesson 6-6)*

91. $\begin{cases} y < -2x + 1 \\ y > 3x - 5 \end{cases}$ **92.** $\begin{cases} y \geq -x + 2 \\ y \leq x - 3 \end{cases}$ **93.** $\begin{cases} y \leq -4x \\ y > 2x - 6 \end{cases}$

Factor each trinomial. Check your answer. *(Lesson 8-3)*

94. $x^2 + 6x + 8$ **95.** $x^2 - 8x - 9$ **96.** $x^2 - 8x + 12$

Use a Graph to Factor Polynomials

You can use a graphing calculator to help factor polynomials.

Activity

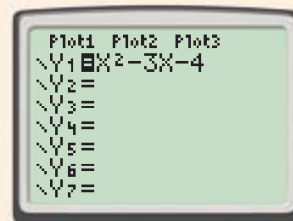Factor $x^2 - 3x - 4$ using algebra and check your factorization using a graphing calculator.

1 $x^2 - 3x - 4$

$(x + \boxed{})(x + \boxed{})$ *$b = -3$ and $c = -4$; look for factors of -4 whose sum is -3.*

$(x - 4)(x + 1)$ *$-4(1) = -4$; $-4 + 1 = -3$*

2 Press [Y=] and enter $x^2 - 3x - 4$ for **Y1**.

```
Plot1 Plot2 Plot3
\Y1◼X²-3X-4
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

3 Press [GRAPH] to view the graph of the equation.

4 Press [TRACE] and use the left and right buttons to move the cursor along the graph. The graph appears to cross the x-axis at $x = -1$ and $x = 4$.

$x = -1$ $x = 4$

```
Y1=X²-3X-4
X=3.1914894  Y=-3.388864
```
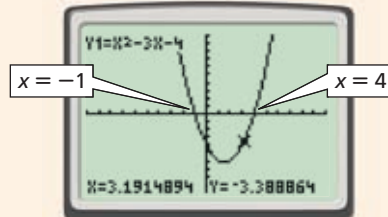
5 To find the value of y at $x = -1$, enter -1 and press [ENTER] while in *Trace* mode. The calculator gives you a value for y. Then enter 4 to find the value of y at $x = 4$.

The calculator tells you that $y = 0$ at $x = -1$ and at $x = 4$.

Notice that for a function with a binomial factor of the form $(x - a)$, it appears that a is an x-intercept.

```
Y1=X²-3X-4
X=-1    Y=0
```
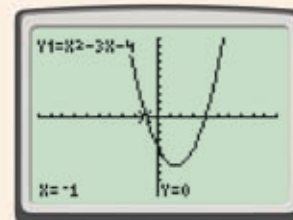
Try This

Graph each trinomial and use the graph to predict the factors. Then factor each trinomial using algebra.

1. $x^2 - x - 2$ **2.** $x^2 + 5x + 6$ **3.** $x^2 + x - 12$

4. $x^2 + 12x - 64$ **5.** $x^2 - 4x - 5$ **6.** $3x^2 + 16x - 12$

MULTI-STEP TEST PREP
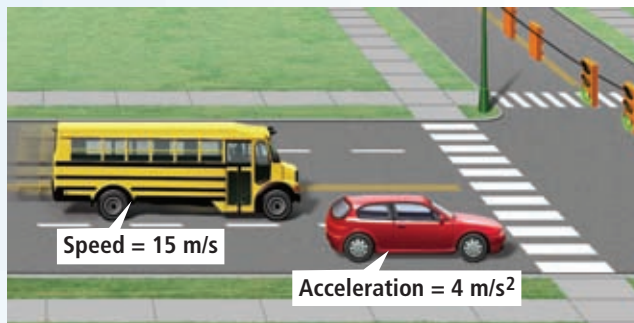
Factoring

Red Light, Green Light The equation for the motion of an object with constant acceleration is $d = vt + \frac{1}{2}at^2$ where d is distance traveled in meters, v is starting velocity in m/s, a is acceleration in m/s^2, and t is time in seconds.

1. A car is stopped at a traffic light. The light changes to green and the driver starts to drive, accelerating at a rate of 4 m/s^2. Write an equation for the distance the car travels in time t.

Speed = 15 m/s

Acceleration = 4 m/s^2

2. A bus is traveling at a speed of 15 m/s. The driver approaches the same traffic light in another traffic lane. He does not brake, and continues at the same speed. Write an equation for the distance the bus travels in time t. (*Hint:* At a constant speed, the acceleration is 0 m/s^2.)

3. Set the equations equal to each other so you can determine when the car and bus are the same distance from the intersection. Collect all the terms on the left side of this new equation, leaving 0 on the right side. Factor the expression on the left side of the equation.

4. Let $t = 0$ be the point at which the car is just starting to drive and the bus is even with the car. Find the other time when the vehicles will be the same distance from the intersection.

5. What distance will the two vehicles have traveled when they are again at the same distance from the intersection?

6. A truck traveling at 16 m/s is 24 meters behind the bus at $t = 0$. The equation $d = -24 + 16t$ gives the position of the truck. At what time will the truck be the same distance from the intersection as the bus? What will that distance be?

READY TO GO ON?

Quiz for Lessons 8-1 Through 8-4

8-1 Factors and Greatest Common Factors

Write the prime factorization of each number.

1. 54   **2.** 42   **3.** 50   **4.** 120   **5.** 44   **6.** 78

Find the GCF of each pair of monomials.

7. $6p^3$ and $2p$ **8.** $12x^3$ and $18x^4$

**9.** -15 and $20s^4$ **10.** $3a$ and $4b^2$

11. Brent is making a wooden display case for his baseball collection. He has 24 balls from American League games and 30 balls from National League games. He wants to display the same number of baseballs in each row and does not want to put American League baseballs in the same row as National League baseballs. How many rows will Brent need in the display case to put the greatest number of baseballs possible in each row?

8-2 Factoring by GCF

Factor each polynomial. Check your answer.

12. $2d^3 + 4d$ **13.** $m^2 - 8m^5$

14. $12x^4 - 8x^3 - 4x^2$ **15.** $3k^2 + 6k - 3$

16. The surface area of a cone can be found using the expression $s\pi r + \pi r^2$, where s represents the slant height and r represents the radius of the base. Factor this expression.
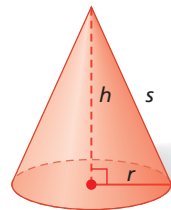
Factor each polynomial by grouping. Check your answer.

17. $w^3 - 4w^2 + w - 4$ **18.** $3x^3 + 6x^2 - 4x - 8$

19. $2p^3 - 6p^2 + 15 - 5p$ **20.** $n^3 - 6n^2 + 5n - 30$

8-3 Factoring $x^2 + bx + c$

Factor each trinomial. Check your answer.

21. $n^2 + 9n + 20$ **22.** $d^2 - 6d - 7$ **23.** $x^2 - 6x + 8$

24. $y^2 + 7y - 30$ **25.** $k^2 - 6k + 5$ **26.** $c^2 - 10c + 24$

27. Simplify and factor the expression $n(n + 3) - 4$. Show that the original expression and the factored form have the same value for $n = 0, 1, 2, 3,$ and 4.

8-4 Factoring $ax^2 + bx + c$

Factor each trinomial. Check your answer.

28. $2x^2 + 11x + 5$ **29.** $3n^2 + 16n + 21$ **30.** $5y^2 - 7y - 6$

31. $4g^2 - 10g + 6$ **32.** $6p^2 - 18p - 24$ **33.** $12d^2 + 7d - 12$

34. The area of a rectangle is $(8x^2 + 8x + 2)$ cm^2. The width is $(2x + 1)$ cm. What is the length of the rectangle?

Factoring Special Products

Objectives

Factor perfect-square trinomials.

Factor the difference of two squares.

Who uses this?

Urban planners can use the area of a square park to find its length and width. (See Example 2.)

You studied the patterns of some special products of binomials in Chapter 7. You can use those patterns to factor certain polynomials.

A trinomial is a perfect square if:
- The **first** and **last** terms are perfect squares.
- The **middle** term is two times one factor from the first term and one factor from the last term.

$$9x^2 \quad + \quad 12x \quad + \quad 4$$
$$3x \cdot 3x \quad 2(3x \cdot 2) \quad 2 \cdot 2$$

Know it! Note

Perfect-Square Trinomials	
PERFECT-SQUARE TRINOMIAL	**EXAMPLES**
$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$	$x^2 + 6x + 9 = (x + 3)(x + 3) = (x + 3)^2$
$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$	$x^2 - 2x + 1 = (x - 1)(x - 1) = (x - 1)^2$

EXAMPLE **1** **Recognizing and Factoring Perfect-Square Trinomials**

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

A $x^2 + 12x + 36$

$$x^2 + 12x + 36$$
$$x \cdot x \quad 2(x \cdot 6) \quad 6 \cdot 6$$

The trinomial is a perfect square. Factor.

Method 1 Factor.

$x^2 + 12x + 36$

Factors of 36	Sum	
1 and 36	37	✗
2 and 18	20	✗
3 and 12	15	✗
4 and 9	13	✗
6 and **6**	12	✓

$(x + 6)(x + 6)$

Method 2 Use the rule.

$x^2 + 12x + 36$ $a = x, b = 6$

$x^2 + 2(x)(6) + 6^2$ *Write the trinomial as $a^2 + 2ab + b^2$.*

$(x + 6)^2$ *Write the trinomial as $(a + b)^2$.*

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

B $4x^2 - 12x + 9$

$$4x^2 - 12x + 9$$

$2x \cdot 2x \quad 2(2x \cdot 3) \quad 3 \cdot 3$ *The trinomial is a perfect square. Factor.*

$4x^2 - 12x + 9$	*a = 2x, b = 3*
$(2x)^2 - 2(2x)(3) + 3^2$	$a^2 - 2ab + b^2$
$(2x - 3)^2$	$(a - b)^2$

C $x^2 + 9x + 16$

$$x^2 + 9x + 16$$

$x \cdot x \quad 2(x \cdot 4) \quad 4 \cdot 4$ $2(x \cdot 4) \neq 9x$

$x^2 + 9x + 16$ is not a perfect-square trinomial because $9x \neq 2(x \cdot 4)$.

CHECK IT OUT! Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

1a. $x^2 + 4x + 4$ **1b.** $x^2 - 14x + 49$ **1c.** $9x^2 - 6x + 4$

EXAMPLE 2   *Problem-Solving Application*

PROBLEM
SOLVING

The park in the center of the Place des Vosges in Paris, France, is in the shape of a square. The area of the park is $(25x^2 + 70x + 49)$ ft². The side length of the park is in the form $cx + d$, where c and d are whole numbers. Find an expression in terms of x for the perimeter of the park. Find the perimeter when $x = 8$ ft.

1 ▸ **Understand the Problem**

The **answer** will be an expression for the perimeter of the park and the value of the expression when $x = 8$.

List the **important information:**
- The park is a square with area $(25x^2 + 70x + 49)$ ft².
- The side length of the park is in the form $cx + d$, where c and d are whole numbers.

2   **Make a Plan**

The formula for the area of a square is area = $(\text{side})^2$.

Factor $25x^2 + 70x + 49$ to find the side length of the park. Write a formula for the perimeter of the park, and evaluate the expression for $x = 8$.

3 **Solve**

$$25x^2 + 70x + 49 \qquad a = 5x,\ b = 7$$
$$(5x)^2 + 2(5x)(7) + 7^2 \quad \textit{Write the trinomial as } a^2 + 2ab + b^2.$$
$$(5x + 7)^2 \qquad \textit{Write the trinomial as } (a + b)^2.$$

$$25x^2 + 70x + 49 = (5x + 7)(5x + 7)$$

The side length of the park is $(5x + 7)$ ft.

Write a formula for the perimeter of the park.

$$P = 4s \qquad\qquad \textit{Write the formula for the perimeter of a square.}$$
$$= 4(5x + 7) \qquad \textit{Substitute the side length for s.}$$
$$= 20x + 28 \qquad \textit{Distribute 4.}$$

An expression for the perimeter of the park in feet is $20x + 28$.

Evaluate the expression when $x = 8$.

$$P = 20x + 28$$
$$= 20(8) + 28 \qquad \textit{Substitute 8 for x.}$$
$$= 188$$

When $x = 8$ ft, the perimeter of the park is 188 ft.

4 **Look Back**

For a square with a perimeter of 188 ft, the side length is $\frac{188}{4} = 47$ ft and the area is $47^2 = 2209$ ft^2.

Evaluate $25x^2 + 70x + 49$ for $x = 8$:
$$25(8)^2 + 70(8) + 49$$
$$1600 + 560 + 49$$
$$2209\ ✓$$

CHECK IT OUT!

2. **What if...?** A company produces square sheets of aluminum, each of which has an area of $(9x^2 + 6x + 1)$ m^2. The side length of each sheet is in the form $cx + d$, where c and d are whole numbers. Find an expression in terms of x for the perimeter of a sheet. Find the perimeter when $x = 3$ m.

In Chapter 7 you learned that the difference of two squares has the form $a^2 - b^2$. The difference of two squares can be written as the product $(a + b)(a - b)$. You can use this pattern to factor some polynomials.

A polynomial is a difference of two squares if:
  • There are two terms, one subtracted from the other.
  • Both terms are perfect squares.

$$4x^2 - 9$$
$$2x \cdot 2x \quad 3 \cdot 3$$

Know it! Note

Difference of Two Squares

DIFFERENCE OF TWO SQUARES	EXAMPLE
$a^2 - b^2 = (a + b)(a - b)$	$x^2 - 9 = (x + 3)(x - 3)$

EXAMPLE 3

Recognizing and Factoring the Difference of Two Squares

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

A $x^2 - 81$

$$x^2 - 81$$

$x \cdot x \qquad 9 \cdot 9$ *The polynomial is a difference of two squares.*

$$x^2 - 9^2$$ *a = x, b = 9*

$$(x + 9)(x - 9)$$ *Write the polynomial as $(a + b)(a - b)$.*

$$x^2 - 81 = (x + 9)(x - 9)$$

B $9p^4 - 16q^2$

$$9p^4 - 16q^2$$

$3p^2 \cdot 3p^2 \qquad 4q \cdot 4q$ *The polynomial is a difference of two squares.*

$$(3p^2)^2 - (4q)^2$$ *a = 3p², b = 4q*

$$(3p^2 + 4q)(3p^2 - 4q)$$ *Write the polynomial as $(a + b)(a - b)$.*

$$9p^4 - 16q^2 = (3p^2 + 4q)(3p^2 - 4q)$$

C $x^6 - 7y^2$

$$x^6 - 7y^2$$

$x^3 \cdot x^3$ *$7y^2$ is not a perfect square.*

$x^6 - 7y^2$ is not the difference of two squares because $7y^2$ is not a perfect square.

✓ CHECK IT OUT! Determine whether the binomial is a difference of two squares. If so, factor. If not, explain.

3a. $1 - 4x^2$ **3b.** $p^8 - 49q^6$ **3c.** $16x^2 - 4y^5$

THINK AND DISCUSS

1. The binomial $1 - x^4$ is a difference of two squares. Use the rule to identify a and b in $1 - x^4$.

2. The polynomial $x^2 + 8x + 16$ is a perfect-square trinomial. Use the rule to identify a and b in $x^2 + 8x + 16$.

3. **GET ORGANIZED** Copy and complete the graphic organizer. Write an example of each type of special product and factor it.

Special Product	Factored Form
Perfect-square trinomial with positive coefficient of middle term	
Perfect-square trinomial with negative coefficient of middle term	
Difference of two squares	

GUIDED PRACTICE

SEE EXAMPLE 1
p. 578

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

1. $x^2 - 4x + 4$ **2.** $x^2 - 4x - 4$ **3.** $9x^2 - 12x + 4$

4. $x^2 + 2x + 1$ **5.** $x^2 - 6x + 9$ **6.** $x^2 - 6x - 9$

SEE EXAMPLE 2
p. 579

7. City Planning A city purchases a square plot of land with an area of $(x^2 + 24x + 144)$ yd² for a park. The dimensions of the plot are of the form $ax + b$, where a and b are whole numbers. Find an expression for the perimeter of the park. Find the perimeter when $x = 10$ yd.

SEE EXAMPLE 3
p. 581

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

8. $1 - 4x^2$ **9.** $s^2 - 4^2$ **10.** $81x^2 - 1$

11. $4x^4 - 9y^2$ **12.** $x^8 - 50$ **13.** $x^6 - 9$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14–19	1
20	2
21–26	3

Extra Practice
Skills Practice p. S19
Application Practice p. S35

Determine whether the trinomial is a perfect square. If so, factor. If not, explain.

14. $4x^2 - 4x + 1$ **15.** $4x^2 - 4x - 1$ **16.** $36x^2 - 12x + 1$

17. $25x^2 + 10x + 4$ **18.** $9x^2 + 18x + 9$ **19.** $16x^2 - 40x + 25$

20. Measurement You are given a sheet of paper and told to cut out a square piece with an area of $(4x^2 - 44x + 121)$ mm². The dimensions of the square have the form $ax - b$, where a and b are whole numbers. Find an expression for the perimeter of the square you cut out. Find the perimeter when $x = 41$ mm.

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

21. $1^2 - 4x^2$ **22.** $25m^2 - 16n^2$ **23.** $4x - 9y$

24. $49p^{12} - 9q^6$ **25.** $9^2 - 100x^4$ **26.** $x^3 - y^3$

Find the missing term in each perfect-square trinomial.

27. $x^2 + 14x + \blacksquare$ **28.** $9x^2 + \blacksquare + 25$ **29.** $\blacksquare - 36y + 81$

Factor each polynomial using the rule for perfect-square trinomials or the rule for a difference of two squares. Tell which rule you used.

30. $x^2 - 8x + 16$ **31.** $100x^2 - 81y^2$ **32.** $36x^2 + 24x + 4$

33. $4r^6 - 25s^6$ **34.** $49x^2 - 70x + 25$ **35.** $x^{14} - 144$

36. Write About It What is similar about a perfect-square trinomial and a difference of two squares? What is different?

37. Critical Thinking Describe two ways to create a perfect-square trinomial.

38. For what value of b would $(x + b)(x + b)$ be the factored form of $x^2 - 22x + 121$?

**39.** For what value of c are the factors of $x^2 + cx + 256$ the same?

40. This problem will prepare you for the Multi-Step Test Prep on page 592.

Juanita designed a vegetable garden in the shape of a square and purchased fencing for that design. Then she decided to change the design to a rectangle.

  a. The square garden had an area of x^2 ft². The area of the rectangular garden is $(x^2 - 25)$ ft². Factor this expression.

  b. The rectangular garden must have the same perimeter as the square garden, so Juanita added a number of feet to the length and subtracted the same number of feet from the width. Use your factors from part **a** to determine how many feet were added to the length and subtracted from the width.

  c. If the original length of the square garden was 8 feet, what are the length and width of the new garden?

41. Multi-Step The area of a square is represented by $25z^2 - 40z + 16$.

  a. What expression represents the length of a side of the square?

  b. What expression represents the perimeter of the square?

  c. What are the length of a side, the perimeter, and the area of the square when $z = 3$?

42. Multi-Step A small rectangle is drawn inside a larger rectangle as shown.
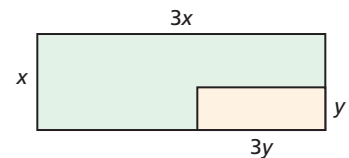
  a. What is the area of each rectangle?

  b. What is the area of the green region?

  c. Factor the expression for the area of the green region. (*Hint:* First factor out the common factor of 3 and then factor the binomial.)
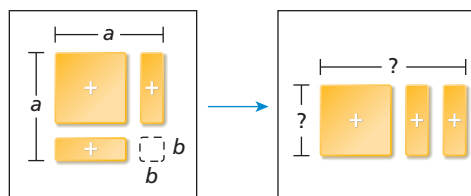
43. Evaluate each expression for the values of x.

	x	$x^2 + 10x + 25$	$(x + 5)^2$	$(x - 5)^2$	$x^2 - 10x + 25$	$x^2 - 25$
a.	−5	■	■	■	■	■
b.	−1	■	■	■	■	■
c.	0	■	■	■	■	■
d.	1	■	■	■	■	■
e.	5	■	■	■	■	■

44. In the table above, which columns have equivalent values? Explain why.

45. Geometry A model for the difference of two squares is shown below. Copy and complete the second figure by writing the missing labels.

46. ⫻**ERROR ANALYSIS**⫻ Two students factored $25x^4 - 9y^2$. Which is incorrect? Explain the error.

A
$25x^4 - 9y^2$
$(5x - 3)(5x + 3)$

B
$25x^4 - 9y^2$
$(5x^2 - 3y)(5x^2 + 3y)$

47. A polynomial expression is evaluated for the x- and y-values shown in the table. Which expression could have been evaluated to give the values shown in the third column?

x	y	Value of Expression
0	0	0
−1	−1	0
1	1	0
1	−1	4

  Ⓐ $x^2 - y^2$
  Ⓑ $x^2 + 2xy + y^2$
  Ⓒ $x^2 - 2xy + y^2$
  Ⓓ None of the above

48. The area of a square is $4x^2 + 20x + 25$. Which expression can also be used to model the area of the square?

  Ⓕ $(2x - 5)(5 - 2x)$ Ⓗ $(2x - 5)^2$
  Ⓖ $(2x + 5)(2x - 5)$ Ⓙ $(2x + 5)^2$

49. Gridded Response Evaluate the polynomial expression $x^2 - 18x + 81$ for $x = 10$.

CHALLENGE AND EXTEND

50. The binomial $81x^4 - 16$ can be factored using the rule for a difference of two squares.

  a. Fill in the factorization: $81x^4 - 16$
                             $\left(9x^2 + \blacksquare\right)\left(\blacksquare - \blacksquare\right)$

  b. One binomial from part **a** can be further factored. Identify the binomial and factor it.

  c. Write your own binomial that can be factored twice as the difference of two squares.

51. The expression $4 - (v + 2)^2$ is the difference of two squares, because it fits the rule $a^2 - b^2$.

  a. Identify a and b in the expression.

  b. Factor and simplify $4 - (v + 2)^2$.

The *difference of cubes* is an expression of the form $a^3 - b^3$. It can be factored according to the rule $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$. For each binomial, identify a and b, and factor using the rule.

52. $x^3 - 1$ **53.** $27y^3 - 64$ **54.** $n^6 - 8$

SPIRAL REVIEW

Find the domain and range for each relation and tell whether the relation is a function. *(Lesson 4-2)*

55. $\{(5, 2), (4, 1), (3, 0), (2, -1)\}$ **56.** $\{(-3, 6), (-1, 6), (1, 6), (3, 6)\}$

57. $\{(2, -8), (2, -2), (2, 4), (2, 10)\}$ **58.** $\{(-2, 4), (-1, 1), (0, 0), (1, 1)\}$

Multiply. *(Lesson 7-8)*
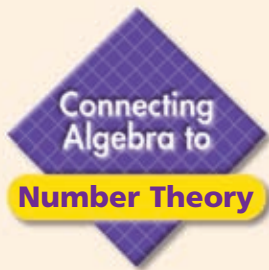
59. $2a(3a^2 + 7a - 5)$ **60.** $(x + 3)(x - 8)$ **61.** $(t - 4)^2$

Find the GCF of each pair of monomials. *(Lesson 8-1)*

62. $9m^2$ and $3m^2$ **63.** $8c^2$ and $8d^2$ **64.** $-12x^3y$ and $16y^2$

Mentally Math

Number Theory Recognizing patterns of special products can help you perform multiplication mentally.

Remember these special products that you studied in Chapters 7 and 8.

Patterns of Special Products	
Difference of Two Squares	$(a + b)(a - b) = a^2 - b^2$
Perfect-Square Trinomial	$(a + b)^2 = a^2 + 2ab + b^2$ $(a - b)^2 = a^2 - 2ab + b^2$

Example 1

Simplify $17^2 - 7^2$.

This expression is a difference of two squares with $a = 17$ and $b = 7$.

$\quad a^2 - b^2 = (a + b)(a - b)$ *Write the rule for a difference of two squares.*

$\quad 17^2 - 7^2 = (17 + 7)(17 - 7)$ *Substitute 17 for a and 7 for b.*

$\quad\quad\quad\quad = (24)(10)$ *Simplify.*

$\quad\quad\quad\quad = 240$

Example 2

Simplify $14^2 + 2(14)(6) + 6^2$.

This expression is a perfect-square trinomial with $a = 14$ and $b = 6$.

$\quad a^2 + 2ab + b^2 = (a + b)^2$ *Write the rule for a perfect-square trinomial.*

$\quad 14^2 + 2(14)(6) + 6^2 = (14 + 6)^2$ *Substitute 14 for a and 6 for b.*

$\quad\quad\quad\quad\quad = (20)^2$ *Simplify.*

$\quad\quad\quad\quad\quad = 400$

Try This

Simplify each expression using the rules for special products.

1. $18^2 - 12^2$ **2.** $11^2 + 2(11)(14) + 14^2$ **3.** $22^2 - 18^2$

4. $38^2 - 2(38)(27) + 27^2$ **5.** $29^2 - 2(29)(17) + 17^2$ **6.** $55^2 + 2(55)(45) + 45^2$

7. $14^2 - 9^2$ **8.** $13^2 - 12^2$ **9.** $14^2 + 2(14)(16) + 16^2$

Choosing a Factoring Method

Objectives

Choose an appropriate method for factoring a polynomial.

Combine methods for factoring a polynomial.

Why learn this?

You will need to factor polynomials to solve quadratic equations, which have many applications in physics. (See Exercise 42.)

The height of a leaping ballet dancer can be modeled by a quadratic polynomial. Solving an equation that contains that polynomial may require factoring the polynomial.

Recall that a polynomial is in its fully factored form when it is written as a product that cannot be factored further.

EXAMPLE 1 | **Determining Whether an Expression Is Completely Factored**

Tell whether each expression is completely factored. If not, factor it.

A $2x(x^2 + 4)$

$2x(x^2 + 4)$ *Neither 2x nor $x^2 + 4$ can be factored further.*

$2x(x^2 + 4)$ is completely factored.

Caution!

$x^2 + 4$ is a *sum* of squares, and cannot be factored.

B $(2x + 6)(x + 5)$

$(\mathbf{2}x + \mathbf{6})(x + 5)$ *2x + 6 can be further factored.*

$2(x + 3)(x + 5)$ *Factor out 2, the GCF of 2x and 6.*

$2(x + 3)(x + 5)$ is completely factored.

CHECK IT OUT! Tell whether each expression is completely factored. If not, factor it.

1a. $5x^2(x - 1)$ **1b.** $(4x + 4)(x + 1)$

To factor a polynomial completely, you may need to use more than one factoring method. Use the steps below to factor a polynomial completely.

Factoring Polynomials
Step 1 Check for a greatest common factor.
Step 2 Check for a pattern that fits the difference of two squares or a perfect-square trinomial.
Step 3 To factor $x^2 + bx + c$, look for two numbers whose sum is b and whose product is c.   To factor $ax^2 + bx + c$, check factors of a and factors of c in the binomial factors. The sum of the products of the outer and inner terms should be b.
Step 4 Check for common factors.

EXAMPLE **2** **Factoring by GCF and Recognizing Patterns**

Factor $-2xy^2 + 16xy - 32x$ completely. Check your answer.

$-2xy^2 + 16xy - 32x$

$-2x(y^2 - 8y + 16)$ *Factor out the GCF. $y^2 - 8y + 16$ is a perfect-square trinomial of the form $a^2 - 2ab + b^2$.*

$-2x(y - 4)^2$ *$a = y$, $b = 4$*

Check $-2x(y - 4)^2 = -2x(y^2 - 8y + 16)$
$= -2xy^2 + 16xy - 32x$ ✓

CHECK IT OUT! Factor each polynomial completely. Check your answer.
2a. $4x^3 + 16x^2 + 16x$ **2b.** $2x^2y - 2y^3$

If none of the factoring methods work, the polynomial is unfactorable.

EXAMPLE **3** **Factoring by Multiple Methods**

Factor each polynomial completely.

A $2x^2 + 5x + 4$

$2x^2 + 5x + 4$ *The GCF is 1 and there is no pattern.*

$(\boxed{}x + \boxed{})(\boxed{}x + \boxed{})$ *$a = 2$ and $c = 4$; Outer + Inner = 5*

Remember!

For a polynomial of the form $ax^2 + bx + c$, if there are no integers whose sum is b and whose product is ac, then the polynomial is unfactorable.

Factors of 2	Factors of 4	Outer + Inner	
1 and 2	1 and 4	$1(4) + 2(1) = 6$	✗
1 and 2	4 and 1	$1(1) + 2(4) = 9$	✗
1 and 2	2 and 2	$1(2) + 2(2) = 6$	✗

$2x^2 + 5x + 4$ is unfactorable.

B $3n^4 - 15n^3 + 12n^2$

$3n^2(n^2 - 5n + 4)$ *Factor out the GCF. There is no pattern.*

$(n + \boxed{})(n + \boxed{})$ *$b = -5$ and $c = 4$; look for factors of 4 whose sum is -5.*

Factors of 4	Sum	
-1 and -4	-5	✓

The factors needed are -1 and -4.

$3n^2(n - 1)(n - 4)$

C $4x^3 + 18x^2 + 20x$

$2x(2x^2 + 9x + 10)$ *Factor out the GCF. There is no pattern.*

$(\boxed{}x + \boxed{})(\boxed{}x + \boxed{})$ *$a = 2$ and $c = 10$; Outer + Inner = 9*

Factors of 2	Factors of 10	Outer + Inner	
1 and 2	1 and 10	$1(10) + 2(1) = 12$	✗
1 and 2	10 and 1	$1(11) + 2(10) = 21$	✗
1 and 2	2 and 5	$1(5) + 2(2) = 9$	✓

$(x + 2)(2x + 5)$

$2x(x + 2)(2x + 5)$

D $p^5 - p$

$\quad p(p^4 - 1)$ *Factor out the GCF.*

$\quad p(p^2 + 1)(p^2 - 1)$ *$p^4 - 1$ is a difference of two squares.*

$\quad p(p^2 + 1)(p + 1)(p - 1)$ *$p^2 - 1$ is a difference of two squares.*

CHECK IT OUT! Factor each polynomial completely. Check your answer.

3a. $3x^2 + 7x + 4$ **3b.** $2p^5 + 10p^4 - 12p^3$

3c. $9q^6 + 30q^5 + 24q^4$ **3d.** $2x^4 + 18$

Know it! Note

Methods to Factor Polynomials

Any Polynomial—Look for the greatest common factor.

$ab - ac = a(b - c)$	$6x^2y + 10xy^2 = 2xy(3x + 5y)$

Binomials—Look for a difference of two squares.

$a^2 - b^2 = (a + b)(a - b)$	$x^2 - 9y^2 = (x + 3y)(x - 3y)$

Trinomials—Look for perfect-square trinomials and other factorable trinomials.

$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$	$x^2 + 4x + 4 = (x + 2)^2$ $x^2 - 2x + 1 = (x - 1)^2$
$x^2 + bx + c = (x + \square)(x + \square)$ $ax^2 + bx + c = (\square x + \square)(\square x + \square)$	$x^2 + 3x + 2 = (x + 1)(x + 2)$ $6x^2 + 7x + 2 = (2x + 1)(3x + 2)$

Polynomials of Four or More Terms—Factor by grouping.

$ax + bx + ay + by = x(a + b) + y(a + b)$ $\qquad\qquad\qquad\qquad = (x + y)(a + b)$	$2x^3 + 4x^2 + x + 2 = (2x^3 + 4x^2) + (x + 2)$ $\qquad\qquad\qquad\qquad = 2x^2(x + 2) + 1(x + 2)$ $\qquad\qquad\qquad\qquad = (x + 2)(2x^2 + 1)$

THINK AND DISCUSS

1. Give an expression that includes a polynomial that is not completely factored.

2. Give an example of an unfactorable binomial and an unfactorable trinomial.

Know it! Note

3. GET ORGANIZED Copy the graphic organizer. Draw an arrow from each expression to the method you would use to factor it.

Factoring Methods	
Polynomial	**Method**
1. $16x^4 - 25y^8$	**A.** Factoring out the GCF
2. $x^2 + 10x + 25$	**B.** Factoring by grouping
3. $9t^2 + 27t + 18t^4$	**C.** Unfactorable
4. $a^2 + 3a - 7a - 21$	**D.** Difference of two squares
5. $100b^2 + 81$	**E.** Perfect-square trinomial

8-6

Exercises

go.hrw.com
Homework Help Online
KEYWORD: MA7 8-6
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

SEE EXAMPLE 1
p. 586

Tell whether each expression is completely factored. If not, factor it.

1. $3x(9x^2 + 1)$ **2.** $2(4x^3 - 3x^2 - 8x)$ **3.** $2k^2(4 - k^3)$

4. $(2x + 3)(3x - 5)$ **5.** $4(4p^4 - 1)$ **6.** $a(a^3 + 2ab + b^2)$

Factor each polynomial completely. Check your answer.

SEE EXAMPLE 2
p. 587

7. $3x^5 - 12x^3$ **8.** $4x^3 + 8x^2 + 4x$ **9.** $8pq^2 + 8pq + 2p$

10. $18rs^2 - 2r$ **11.** $mn^5 - m^3n$ **12.** $2x^2y - 20xy + 50y$

SEE EXAMPLE 3
p. 587

13. $6x^4 - 3x^3 - 9x^2$ **14.** $3y^2 + 14y + 4$ **15.** $p^5 + 3p^3 + p^2 + 3$

16. $7x^5 + 21x^4 - 28x^3$ **17.** $2z^2 + 11z + 6$ **18.** $9p^2 - q^2 + 3p$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
19–24	1
25–30	2
31–36	3

Extra Practice
Skills Practice p. S19
Application Practice p. S35

Tell whether each expression is completely factored. If not, factor it.

19. $2x(y^3 - 4y^2 + 5y)$ **20.** $2r(25r^6 - 36)$ **21.** $3n^2(n^2 - 25)$

22. $2m(m + 1)(m + 4)$ **23.** $2y^2(4x^2 + 9)$ **24.** $4(7g + 9h^2)$

Factor each polynomial completely. Check your answer.

25. $-4x^3 + 24x^2 - 36x$ **26.** $24r^2 - 6r^4$ **27.** $5d^2 - 60d + 135$

28. $4y^8 + 36y^7 + 81y^6$ **29.** $98x^3 - 50xy^2$ **30.** $4x^3y - 4x^2y - 8xy$

31. $5x^2 - 10x + 14$ **32.** $121x^2 + 36y^2$ **33.** $p^4 - 16$

34. $4m^6 - 30m^5 + 36m^4$ **35.** $2k^3 + 3k^2 + 6k + 9$ **36.** $ab^4 - 16a$

Write an expression for each situation. Factor your expression.

37. the square of Ella's age plus 12 times Ella's age plus 36

38. the square of the distance from point A to point B minus 81

39. the square of the number of seconds Bob can hold his breath minus 16 times the number of seconds plus 28

40. three times the square of the number of apples on a tree minus 22 times the number of apples plus 35

41. the square of Beth's score minus 49

42. Physics The height in meters of a ballet dancer's center of mass when she leaps can be modeled by the polynomial $-5t^2 + 30t + 1$, where t is time in seconds after the jump. Tell whether the polynomial is fully factored when written as $-1(5t^2 - 30t - 1)$. Explain.

43. Write About It When asked to factor a polynomial completely, you first determine that the terms in the polynomial do not share any common factors. What would be your next step?

Factor and simplify each expression.

44. $12(x + 1)^2 + 60(x + 1) + 75$ **45.** $(2x + 3)^2 - (x - 4)^2$

46. $45x(x - 2)^2 + 60x(x - 2) + 20x$ **47.** $(3x - 5)^2 - (y + 2)^2$

MULTI-STEP TEST PREP

48. This problem will prepare you for the Multi-Step Test Prep on page 588.

    a. The area of a Marci's rectangular flower garden is $(x^2 + 2x - 15)$ ft². Factor this expression.

    b. Draw a diagram of the garden and label the length and width with your factors from part **a.**

    c. Find the length and width of the flower garden if $x = 7$ ft.

49. Critical Thinking Show two methods of factoring $4x^2 - 100$.

50. Estimation Estimate the value of $2x^2 + 5xy + 3y^2$ when $x = -10.1$ and $y = 10.05$. (*Hint:* Factor the expression first.)

51. /// ERROR ANALYSIS /// Examine the factorization shown. Explain why the factorization is incorrect.

$12x^2 - 12x - 3$
$3(4x^2 - 4x - 1)$
$3(2x - 1)(2x - 1)$

Math History

Blaise Pascal was a French mathematician who lived in the 1600s.

Math History Use the following information for Exercises 52–54.

The triangle at right is called *Pascal's Triangle.* The triangle starts with 1 and each of the other numbers in the triangle is the sum of the two numbers in the row above it.

0					1					
1						1		1		
2				1		2		1		
3			1		3		3		1	
4		1		4		6		4		1
5	1		5		10		10		5	1

Pascal's Triangle can be used to write the product of a binomial raised to an integer power. The numbers in each row give you the coefficients of each term in the product.

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

The numbers in row **3** are 1, 3, 3, 1. These are the coefficients of the terms in the product $(a + b)^3$. The power of a decreases in each term and the power of b increases in each term.

Use the patterns you see in Pascal's Triangle to write the power of the binomial $a + b$ given by each polynomial.

52. $a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6 = (a + b)^\blacksquare$

53. $a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 + 28a^2b^6 + 8ab^7 + b^8 = (a + b)^\blacksquare$

54. $a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7 = (a + b)^\blacksquare$

TEST PREP

55. Which expression equals $6x^2 + 7x - 10$?

    Ⓐ $(6x + 2)(x - 5)$ Ⓒ $(x + 2)(6x - 5)$

    Ⓑ $(2x + 5)(3x - 2)$ Ⓓ $(3x + 2)(2x - 5)$

56. What is the complete factorization of $16x^{12} - 256$?

    Ⓕ $16(x^6 + 4)(x^6 - 4)$ Ⓗ $16(x^6 + 4)(x^3 + 2)(x^3 - 2)$

    Ⓖ $(4x^6 + 16)(4x^6 - 16)$ Ⓙ $(4x^6 + 16)(2x^3 + 4)(2x^3 - 4)$

57. Which of the expressions below represents the fifth step of the factorization?

Step 1: $40a^3 - 60a^2 - 10a + 15$

Step 2: $5(8a^3 - 12a^2 - 2a + 3)$

Step 3: $5[(8a^3 - 12a^2) - (2a - 3)]$

Step 4: $5[4a^2(2a - 3) - 1(2a - 3)]$

Step 5: ▉

Step 6: $5(2a - 3)(2a + 1)(2a - 1)$

  (A) $5(2a - 3)(2a + 3)(4a^2 - 1)$ (C) $5(2a - 3)(4a^2 - 1)$

  (B) $5(2a - 3)(4a^2 + 1)$ (D) $5(2a - 3)(2a - 3)(4a^2 - 1)$

58. Short Response Use the polynomial $8x^3 + 24x^2 + 18x$ for the following.

  a. Factor the polynomial. Explain each step and tell whether you used any rules for special products.

  b. Explain another set of steps that could be used to factor the polynomial.

CHALLENGE AND EXTEND

59. Geometry The volume of the cylinder shown is represented by the expression $72\pi p^3 + 48\pi p^2 + 8\pi p$. The height of the cylinder is $8p$.

  a. Factor the expression for volume.

  b. What expression represents the radius of the cylinder?

  c. If the radius is 4 cm, what are the height and volume of the cylinder?
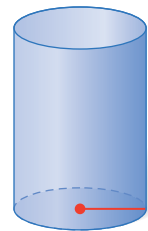
$V = \pi r^2 h$

Factor.

60. $g^7 + g^3 + g^5 + g^4$ **61.** $h^2 + h^8 + h^6 + h^4$
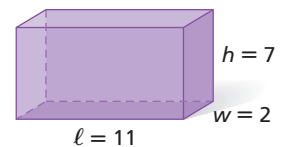
62. $x^{n+2} + x^{n+1} + x^n$ **63.** $x^{n+5} + x^{n+4} + x^{n+3}$

64. Geometry The rectangular prism has the dimensions shown.

  a. Write expressions for the height and length of the prism using w.

  b. Write a polynomial that represents the volume of the prism using w.

$h = 7$

$w = 2$

$\ell = 11$

SPIRAL REVIEW

Simplify each expression by combining like terms. *(Lesson 1-7)*

65. $-6n + 4n$ **66.** $5x^2 - 8x + 4x^2$ **67.** $2.6r + 9.7r$

Write and solve a proportion to answer each question. *(Lesson 2-7)*

68. The ratio of fiction to nonfiction books on Jessika's shelf is 3 to 4. Jessika has 12 nonfiction books. How many fiction books does she have?

69. The scale of a model car is $23:2$. The steering wheel on the model car has a diameter of 3 cm. What is the diameter of the steering wheel on the actual car?

Factor each trinomial. *(Lesson 8-4)*

70. $2x^2 + 13x + 15$ **71.** $4x^2 + 4x - 3$ **72.** $6x^2 - 11x - 10$

Factoring

Shaping the Environment The Environmental Awareness Club is going to plant a garden on the front lawn of the school. Henry suggests a garden in the shape of a square. Theona suggests a rectangular shape.

Width = ?

1. Henry's plans include a square garden with an area of $(x^2 + 12x + 36)$ m^2. Write expressions for the length and width of the square garden.

2. A drawing of the square garden shows a length of 12 m. What is the width of the square garden? What is the value of x? What is the total area of the square garden?

Length = 12 m

3. Theona's plans include a rectangular garden with an area of $(x^2 + 14x + 24)$ m^2. Write expressions for the length and width of the rectangular garden.

Width = ?

4. A drawing of the rectangular garden shows that the length is 6 m longer than the length of the square garden. What is the width of the rectangular garden? How much shorter is the width of the rectangular garden than the square garden?

Length = (12 + 6) m

5. Find the perimeter of each garden in terms of x.

6. Which plan should the club choose if they want the garden that covers the most area? Which plan should the club choose if they want the garden that requires the least fencing around it? Explain your reasoning.

READY TO GO ON?

Quiz for Lessons 8-5 Through 8-6

8-5 Factoring Special Products

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

1. $x^2 + 8x + 16$ 　　　　**2.** $4x^2 - 20x + 25$ 　　　　**3.** $x^2 + 3x + 9$

4. $2x^2 - 4x + 4$ 　　　　**5.** $9x^2 - 12x + 4$ 　　　　**6.** $x^2 - 12x - 36$

7. An architect is designing square windows with an area of $(x^2 + 20x + 100)$ ft^2. The dimensions of the windows are of the form $ax + b$, where a and b are whole numbers. Find an expression for the perimeter of the windows. Find the perimeter of a window when $x = 4$ ft.

Determine whether each trinomial is a difference of two squares. If so, factor. If not, explain.

8. $x^2 - 121$ 　　　　**9.** $4t^2 - 20$ 　　　　**10.** $1 - 9y^4$

11. $25m^2 - 4m^6$ 　　　　**12.** $16x^2 + 49$ 　　　　**13.** $r^4 - t^2$

14. The area of a square is $(36d^2 - 36d + 9)$ in^2.

    a. What expression represents the length of a side of the square?

    b. What expression represents the perimeter of the square?

    c. What are the length of a side, the perimeter, and the area of the square when $d = 2$ in.?

8-6 Choosing a Factoring Method

**Tell whether each expression is completely factored. If not, factor it.**

15. $5(x^2 + 3x + 1)$ 　　　　　　　**16.** $6x(5x^2 - x)$

17. $3t(t^4 - 9)$ 　　　　　　　　　**18.** $2(m^2 - 10m + 25)$

19. $3(2y^2 - 5)(y + 1)$ 　　　　　**20.** $(2n + 6)(n - 4)$

Factor each polynomial completely. Check your answer.

21. $3x^3 - 12x^2 + 12x$ 　　　　**22.** $16m^3 - 4m$ 　　　　**23.** $5x^3y - 45xy$

24. $3t^2 + 5t - 1$ 　　　　　　　**25.** $3c^2 + 12c - 63$ 　　　**26.** $x^5 - 81x$

Write an expression for each situation. Then factor your expression.

27. the difference of the square of a board's length and 36

28. the square of Michael's age minus 8 times Michael's age plus 16

29. two times the square of a car's speed plus 2 times the car's speed minus 12

30. three times the cube of Jessie's height plus 3 times the square of Jessie's height minus 6 times Jessie's height

31. Write an expression for the area of the shaded region. Then factor the expression.
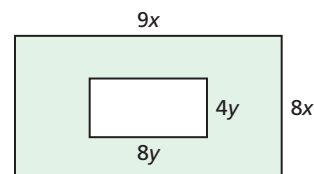
Vocabulary

greatest common factor 544   prime factorization 544

Complete the sentences below with vocabulary words from the list above.

1. A number written as a product so that each of its factors has no factors other than 1 and itself is the ____?____.

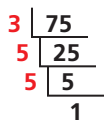**2.** The ____?____ of two monomials is the greatest of the factors that the monomials share.

8-1 Factors and Greatest Common Factors (pp. 544–549)

EXAMPLES

■ Write the prime factorization of 84.

84
4 • 21
2 • 2 • 3 • 7
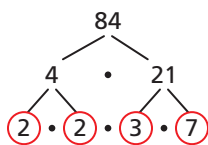
Write as a product. Continue until all factors are prime.

■ Write the prime factorization of 75.

3 | 75
5 | 25
5 | 5
   1

Keep dividing by prime factors until the quotient is 1.

$75 = 3 \cdot 5 \cdot 5 = 3 \cdot 5^2$

■ **Find the GCF of 36 and 90.**

$36 = 2 \cdot 2 \cdot 3 \cdot 3$
$90 = 2 \cdot 3 \cdot 3 \cdot 5$

Write the prime factorization of each number.

$2 \cdot 3 \cdot 3 = 18$

Find the product of the common factors.

The GCF of 36 and 90 is 18.

■ **Find the GCF of $10x^5$ and $4x^2$.**

$10x^5 = 2 \cdot 5 \cdot x \cdot x \cdot x \cdot x \cdot x$
$4x^2 = 2 \cdot 2 \cdot x \cdot x$

Write the prime factorization of each coefficient. Write powers as products.

$2 \cdot x \cdot x = 2x^2$

Find the product of the common factors.

The GCF of $10x^5$ and $4x^2$ is $2x^2$.

EXERCISES

Write the prime factorization of each number.

3. 12   **4.** 20

5. 32   **6.** 23

7. 40   **8.** 64

9. 66   **10.** 114

Find the GCF of each pair of numbers.

11. 15 and 50

12. 36 and 132

13. 29 and 30

14. 54 and 81

15. 20 and 48

Find the GCF of each pair of monomials.

16. $9m$ and 3

17. $4x$ and $2x^2$

18. $-18b^4$ and $27b^2$

19. $100r$ and $25r^5$

20. A hardware store carries 42 types of boxed nails and 36 types of boxed screws. The store manager wants to build a rack so that he can display the hardware in rows. He wants to put the same number of boxes in each row, but he wants no row to contain both nails and screws. What is the greatest number of boxes that he can display in one row? How many rows will there be if the manager puts the greatest number of boxes in each row?

8-2 Factoring by GCF (pp. 551–557)

EXAMPLES

■ Factor $3t^3 - 9t^2$. Check your answer.

$3t^3 = 3 \cdot t \cdot t \cdot t$
$9t^2 = 3 \cdot 3 \cdot t \cdot t$ *Find the GCF.*

GCF: $3 \cdot t \cdot t = 3t^2$

$3t^3 - 9t^2 = \mathbf{3t^2}(t) - \mathbf{3t^2}(3)$
    $= \mathbf{3t^2}(t - 3)$ *Factor out the GCF.*

Check $3t^2(t - 3) = 3t^3 - 9t^2$ ✓

■ Factor $-12s - 6s^3$. Check your answer.

$-1(12s + 6s^3)$ *Factor out −1.*

$12s = 2 \cdot 2 \cdot 3 \cdot s$
$6s^3 = 2 \cdot \quad 3 \cdot s \cdot s \cdot s$ *Find the GCF.*

  GCF: $2 \cdot 3 \cdot s = 6s$

$-1(12s + 6s^3)$

$-1[(6s)(2) + (6s)(s^2)]$

$-1[(6s)(2 + s^2)]$

$-6s(2 + s^2)$ *Factor out the GCF.*

Check $-6s(2 + s^2) = -12s - 6s^3$ ✓

■ Factor $5(x - 7) + 3x(x - 7)$ by grouping.

$5(x - 7) + 3x(x - 7)$ *(x − 7) is a common factor.*

$(x - 7)(5 + 3x)$ *Factor out (x − 7).*

■ Factor $6b^3 + 8b + 15b^2 + 20$ by grouping.

$(6b^3 + 8b) + (15b^2 + 20)$ *Group terms that have a common factor.*

$2b(3b^2 + 4) + 5(3b^2 + 4)$ *Factor each group.*

$(3b^2 + 4)(2b + 5)$ *Factor out $(3b^2 + 4)$.*

■ Factor $2m^3 - 6m^2 + 15 - 5m$. Check your answer.

$(2m^3 - 6m^2) + (15 - 5m)$ *Group terms.*
$2m^2(m - 3) + 5(3 - m)$ *Factor each group.*

$2m^2(m - 3) + 5(-1)(m - 3)$ *Rewrite (3 − m) as (−1)(m − 3).*

$2m^2(m - 3) - 5(m - 3)$ *Simplify.*
$(m - 3)(2m^2 - 5)$ *Factor out (m − 3).*

Check $(m - 3)(2m^2 - 5)$

    $2m^3 - 5m - 6m^2 + 15$
    $2m^3 - 6m^2 + 15 - 5m$ ✓

EXERCISES

Factor each polynomial. Check your answer.

21. $5x - 15x^3$ **22.** $-16b + 32$

23. $-14v - 21$ **24.** $4a^2 - 12a - 8$

25. $5g^5 - 10g^3 - 15g$ **26.** $40p^2 - 10p + 30$

27. A civil engineer needs the area of a rectangular lot to be $(6x^2 + 5x)$ ft². Factor this polynomial to find possible expressions for the dimensions of the lot.

Factor each expression.

28. $2x(x - 4) + 9(x - 4)$

29. $t(3t + 5) - 6(3t + 5)$

30. $5(6 - n) - 3n(6 - n)$

31. $b(b + 4) + 2(b + 4)$

32. $x^2(x - 3) + 7(x - 3)$

Factor each polynomial. Check your answer.

33. $n^3 + n - 4n^2 - 4$

34. $6b^2 - 8b + 15b - 20$

35. $2h^3 - 7h + 14h^2 - 49$

36. $3t^2 + 18t + t + 6$

37. $10m^3 + 15m^2 - 2m - 3$

38. $8p^3 + 4p - 6p^2 - 3$

39. $5r - 10 + 2r - r^2$

40. $b^3 - 5b + 15 - 3b^2$

41. $6t - t^3 - 4t^2 + 24$

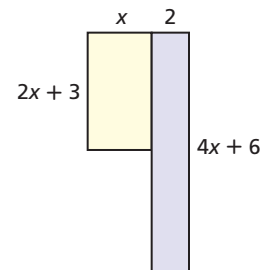**42.** $12h - 3h^2 + h - 4$

43. $d - d^2 + d - 1$

44. $6b - 5b^2 + 10b - 12$

45. $5t - t^2 - t + 5$

46. $8b^2 - 2b^3 - 5b + 20$

47. $3r - 3r^2 - 1 + r$

48. Write an expression for the area of each of the two rectangles shown. Then write and factor an expression for the combined area.

8-3 Factoring $x^2 + bx + c$ (pp. 560–567)

EXAMPLES

Factor each trinomial. Check your answer.

■ $x^2 + 14x + 45$

$(x + \boxed{})(x + \boxed{})$ *Look for factors of 45*

$(x + 9)(x + 5)$ *whose sum is 14.*

Check $(x + 9)(x + 5) = x^2 + 5x + 9x + 45$

$= x^2 + 14x + 45 ✓$

■ $x^2 + 6x - 27$

$(x + \boxed{})(x - \boxed{})$ *Look for factors of −27*

$(x + 9)(x - 3)$ *whose sum is 6.*

Check $(x + 9)(x - 3) = x^2 - 3x + 9x - 27$

$= x^2 + 6x - 27 ✓$

EXERCISES

Factor each trinomial. Check your answer.

49. $x^2 + 6x + 5$ **50.** $x^2 + 6x + 8$

51. $x^2 + 8x + 15$ **52.** $x^2 - 8x + 12$

53. $x^2 + 10x + 25$ **54.** $x^2 - 13x + 22$

55. $x^2 + 24x + 80$ **56.** $x^2 - 26x + 120$

57. $x^2 + 5x - 84$ **58.** $x^2 - 5x - 24$

59. $x^2 - 3x - 28$ **60.** $x^2 + 4x - 5$

61. $x^2 + x - 6$ **62.** $x^2 + x - 20$

63. $x^2 - 2x - 48$ **64.** $x^2 - 5x - 36$

65. $x^2 - 6x - 72$ **66.** $x^2 - 3x - 70$

67. $x^2 + 14x - 120$ **68.** $x^2 + 6x - 7$

69. The rectangle shown has an area of $(y^2 + 8y + 15)$ m². What is the width of the rectangle?

y + 5

$?$

8-4 Factoring $ax^2 + bx + c$ (pp. 568–574)

EXAMPLES

Factor each trinomial.

■ $6x^2 + 17x + 5$

$(\boxed{}x + \boxed{})(\boxed{}x + \boxed{})$ *a = 6 and c = 5;*

   Outer + Inner = 17

Factors of 6	Factors of 5	Outer + Inner	
1 and 6	5 and 1	$1(1) + 6(5) = 31$	✗
2 and 3	1 and 5	$2(5) + 3(1) = 13$	✗
2 and 3	5 and 1	$1(1) + 3(5) = 17$	✓

$(2x + 5)(3x + 1)$

■ $2n^2 - n - 10$

$(\boxed{}n + \boxed{})(\boxed{}n + \boxed{})$ *a = 2 and c = −10;*

   Outer + Inner = −1

Factors of 2	Factors of −10	Outer + Inner	
1 and 2	1 and −10	$1(-10) + 2(1) = -8$	✗
1 and 2	−1 and 10	$1(10) + 2(-1) = 8$	✗
1 and 2	2 and −5	$1(-5) + 2(2) = -1$	✓

$(1n + 2)(2n - 5) = (n + 2)(2n - 5)$

EXERCISES

Factor each trinomial. Check your answer.

70. $2x^2 + 11x + 5$ **71.** $3x^2 + 10x + 7$

72. $2x^2 - 3x + 1$ **73.** $3x^2 + 8x + 4$

74. $5x^2 + 28x + 15$ **75.** $6x^2 - 19x + 15$

76. $4x^2 + 13x + 10$ **77.** $3x^2 + 10x + 8$

78. $7x^2 - 37x + 10$ **79.** $9x^2 + 18x + 8$

80. $2x^2 - x - 1$ **81.** $3x^2 - 11x - 4$

82. $2x^2 - 11x + 5$ **83.** $7x^2 - 19x - 6$

84. $5x^2 - 9x - 2$ **85.** $-6x^2 - x + 2$

86. $6x^2 - x - 5$ **87.** $6x^2 + 17x - 14$

88. $-4x^2 + 8x + 5$ **89.** $-10x^2 + 11x + 6$

90. Write the polynomial modeled and then factor.

$12x^2$ 4x

$-15x$ −5

8-5 Factoring Special Products (pp. 578–584)

EXAMPLES

■ Determine whether $x^2 + 18x + 81$ is a perfect square. If so, factor. If not, explain.

$x^2 + 18x + 81$ *The trinomial is of the form $a^2 + 2ab + b^2$, so it is a perfect-square trinomial.*

$x \cdot x \quad 2(x \cdot 9) \quad 9 \cdot 9$

$x^2 + 18x + 81 = (x + 9)^2$

■ Determine whether $49x^4 - 25y^6$ is a difference of two squares. If so, factor. If not, explain.

$49x^4 - 25y^6$ *The binomial is a difference of two squares.*

$7x^2 \cdot 7x^2 \quad 5y^3 \cdot 5y^3$

$(7x^2)^2 - (5y^3)^2$ *$a = 7x^2, b = 5x^3$*

$(7x^2 + 5y^3)(7x^2 - 5y^3)$ *Write the binomial as $(a+b)(a-b)$.*

$49x^4 - 25y^6 = (7x^2 + 5y^3)(7x^2 - 5y^3)$

EXERCISES

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

91. $x^2 + 12x + 36$ **92.** $x^2 + 5x + 25$

93. $4x^2 - 2x + 1$ **94.** $9x^2 + 12x + 4$

95. $16x^2 + 8x + 4$ **96.** $x^2 + 14x + 49$

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

97. $100x^2 - 81$ **98.** $x^2 - 2$

99. $5x^4 - 10y^6$ **100.** $(-12)^2 - (x^3)^2$

101. $121b^2 + 9c^8$ **102.** $100p^2 - 25q^2$

Factor each polynomial using the rule for perfect-square trinomials or the rule for a difference of two squares. Tell which rule you used.

103. $x^2 - 25$ **104.** $x^2 + 20x + 100$

105. $j^2 - k^4$ **106.** $9x^2 - 42x + 49$

107. $81x^2 + 144x + 64$ **108.** $16b^4 - 121c^6$

8-6 Choosing a Factoring Method (pp. 586–591)

EXAMPLES

■ Tell whether $(3x - 9)(x + 4)$ is completely factored. If not, factor it.

$(3x - 9)(x + 4)$ *$3x - 9$ can be factored.*

$3(x - 3)(x + 4)$ *Factor out 3, the GCF of $3x$ and 9.*

■ $3ab^2 - 48a$

$3a(b^2 - 16)$ *Factor out the GCF.*

$3a(b + 4)(b - 4)$ *Factor the difference of two squares.*

Check $3a(b + 4)(b - 4) = 3a(b^2 - 16)$

            $= 3ab^2 - 48a$ ✓

■ $2m^3 + 4m^2 - 48m$

$2m(m^2 + 2m - 24)$ *Factor out the GCF.*

$2m(m - 4)(m + 6)$ *Factor the trinomial.*

Check $2m(m - 4)(m + 6)$

        $2m(m^2 + 2m - 24)$

        $2m^3 + 4m^2 - 48m$ ✓

EXERCISES

Tell whether each polynomial is completely factored. If not, factor it.

109. $4x^2 + 10x + 6 = (4x + 6)(x + 1)$

110. $3y^2 + 75 = 3(y^2 + 25)$

111. $b^4 - 81 = (b^2 + 9)(b^2 - 9)$

112. $x^2 - 6x + 9 = (x - 3)^2$

Factor each polynomial completely. Check your answer.

113. $4x^2 - 64$ **114.** $3b^5 - 6b^4 - 24b^3$

115. $a^4b^3 - a^2b^5$ **116.** $t^{20} - t^4$

117. $5x^2 + 20x + 15$ **118.** $2x^4 - 50x^2$

119. $8t + 32 + 2st + 8s$

120. $25m^3 - 90m^2 - 40m$

121. $32x^4 - 48x^3 + 8x^2 - 12x$

122. $6s^4t + 12s^3t^2 + 6s^2t^3$

123. $10m^3 + 4m^2 - 90m - 36$

CHAPTER TEST

Find the GCF of each pair of monomials.

1. $3t^4$ and $8t^2$

2. $2y^3$ and $-12y$

3. $15n^5$ and $9n^4$

4. Write the prime factorization of 360.

5. A coin collector is arranging a display of three types of nickels. The types of nickels and number of each type are shown in the table. The collector wants to arrange them in rows with the same number in each row without having different types in the same row. How many rows will she need if she puts the greatest possible number of nickels in each row?

Type of Nickel	Number of Nickels
Liberty	16
Buffalo	24
Jefferson	40

Factor each expression.

6. $24m^2 + 4m^3$

7. $9x^5 - 12x$

8. $-2r^4 - 6$

9. $3(c - 5) + 4c(c - 5)$

10. $10x^3 + 4x - 25x^2 - 10$

11. $4y^3 - 4y^2 - 3 + 3y$

12. A model rocket is shot vertically from a deck into the air at a speed of 50 m/s. The expression $-5t^2 + 50t + 5$ gives the approximate height of the rocket after t seconds. Factor this expression.

Factor each trinomial.

13. $x^2 + 6x + 5$

14. $x^2 - 4x - 21$

15. $x^2 - 8x + 15$

16. $2x^2 + 9x + 7$

17. $2x^2 + 9x - 18$

18. $-3x^2 - 2x + 8$

Determine whether each trinomial is a perfect square. If so, factor. If not, explain.

19. $a^2 + 14a + 49$

20. $2x^2 + 10x + 25$

21. $9t^2 - 6t + 1$

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

22. $b^2 - 16$

23. $25y^2 - 10$

24. $9a^2 - b^{10}$

25. A company is producing square sheets of plastic. Each has an area of $(9x^2 + 30x + 25)$ ft². The dimensions of each sheet are of the form $ax + b$, where a and b are whole numbers. Find an expression for the perimeter of a sheet. Find the perimeter when $x = 4$ ft.

Tell whether each expression is completely factored. If not, factor it.

26. $(6x - 3)(x + 5)$

27. $(v^5 + 10)(v^5 - 10)$

28. $(2b + 3)(3b - 2)$

Factor each polynomial completely.

29. $8x^3 + 72x^2 + 160x$

30. $3x^5 - 27x^3$

31. $8x^3 + 64x^2 - 20x - 160$

32. $cd^4 - c^7d^6$

33. $100x^2 - 80x + 16$

34. $7m^8 - 7$

COLLEGE ENTRANCE EXAM PRACTICE

FOCUS ON ACT

The ACT Mathematics test booklet usually has writing space for scratch work. You may not bring your own scratch paper to the testing center. Remember that any scratch work done in the test booklet is for your use only and will not be scored. Be sure to transfer your final answer to the answer sheet.

HOT TIP! If you are unsure how to solve a problem, look through the answer choices. They may provide you with a clue to the solution method. It may take longer to work backward from the answer choices, so make sure you monitor your time.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

---

1. What is the value of $c^2 - d^2$ if $c + d = 7$ and $c - d = -2$?

(A) -14

(B) -5

(C) 5

(D) 14

(E) 45

---

2. Which of the following is the complete factorization of $6a^3b + 3a^2b^3$?

(F) $6a^3b^3$

(G) $9a^5b^4$

(H) $3ab(2a^2 + ab^2)$

(J) $3a^2b(2a + b^2)$

(K) $(6a^3b)(3a^2b^3)$

---

3. Which of the following is a factor of $x^2 + 3x - 18$?

(A) $x + 2$

(B) $x + 3$

(C) $x + 6$

(D) $x + 9$

(E) $x + 18$

---

4. The binomial $x - 3$ is NOT a factor of which of the following trinomials?

(F) $2x^2 - x - 3$

(G) $2x^2 - 5x - 3$

(H) $2x^2 - 8x + 6$

(J) $3x^2 - 6x - 9$

(K) $3x^2 - 10x + 3$

---

5. For what value of n is $4x^2 + 20x + n^2 = (2x + n)^2$ true for any real number x?

(A) 4

(B) 5

(C) 8

(D) 10

(E) 25

---

6. What is the factored form of $x^2 + \frac{2x}{3} + \frac{x}{2} + \frac{2}{6}$?

(F) $\left(x + \frac{1}{3}\right)\left(x + \frac{1}{2}\right)$

(G) $\left(x + \frac{1}{2}\right)\left(x + \frac{2}{3}\right)$

(H) $\left(x + \frac{2}{3}\right)\left(x + \frac{1}{6}\right)$

(J) $(x + 2)\left(x + \frac{1}{3}\right)$

(K) $\left(x + \frac{1}{3}\right)\left(x + \frac{2}{3}\right)$

TEST TACKLER

Standardized Test Strategies

Any Question Type: Translate Words to Math

When reading a word problem, look for key words and context clues to help you translate the words into a mathematical equation or expression.

Some key words, such as those shown in this table, represent certain mathematical operations.

Action	Math Operation
Combining, increasing	Addition
Decreasing, reducing	Subtraction
Increasing or decreasing by a factor	Multiplication
Separating	Division

EXAMPLE

Short Response The polynomial $x^2 + 7x + 12$ represents the area of a rectangle in square meters. The width is $(x + 3)$ meters. Find the combined measure of the length and the width.

Use action words and context clues to translate the words into equations.

$x^2 + 7x + 12$ **represents** the **area of a rectangle** in square meters.
$x^2 + 7x + 12 \qquad = \qquad\qquad\qquad A$

The **width is** $(x + 3)$ meters.
$\qquad w \;\; = (x + 3)$

Find the **combined measure** of the **length and** the **width.**
$\qquad\qquad m \qquad\qquad = \quad \ell \quad + \quad\quad w$

Now use the equations to solve the problem.

$A = \ell w$ *Write the formula for area of a rectangle.*

$x^2 + 7x + 12 = \ell(x + 3)$ *Substitute $x^2 + 7x + 12$ for A and $(x + 3)$ for w.*

$\left(x + \boxed{?}\,\right)(x + 3)$ *Factor $x^2 + 7x + 12$ to find an expression for the length.*

$(x + 4)(x + 3)$ *$3(4) = 12; 3 + 4 = 7$*

The length is $(x + 4)$.

$m = \ell + w$ *Write the equation for the combined measure of the length and width.*

$m = (x + 4) + (x + 3)$ *Substitute $(x + 4)$ for ℓ and $(x + 3)$ for w.*

$m = 2x + 7$ *Combine like terms.*

The combined measure of the length and width is $(2x + 7)$ meters.

Read each test item and answer the questions that follow.

Item A

Short Response The width of Alvin's rectangular mural is 6 times the length x. Alvin plans to make a new mural with an area of $(6x^2 - 24x + 24)$ square meters. By how much did Alvin decrease the area of the mural? Show your work.

---

1. What key words or context clues are in the first sentence of the test item? Use these clues to write an expression that represents the width of the rectangle.

2. Write an equation to represent the area of Alvin's first mural.

3. What math operation does the key word *decrease* represent?

Item B

Multiple Choice Which factored expression represents the phrase shown below?

the square of the number of hours it takes to empty a cistern minus 20 times the number of hours plus 64

  Ⓐ $(h - 16)(h - 4)$ Ⓒ $(h - 8)(h - 8)$

  Ⓑ $(h^2 - 20)(h - 64)$ Ⓓ $(h - 16)(h + 4)$

---

4. Which word in the phrase tells you to use an exponent in your expression?

5. What is the unknown value in the expression? Define a variable to represent this value.

6. Identify other key words and the mathematical operation phrase each one represents.

Item C

Multiple Choice A company owns two packaging plants. The polynomial $0.05x^2 + 16x - 9400$ models one plant's profit, where x is the number of units packaged. The polynomial $-0.01x^2 + 17x - 5400$ models the other plant's profit. If x is 25,000, what is the total profit of both plants?

  Ⓐ $-\$5,830,300$

  Ⓑ $\$25,810,200$

  Ⓒ $\$31,640,500$

  Ⓓ $\$37,471,000$

---

7. What mathematical symbol does the word *models* represent?

8. Write an equation for each plant that can be used to determine its profit P.

9. What mathematical operation does the term "total profit" represent?

Item D

Gridded Response One of the bases of a trapezoid is 12 meters greater than its height. The other base is 4 meters less than its height. Find the area of the trapezoid when the height is 6 meters.

---

10. Identify the unknown dimension, and assign it a variable.

11. A student is unsure how many bases a trapezoid has. Identify the context clues that can help this student.

12. Make a list of the key words in the problem, and link each word to its mathematical meaning.

13. Write an expression for each base of the trapezoid.

CUMULATIVE ASSESSMENT, CHAPTERS 1–8

Multiple Choice

1. A rectangle has an area of $(x^2 + 5x - 24)$ square units. Which of the following are possible expressions for the length and the width of the rectangle?

Ⓐ Length: $(x - 24)$ units; width: $(x + 1)$ units

Ⓑ Length: $(x - 4)$ units; width: $(x + 6)$ units

Ⓒ Length: $(x - 3)$ units; width: $(x + 8)$ units

Ⓓ Length: $(x + 12)$ units; width: $(x - 2)$ units

2. Which property of real numbers is used to transform the equation in Step 1 into the equation in Step 2?

Step 1: $4(x - 5) + 8 = 88$
Step 2: $4x - 20 + 8 = 88$
Step 3: $\quad 4x - 12 = 88$
Step 4: $\quad\quad 4x = 100$
Step 5: $\quad\quad 4x = 25$

Ⓕ Commutative Property of Multiplication

Ⓖ Associative Property of Multiplication

Ⓗ Multiplication Property

Ⓙ Distributive Property

3. If $\frac{2}{3}x - 9 = 3$, what is the value of the expression $8x - 3$?

Ⓐ -75 Ⓒ 61

Ⓑ -35 Ⓓ 141

4. Carlos and Bonita were just hired at a manufacturing plant. Carlos will earn $12.50 per hour. He will receive a hiring bonus of $300. Bonita will not get a hiring bonus, but she will earn $14.50 per hour. Which equation can you use to determine the number of hours h when both employees will have earned the same total amount?

Ⓕ $300 + 14.50h = 12.50h$

Ⓖ $14.50h + 300 = 12.50h$

Ⓗ $14.50h + 12.50h = 300$

Ⓙ $300 + 12.50h = 14.50h$

5. Which of the following expressions is equivalent to $x^2 - 8x + 16$?

Ⓐ $(x + 4)^2$ Ⓒ $(x + 8)(x + 2)$

Ⓑ $(x + 4)(x - 4)$ Ⓓ $(x - 4)^2$

6. An amusement park has two jumping attractions called moonbounces in the shape of similar cubes. The larger moonbounce has a volume of 4800 cubic feet. The smaller moonbounce is half the length of the larger one. What is the volume of the smaller moonbounce?

Ⓕ 300 cubic feet
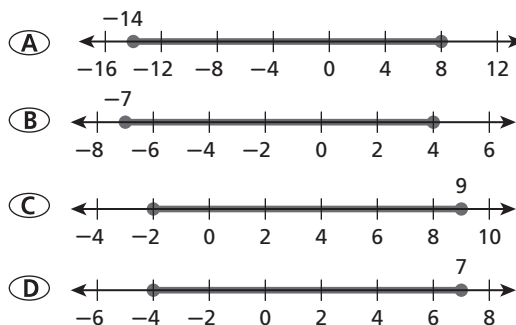
Ⓖ 600 cubic feet

Ⓗ 1200 cubic feet

Ⓙ 2400 cubic feet

7. What is the value of y if the line through $(1, -1)$ and $(2, 2)$ is parallel to the line through $(-2, 1)$ and $(-1, y)$?

Ⓐ -8 Ⓒ 3

Ⓑ -2 Ⓓ 4

8. Which of the following shows the complete factorization of $2x^3 + 4x^2 - 6x$?

Ⓕ $(2x^2 - 2x)(x + 3)$

Ⓖ $2x(x^2 + 2x - 3)$

Ⓗ $2x(x - 1)(x + 3)$

Ⓙ $2(x^3 + 2x^2 - 3x)$

9. Which graph shows the solution set of the compound inequality $-9 \leq 5 - 2x \leq 13$?

Ⓐ

Ⓑ

Ⓒ

Ⓓ

10. Which point lies on the graph of both functions?

$f(x) = 2x - 10$
$g(x) = 10 - 2x$

Ⓕ $(5, 0)$ Ⓗ $(0, 0)$

Ⓖ $(1, -8)$ Ⓙ $(2, 6)$

11. Hayley plans to solve the system of equations below.

$$\begin{cases} x + 3y = 8 \\ 5x - y = 8 \end{cases}$$

Which of the following does NOT show an equation Hayley can use to solve the system of equations?

Ⓐ $x + 3(5x - 8) = 8$

Ⓑ $5(8 - 3y) - y = 8$

Ⓒ $x = 8 - 3y$

Ⓓ $5x - (-x + 8) = 8$

12. Which value of b would make $x^2 + bx - 2$ factorable?

Ⓕ -2 Ⓗ 0

Ⓖ -1 Ⓙ 3

13. Which expression is equivalent to $5(x - 2) + 4x$?

Ⓐ $5x - 10$ Ⓒ $9x - 10$

Ⓑ $9x - 2$ Ⓓ $20x - 2$

Gridded Response

14. The complete factorization of $-12x^3 + 14x^2 + 6x$ is $-2x(ax + 1)(2x - 3)$. What is the value of a?

15. The expression $x^2 + x + b$ is a perfect-square trinomial. What is the value of b?

16. Margaret is buying a $35 sweater that is on sale for 20% off. What is the total price of the sweater, in dollars, when 5% sales tax is added?

17. A car is traveling at a constant speed. In 4 hours, the car travels 220 miles. How many hours will it take the car to travel 550 miles?

Short Response

18. The area of a certain circle is $\pi(9x^2 + 6x + 1)$ square centimeters. Find an expression for the length of the circle's radius. Explain how you found your answer.

19. A rectangle has an area of $(x^2 - 25)$ square feet.

   a. Use factoring to write possible expressions for the length and width of the rectangle.

   b. Use your expressions from part **a** to write an expression for the perimeter of the rectangle. Simplify the expression.

   c. Use your expressions from parts **a** and **b** to find the perimeter and the area of the rectangle when $x = 10$ feet. Show your work.

20. Write the numbers 57,000,000,000 and 19,000 in scientific notation. Then show how to divide 57,000,000,000 by 19,000 using properties of exponents.

21. Show that you can factor the expression $x^2y - 12 + 3y - 4x^2$ by grouping in two different ways.

Extended Response

22. The diagram below can be used to show that the expression $(a + b)^2$ is equivalent to the expression $a^2 + 2ab + b^2$.
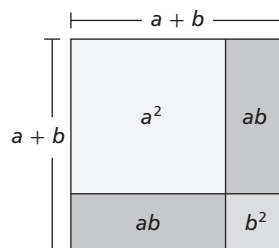
   a. Make a diagram similar to the one above to model the expression $(a + b + c)^2$. Label each distinct area.

   b. Use the labels from your diagram to write an expression equivalent to $(a + b + c)^2$.

   c. Show that your expression in part **b** is equivalent to $(a + b + c)^2$ by evaluating each expression for $a = 4$, $b = 2$, and $c = 1$.

   d. Factor $x^2 + y^2 + 9 + 2xy + 6x + 6y$. Show or explain how you found your answer.

South Carolina

Columbia
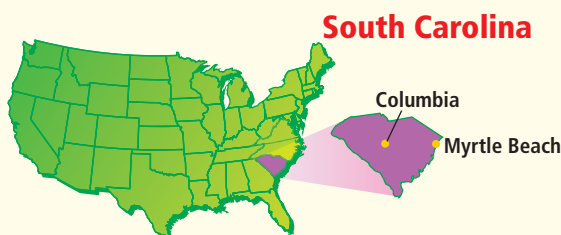
Myrtle Beach

⭐ Alligator Adventure

Alligator Adventure is a tourist attraction in North Myrtle Beach, South Carolina. Its exhibits feature alligators, snakes, Galapagos tortoises, bears, frogs, lizards, tropical birds, and crocodiles.

Choose one or more strategies to solve each problem.

1. At birth, alligators are about 8 inches long. Adult male alligators can grow up to 18 feet long. How many times as long as a newborn alligator is an alligator that measures 18 feet?

2. Alligators build their nests out of dirt, grass, and leaves. The female alligator will deposit 40 to 50 eggs in her nest. Suppose there were 40 eggs in a nest and 80 percent of them hatched. The baby alligators each weigh 3 ounces. What is the total weight of the hatched alligators?

3. Most alligators have U-shaped snouts while most crocodiles have V-shaped snouts. Which of the equations below has a graph that could model the shape of an alligator's snout? Explain.

$y = 3x - 4$

$y = x^2 - 12$

$y = |x^2 - 4|$
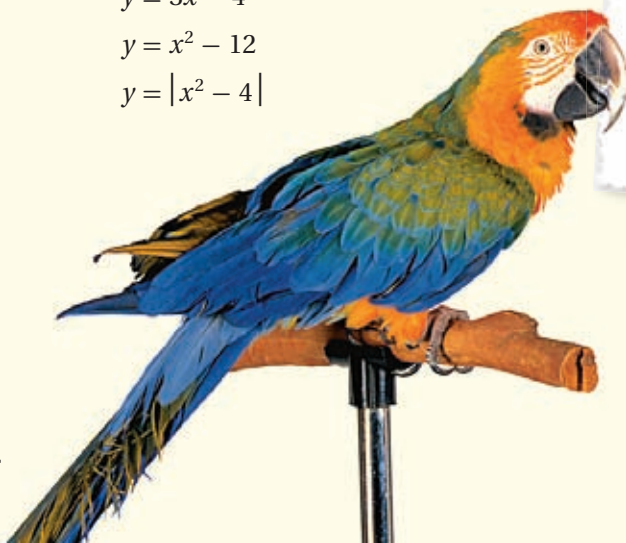
⭐ Melton Memorial Observatory

Melton Memorial Observatory is run by the University of South Carolina in Columbia, South Carolina. The table lists some of the stars that can be seen from the observatory.

A light-year is the distance light can travel in one year, which is 9,500,000,000,000 kilometers.

Eight of the Brightest Stars as Seen from Earth				
Name	Rank	Apparent Magnitude	Distance from Earth (light-years)	Temperature (kelvins)
Sirius	1	−1.46	8.6	9,400
Arcturus	4	−0.04	34	4,290
Vega	5	0.03	25	9,600
Procyon	8	0.38	11.4	6,500
Altair	12	0.77	16	7,550
Spica	14	0.98	220	22,400
Deneb	20	1.25	1500	8,400
Regulus	25	1.35	69	12,000
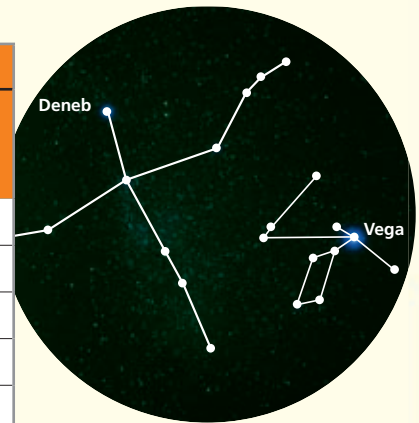
Choose one or more strategies and use the table to solve each problem.

1. Write 1 light-year as a distance in kilometers in scientific notation. Write the distance in kilometers of each star from Earth in scientific notation.

2. Order the stars from closest to Earth to farthest from Earth.

3. Identify which star is farthest from and which is closest to Earth. Approximate how many times as far as the closest star the farthest star is.

Apparent magnitude is a measure of how bright a star (or other object) appears from Earth. The lower the apparent magnitude of a star, the brighter the star appears.

4. Describe the correlation between apparent magnitude and distance from Earth in light-years. Make a scatter plot to support your answer.