CHAPTER 12

Rational Functions and Equations

Chapter Focus

- Graph and use rational functions to solve real-world problems.
- Simplify rational expressions.
- Use rational equations to solve real-world problems.

By Design

Ratios and rational expressions can be used to explore perspective in art and dimensions in package design. Try your hand at both.

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ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. perfect-square trinomial
2. greatest common factor
3. monomial
4. polynomial
5. reciprocals

A. the greatest factor that is shared by two or more terms
B. a number, a variable, or a product of numbers and variables with whole-number exponents
C. two numbers whose product is 1
D. a polynomial with three terms
E. the sum or difference of monomials
F. a trinomial that is the result of squaring a binomial

Simplify Fractions

Simplify.

6. $\frac{12}{4}$ **7.** $\frac{100}{36}$ **8.** $\frac{240}{18}$ **9.** $\frac{121}{66}$

Add and Subtract Fractions

Add or subtract.

10. $\frac{1}{3}+\frac{1}{2}$ **11.** $\frac{7}{8}-\frac{1}{6}$ **12.** $\frac{3}{4}+\frac{2}{3}+\frac{1}{2}$ **13.** $\frac{5}{9}+\frac{1}{12}-\frac{1}{3}$

Factor GCF from Polynomials

Factor each polynomial.

14. x^2+2x **15.** x^2+x **16.** $2x^2+x$ **17.** x^2-x

18. $3x^2+2x$ **19.** $4x^2-4$ **20.** $3x^2-6x$ **21.** x^3-x^2

Properties of Exponents

Simplify each expression.

22. $4x \cdot 3x^2$ **23.** $-5 \cdot 2jk$ **24.** $-2a^3 \cdot 3a^4$ **25.** $3ab \cdot 4a^2b$

26. $2x \cdot 3y \cdot xy$ **27.** $a^2b \cdot 3ab^3$ **28.** $3rs \cdot 3rs^3$ **29.** $5m^2n^2 \cdot 4mn^2$

Simplify Polynomial Expressions

Simplify each expression.

30. $4x-2y-8y$ **31.** $2r-4s+3s-8r$

32. $ab^2-ab+4ab^2+2a^2b+a^2b^2$ **33.** $3g(g-4)+g^2+g$

CHAPTER 12

Study Guide: Preview

Where You've Been

Previously you

- identified, wrote, and graphed equations of direct variation.
- identified and graphed quadratic, exponential, and square-root functions.
- used factoring to solve quadratic equations.
- simplified radical expressions and solved radical equations.

In This Chapter

You will study

- how to identify, write, and graph equations of inverse variation.
- how to graph rational functions and simplify rational expressions.
- how to solve rational equations.

Where You're Going

You can use the skills in this chapter

- to build upon your knowledge of graphing and transforming various types of functions.
- to solve problems involving inverse variation in classes such as Physics and Chemistry.
- to calculate costs when working with a fixed budget.

Key Vocabulary/Vocabulario

asymptote	asíntota
discontinuous function	función discontinua
excluded values	valores excluidos
inverse variation	variación inversa
rational equation	ecuación racional
rational expression	expresión racional
rational function	función racional

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. What are some other words that mean the same as *continuous*? The prefix *dis-* generally means "not." Describe what the graph of a **discontinuous function** might look like.
2. What does it mean for someone or something to be *included* in a group? What about *excluded*? What might it mean for some values to be **excluded values** for a particular function?
3. A *direct variation* is a relationship between two variables, x and y, that can be written in the form $y = kx$ where k is a nonzero constant. The inverse of a number x is $\frac{1}{x}$. Use this information to write the form of an **inverse variation**.
4. You learned in Chapter 1 that an *algebraic expression* is an expression that contains one or more variables, numbers, or operations. You also learned that a *rational number* is a number that can be written in the form of a fraction. Combine these terms to define **rational expression**. Give an example.

Reading and Writing Math

Study Strategy: Prepare for Your Final Exam

Math is a cumulative subject, so your final exam will probably cover all of the material you have learned since the beginning of the course. Preparation is essential for you to be successful on your final exam. It may help you to make a study timeline like the one below.

2 weeks before the final:

- Look at previous exams and homework to determine areas I need to focus on; rework problems that were incorrect or incomplete.
- Make a list of all formulas, definitions, and properties I need to know for the final.
- Create a practice exam using problems from the book that are similar to problems from each exam.

1 week before the final:

- Take the practice exam and check it. For each problem I miss, find 2 or 3 similar ones and work those.
- Work with a friend in the class to quiz each other on formulas, definitions, and properties from my list.

1 day before the final:

- Make sure I have pencils, calculator (check batteries!), and ruler.

Try This

1. Create a timeline that you will use to study for your final exam.

12-1 Algebra Lab

Use with Lesson 12-1

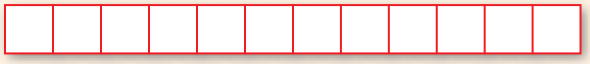
Model Inverse Variation

The relationship between the width and the length of a rectangle with a constant area is an inverse variation. In this activity, you will study this relationship by modeling rectangles with square tiles or grid paper.

Activity

Use 12 square tiles to form a rectangle with an area of 12 square units, or draw the rectangle on grid paper. Use a width of 1 unit and a length of 12 units.

Your rectangle should look like the one shown.



Using the same 12 square tiles, continue forming rectangles by changing the width and length until you have formed all the different rectangles you can that have an area of 12 square units. Copy and complete the table as you form each rectangle.

Width (x)	Length (y)	Area (xy)
1	12	12
		12
		12
		12
		12
		12

Plot the ordered pairs from the table on a graph. Draw a smooth curve through the points.

Try This

1. Look at the table and graph above. What happens to the length as the width increases? Why?
2. This type of relationship is called an *inverse variation*. Why do you think it is called that?
3. For each point, what does xy equal? Complete the equation $xy = \square$. Solve this equation for y.
4. Form all the different rectangles that have an area of 24 square units. Record their widths and lengths in a table. Graph your results. Write an equation relating the width x and length y.
5. **Make a Conjecture** Using the equations you wrote in 3 and 4, what do you think the equation of any inverse variation might look like when solved for y?

12-1 Inverse Variation

Objective
Identify, write, and graph inverse variations.

Vocabulary
inverse variation

Remember!
A direct variation is an equation that can be written in the form $y = kx$, where k is a nonzero constant.

Why learn this?
Inverse variation can be used to find the frequency at which a guitar string vibrates. (See Example 3.)

A relationship that can be written in the form $y = \frac{k}{x}$, where k is a nonzero constant and $x \neq 0$, is an **inverse variation**. The constant k is the constant of variation.

Multiplying both sides of $y = \frac{k}{x}$ by x gives $xy = k$. So, for any inverse variation, the product of x and y is a nonzero constant.

Know it! Note

Inverse Variations

WORDS	NUMBERS	ALGEBRA
y varies inversely as x.	$y = \frac{3}{x}$	$y = \frac{k}{x}$
y is inversely proportional to x.	$xy = 3$	$xy = k \ (k \neq 0)$

There are two methods to determine whether a relationship between data is an inverse variation. You can write a function rule in $y = \frac{k}{x}$ form, or you can check whether xy is constant for each ordered pair.

EXAMPLE 1 Identifying an Inverse Variation

Tell whether each relationship is an inverse variation. Explain.

A

x	y
1	20
2	10
4	5

Method 1 Write a function rule.

$y = \frac{20}{x}$ *Can write in $y = \frac{k}{x}$ form.*

The relationship is an inverse variation.

Method 2 Find xy for each ordered pair.

$1(20) = 20, \ 2(10) = 20, \ 4(5) = 20$

The product xy is constant, so the relationship is an inverse variation.

B

x	y
2	6
3	9
6	18

Method 1 Write a function rule.

$y = 3x$ *Cannot write in $y = \frac{k}{x}$ form.*

The relationship is not an inverse variation.

Method 2 Find xy for each ordered pair.

$2(6) = 12, \ 3(9) = 27, \ 6(18) = 108$

The product xy is not constant, so the relationship is not an inverse variation.

Tell whether each relationship is an inverse variation. Explain.

C $5xy = -21$

$$\frac{5xy}{5} = \frac{-21}{5}$$ *Find xy. Since xy is multiplied by 5, divide both sides by 5 to undo the multiplication.*

$$xy = \frac{-21}{5}$$ *Simplify.*

xy equals the constant $\frac{-21}{5}$, so the relationship is an inverse variation.

CHECK IT OUT! **Tell whether each relationship is an inverse variation. Explain.**

1a.

x	y
–12	24
1	–2
8	–16

1b.

x	y
3	3
9	1
18	0.5

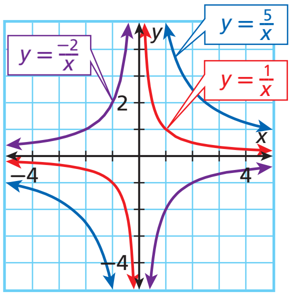
1c. $2x + y = 10$

Helpful Hint

Since k is a nonzero constant, $xy \neq 0$. Therefore, neither x nor y can equal 0, and no solution points will be on the x- or y-axes.

An inverse variation can also be identified by its graph. Some inverse variation graphs are shown. Notice that each graph has two parts that are not connected.

Also notice that none of the graphs contain $(0, 0)$. This is because $(0, 0)$ can never be a solution of an inverse variation equation.



EXAMPLE 2 **Graphing an Inverse Variation**

Write and graph the inverse variation in which $y = 2$ when $x = 4$.

Step 1 Find k.

$k = xy$ *Write the rule for constant of variation.*

$= 4(2)$ *Substitute 4 for x and 2 for y.*

$= 8$

Step 2 Use the value of k to write an inverse variation equation.

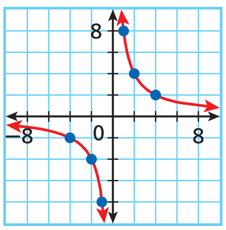
$y = \frac{k}{x}$ *Write the rule for inverse variation.*

$y = \frac{8}{x}$ *Substitute 8 for k.*

Step 3 Use the equation to make a table of values.

x	–4	–2	–1	0	1	2	4
y	–2	–4	–8	undef.	8	4	2

Step 4 Plot the points and connect them with smooth curves.



CHECK IT OUT! **2.** Write and graph the inverse variation in which $y = \frac{1}{2}$ when $x = 10$.

EXAMPLE 3 Music Application

The inverse variation $xy = 2400$ relates the vibration frequency y in hertz (Hz) to the length x in centimeters of a guitar string. Determine a reasonable domain and range and then graph this inverse variation. Use the graph to estimate the frequency of vibration when the string length is 100 centimeters.

Step 1 Solve the function for y so you can graph it.

$xy = 2400$

$y = \frac{2400}{x}$ *Divide both sides by x.*

Step 2 Decide on a reasonable domain and range.

$x > 0$ *Length is never negative and $x \neq 0$.*

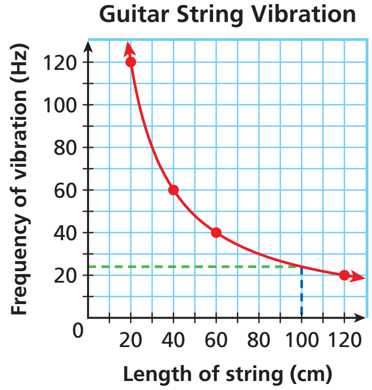
$y > 0$ *Because x and xy are both positive, y is also positive.*

Step 3 Use values of the domain to generate reasonable ordered pairs.

x	20	40	60	120
y	120	60	40	20

Step 4 Plot the points. Connect them with a smooth curve.

Step 5 Find the y-value where $x = 100$. When the string length is 100 cm, the frequency of vibration is about 24 Hz.



Remember!

Recall that sometimes domain and range are restricted in real-world situations.

CHECK IT OUT!

3. The inverse variation $xy = 100$ represents the relationship between the pressure x in atmospheres (atm) and the volume y in mm^3 of a certain gas. Determine a reasonable domain and range and then graph this inverse variation. Use the graph to estimate the volume of the gas when the pressure is 40 atmospheric units.

The fact that $xy = k$ is the same for every ordered pair in any inverse variation can help you find missing values in the relationship.

Know it! Note

Product Rule for Inverse Variation

If (x_1, y_1) and (x_2, y_2) are solutions of an inverse variation, then $x_1y_1 = x_2y_2$.

EXAMPLE 4 Using the Product Rule

Let $x_1 = 3$, $y_1 = 2$, and $y_2 = 6$. Let y vary inversely as x. Find x_2.

$x_1y_1 = x_2y_2$ *Write the Product Rule for Inverse Variation.*

$(3)(2) = x_2(6)$ *Substitute 3 for x_1, 2 for y_1, and 6 for y_2.*

$6 = 6x_2$ *Simplify.*

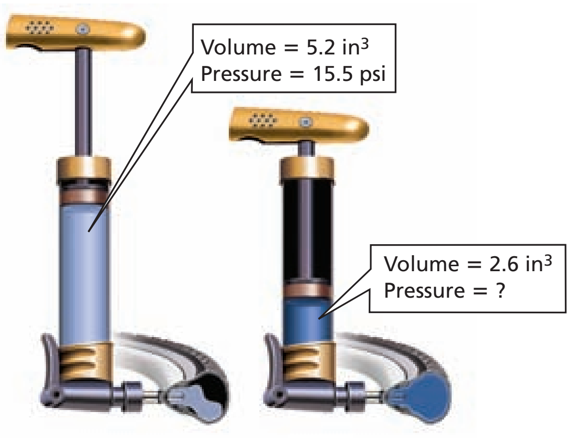
$\frac{6}{6} = \frac{6x_2}{6}$ *Solve for x_2 by dividing both sides by 6.*

$1 = x_2$ *Simplify.*

CHECK IT OUT! 4. Let $x_1 = 2$, $y_1 = -6$, and $x_2 = -4$. Let y vary inversely as x. Find y_2.

EXAMPLE 5 *Physics Application*

Boyle's law states that the pressure of a quantity of gas x varies inversely as the volume of the gas y. The volume of air inside a bicycle pump is 5.2 in^3, and the pressure is 15.5 psi. Assuming no air escapes, what is the pressure of the air inside the pump after the handle is pushed in, and the air is compressed to a volume of 2.6 in^3?



Reading Math

In Example 5, x_1 and y_1 represent volume and pressure **before** the handle is pushed in, and x_2 and y_2 represent volume and pressure **after** the handle is pushed in.

$x_1y_1 = x_2y_2$ *Use the Product Rule for Inverse Variation.*

$(5.2)(15.5) = (2.6)y_2$ *Substitute 5.2 for x_1, 15.5 for y_1, and 2.6 for x_2.*

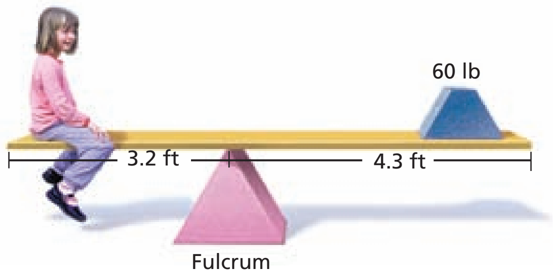
$80.6 = 2.6y_2$ *Simplify.*

$\frac{80.6}{2.6} = \frac{2.6y_2}{2.6}$ *Solve for y_2 by dividing both sides by 2.6.*

$31 = y_2$ *Simplify.*

The pressure after the handle is pushed in is 31 psi.

CHECK IT OUT! 5. On a balanced lever, weight varies inversely as the distance from the fulcrum to the weight. The diagram shows a balanced lever. How much does the child weigh?



THINK AND DISCUSS

1. Name two ways you can identify an inverse variation.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of the parts of the given inverse variation.

$y = \frac{8}{x}$

Constant of variation | Graph | Solutions

12-1 Exercises

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GUIDED PRACTICE

1. **Vocabulary** Describe the graph of an *inverse variation.*

SEE EXAMPLE 1 p. 871

Tell whether each relationship is an inverse variation. Explain.

2.

x	y
1	8
4	2
2	4

3.

x	y
$\frac{1}{6}$	1
$\frac{1}{3}$	2
2	12

4. $x + y = 8$

5. $4xy = 3$

SEE EXAMPLE 2 p. 872

6. Write and graph the inverse variation in which $y = 2$ when $x = 2$.
7. Write and graph the inverse variation in which $y = 6$ when $x = -1$.

SEE EXAMPLE 3 p. 873

8. **Travel** The inverse variation $xy = 30$ relates the constant speed x in mi/h to the time y in hours that it takes to travel 30 miles. Determine a reasonable domain and range and then graph this inverse variation. Use the graph to estimate how many hours it would take traveling 4 mi/h.

SEE EXAMPLE 4 p. 873

9. Let $x_1 = 3$, $y_1 = 12$, and $x_2 = 9$. Let y vary inversely as x. Find y_2.
10. Let $x_1 = 1$, $y_1 = 4$, and $y_2 = 16$. Let y vary inversely as x. Find x_2.

SEE EXAMPLE 5 p. 874

11. **Mechanics** The rotational speed of a gear varies inversely as the number of teeth on the gear. A gear with 12 teeth has a rotational speed of 60 rpm. How many teeth are on a gear that has a rotational speed of 45 rpm?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
12–15	1
16–17	2
18	3
19–20	4
21	5

Extra Practice

Skills Practice p. S26
Application Practice p. S39

Tell whether each relationship is an inverse variation. Explain.

12.

x	y
3	−3
−5	5
7	−7

13.

x	y
2	5
0.5	20
8	1.25

14. $x = \frac{13}{y}$

15. $y = 5x$

16. Write and graph the inverse variation in which $y = -2$ when $x = 5$.
17. Write and graph the inverse variation in which $y = -6$ when $x = -\frac{1}{3}$.
18. **Engineering** The inverse variation $xy = 12$ relates the current x in amps to the resistance y in ohms of a circuit attached to a 12-volt battery. Determine a reasonable domain and range and then graph this inverse variation. Use the graph to estimate the resistance of a circuit when the current is 5 amps.
19. Let $x_1 = -3$, $y_1 = -4$, and $y_2 = 6$. Let y vary inversely as x. Find x_2.
20. Let $x_1 = 7$, $y_1 = 9$, and $x_2 = 6$. Let y vary inversely as x. Find y_2.

LINK

Winter Sports

Snowshoes were originally made of wooden frames strung with animal intestines. Modern snowshoes are made with steel frames and may have cleats for gripping the snow.

21. **Home Economics** The length of fabric that June can afford varies inversely as the price per yard of the fabric. June can afford exactly 5 yards of fabric that costs $10.50 per yard. How many yards of fabric that costs $4.25 per yard can June buy? (Assume that she can only buy whole yards.)

22. **Winter Sports** When a person is snowshoeing, the pressure on the top of the snow in psi varies inversely as the area of the bottom of the snowshoe in square inches. The constant of variation is the weight of the person wearing the snowshoes in pounds.

a. Helen weighs 120 pounds. About how much pressure does she put on top of the snow if she wears snowshoes that cover 360 in^2?

b. Max weighs 207 pounds. If he exerts 0.4 psi of pressure on top of the snow, what is the area of the bottom of his snowshoes in square inches?

Determine if each equation represents a direct variation, an inverse variation, or neither. Find the constant of variation when one exists.

23. $y = 8x$ **24.** $y = \frac{14}{x}$ **25.** $y = \frac{1}{3}x - 2$ **26.** $y = \frac{1}{5}x$

27. $y = 4\frac{3}{x}$ **28.** $y = \frac{x}{2} + 7$ **29.** $y = \frac{15}{x}$ **30.** $y = 5x$

31. **Multi-Step** A track team is competing in a 10 km race. The distance will be evenly divided among the team members. Write an equation that represents the distance d each runner will run if there are n runners. Does this represent a direct variation, inverse variation, or neither?

Determine whether each data set represents a direct variation, an inverse variation, or neither.

32.

x	2	4	8
y	5	10	20

33.

x	6	12	15
y	6	8	9

34.

x	1	2	3
y	12	6	4

35. **Multi-Step** Your club awards one student a $2000 scholarship each year, and each member contributes an equal amount. Your contribution y depends on the number of members x. Write and graph an inverse variation equation that represents this situation. What are a reasonable domain and range?

36. **Estimation** Estimate the value of y if y is inversely proportional to x, $x = 4$, and the constant of variation is 6π.

37. **Critical Thinking** Why will the point $(0, 0)$ never be a solution to an inverse variation?

38. **Write About It** Explain how to write an inverse variation equation of the form $y = \frac{k}{x}$ when values of x and y are known.

39. **Write About It** List all the mathematical terms you know that contain the word *inverse*. How are these terms all similar? How is *inverse variation* similar to these terms?

MULTI-STEP TEST PREP

40. This problem will prepare you for the Multi-Step Test Prep on page 896. The total number of workdays it takes to build the frame of a house varies inversely as the number of people working in a crew. Let x be the number of people in the crew and let y be the number of workdays.

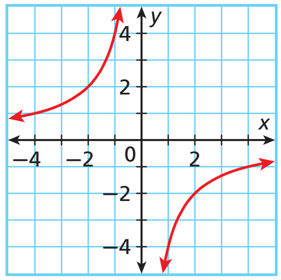
a. Find the constant of variation when $y = 75$ and $x = 2$.

b. Write the rule for the inverse of variation equation.

c. Graph the equation of this inverse variation.

TEST PREP

41. Which equation best represents the graph?



Ⓐ $y = -\frac{1}{4}x$ Ⓒ $y = -\frac{4}{x}$

Ⓑ $y = \frac{1}{4}x$ Ⓓ $y = \frac{4}{x}$

42. Determine the constant of variation if y varies inversely as x and $y = 2$ when $x = 7$.

Ⓕ $\frac{2}{7}$ Ⓖ $\frac{7}{2}$ Ⓗ 3.5 Ⓙ 14

43. Which of the following relationships does NOT represent an inverse variation?

Ⓐ

x	2	4	5
y	10	5	4

Ⓒ

x	2	4	5
y	8	16	20

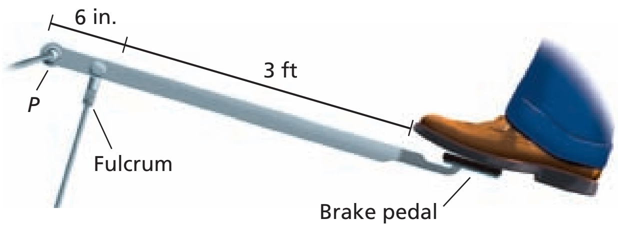
Ⓑ $y = \frac{17.5}{x}$ Ⓓ $\frac{11}{2} = xy$

44. **Gridded Response** At a carnival, the number of tickets Brad can buy is inversely proportional to the price of the tickets. He can afford 12 tickets that cost $2.50 each. How many tickets can Brad buy if each costs $3.00?

CHALLENGE AND EXTEND

45. The definition of inverse variation says that k is a nonzero constant. What function would $y = \frac{k}{x}$ represent if $k = 0$?

46. **Mechanics** A part of a car's braking system uses a lever to multiply the force applied to the brake pedal. The force at the end of a lever varies inversely with the distance from the fulcrum. Point P is the end of the lever. A force of 2 lb is applied to the brake pedal. What is the force created at the point P?



47. **Communication** The strength of a radio signal varies inversely with the square of the distance from the transmitter. A signal has a strength of 2000 watts when it is 4 kilometers from the transmitter. What is the strength of the signal 6 kilometers from the transmitter?

SPIRAL REVIEW

Find the domain and range for each relation. Tell whether the relation is a function. *(Lesson 4-2)*

48. $\{(-2, -4), (-2, -2), (-2, 0), (-2, 2)\}$ **49.** $\{(-4, 5), (-2, 3), (0, 1), (2, 3), (4, 5)\}$

Solve by completing the square. *(Lesson 9-8)*

50. $x^2 + 12x = 45$ **51.** $d^2 - 6d - 7 = 0$ **52.** $2y^2 + 6y = -\frac{5}{2}$

53. A rectangle has a length of 6 cm and a width of 2 cm. Find the length of the diagonal and write it as a simplified radical expression. *(Lesson 11-6)*

12-2 Rational Functions

Objectives

Identify excluded values of rational functions.

Graph rational functions.

Vocabulary

rational function
excluded value
discontinuous function
asymptote

Who uses this?

Gemologists can use rational functions to maximize reflected light. (See Example 4.)

A **rational function** is a function whose rule is a quotient of polynomials. The inverse variations you studied in the previous lesson are a special type of rational function.

Rational functions: $y = \frac{2}{x}, y = \frac{3}{4 - 2x}, y = \frac{1}{x^2}$

For any function involving x and y, an **excluded value** is any x-value that makes the function value y undefined. For a rational function, an excluded value is any value that makes the denominator equal 0.

EXAMPLE 1 Identifying Excluded Values

Identify the excluded value for each rational function.

A $y = \frac{8}{x}$

$x = 0$ *Set the denominator equal to 0.*

The excluded value is 0.

B $y = \frac{3}{x+3}$

$x + 3 = 0$ *Set the denominator equal to 0.*

$x = -3$ *Solve for x.*

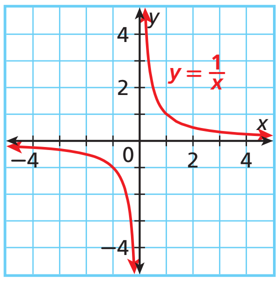
The excluded value is -3.

CHECK IT OUT! **Identify the excluded value for each rational function.**

1a. $y = \frac{10}{x}$ **1b.** $y = \frac{4}{x-1}$ **1c.** $y = -\frac{5}{x+4}$

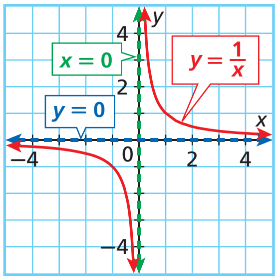
Most rational functions are **discontinuous functions**, meaning their graphs contain one or more jumps, breaks, or holes. This occurs at an excluded value.

One place that a graph of a rational function is discontinuous is at an *asymptote*. An **asymptote** is a line that a graph gets closer to as the absolute value of a variable increases. In the graph shown, both the x- and y-axes are asymptotes. The graphs of rational functions will get closer and closer to but never touch the asymptotes.

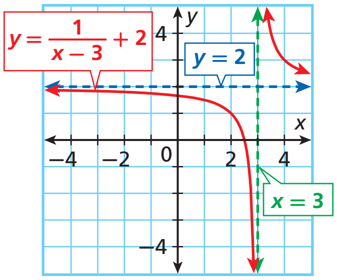


For rational functions, vertical asymptotes will occur at excluded values.

Look at the graph of $y = \frac{1}{x}$. The denominator is 0 when $x = 0$ so **0** is an excluded value. This means there is a vertical asymptote at $x = 0$. Notice the horizontal asymptote at $y = 0$.



Look at the graph of $y = \frac{1}{x-3} + 2$. Notice that the graph of the parent function $y = \frac{1}{x}$ has been translated **3 units right** and there is a vertical asymptote at $x = 3$. The graph has also been translated **2 units up** and there is a horizontal asymptote at $y = 2$.



These translations lead to the following formulas for identifying asymptotes in rational functions.

Writing Math

Vertical lines are written in the form $x = b$, and horizontal lines are written in the form $y = c$.

Know it! Note

Identifying Asymptotes

WORDS	EXAMPLES	
A rational function in the form $y = \frac{a}{x - b} + c$ has a vertical asymptote at the excluded value, or $x = b$, and a horizontal asymptote at $y = c$.	$y = \frac{2}{x}$ $= \frac{2}{x - 0} + 0$ Vertical asymptote: $x = 0$ Horizontal asymptote: $y = 0$	$y = \frac{1}{x + 2} + 4$ $= \frac{1}{x - (-2)} + 4$ Vertical asymptote: $x = -2$ Horizontal asymptote: $y = 4$

EXAMPLE 2 Identifying Asymptotes

Identify the asymptotes.

A $y = \frac{1}{x - 6}$

Step 1 Write in $y = \frac{1}{x - b} + c$ form.

$$y = \frac{1}{x - 6} + 0$$

Step 2 Identify the asymptotes.

vertical: $x = 6$

horizontal: $y = 0$

B $y = \frac{2}{3x - 10} - 7$

Step 1 Identify the vertical asymptote.

$$\begin{aligned} 3x - 10 &= 0 \\ \underline{+10} \quad & \underline{+10} \\ 3x &= 10 \\ x &= \frac{10}{3} \end{aligned}$$

Find the excluded value. Set the denominator equal to 0.

Add 10 to both sides.

Solve for x. $\frac{10}{3}$ is an excluded value.

Step 2 Identify the horizontal asymptote.

$c = -7$ *-7 can be written as $+(-7)$*

$y = -7$ *$y = c$*

vertical asymptote: $x = \frac{10}{3}$; horizontal asymptote: $y = -7$

CHECK IT OUT! **Identify the asymptotes.**

2a. $y = \dfrac{2}{x-5}$ **2b.** $y = \dfrac{1}{4x+16} + 5$ **2c.** $y = \dfrac{3}{x+77} - 15$

To graph a rational function in the form $y = \frac{a}{x-b} + c$ when $a = 1$, you can graph the asymptotes and then translate the parent function $y = \frac{1}{x}$. However, if $a \neq 1$, the graph is not a translation of the parent function. In this case, you can use the asymptotes and a table of values.

EXAMPLE 3 Graphing Rational Functions Using Asymptotes

Graph each function.

A $y = \dfrac{2}{x+1}$

Since the numerator is not 1, use the asymptotes and a table of values.

Step 1 Identify the vertical and horizontal asymptotes.

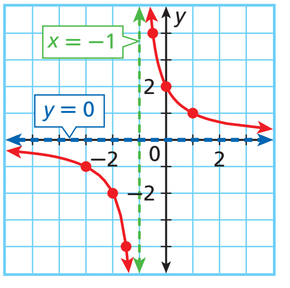
vertical: $x = -1$ *Use $x = b$. $x + 1 = x - (-1)$, so $b = -1$.*

horizontal : $y = 0$ *Use $y = c$. $c = 0$*

Step 2 Graph the asymptotes using dashed lines.

Step 3 Make a table of values. Choose x-values on both sides of the vertical asymptote.

x	-3	-2	$-\frac{3}{2}$	$-\frac{1}{2}$	0	1
y	-1	-2	-4	4	2	1



Step 4 Plot the points and connect them with smooth curves. The curves will get very close to the asymptotes, but will not touch them.

B $y = \dfrac{1}{x-2} - 4$

Since the numerator is 1, use the asymptotes and translate $y = \frac{1}{x}$.

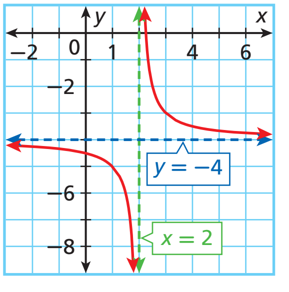
Step 1 Identify the vertical and horizontal asymptotes.

vertical: $x = 2$ *Use $x = b$. $b = 2$*

horizontal: $y = -4$ *Use $y = c$. $c = -4$*

Step 2 Graph the asymptotes using dashed lines.

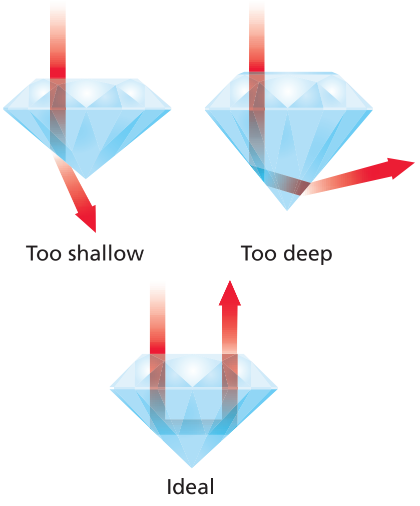
Step 3 Draw smooth curves to show the translation.



CHECK IT OUT! **Graph each function.**

3a. $y = \dfrac{1}{x+7} + 3$ **3b.** $y = \dfrac{2}{x-3} + 2$

EXAMPLE 4 Gemology Application



Some diamonds are cut using ratios calculated by a mathematician, Marcel Tolkowsky, in 1919. The amount of light reflected up through the top of a diamond (brilliancy) can be maximized using the ratio between the width of the diamond and the depth of the diamond. A gemologist has a diamond with a width of 90 millimeters. If x represents the depth of the diamond, then $y = \frac{90}{x}$ represents the brilliancy ratio y.

a. Describe the reasonable domain and range values.

Both the depth of the diamond and the brilliancy ratio will be nonnegative, so nonnegative values are reasonable for the domain and range.

b. Graph the function.

Step 1 Identify the vertical and horizontal asymptotes.

vertical: $x = 0$ *Use $x = b$. $b = 0$*

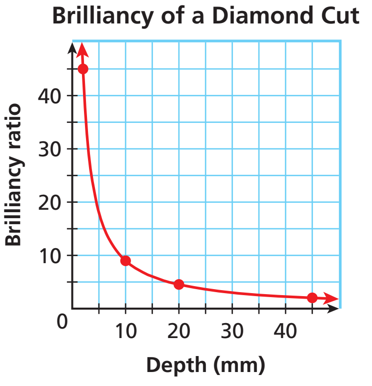
horizontal: $y = 0$ *Use $y = c$. $c = 0$*

Step 2 Graph the asymptotes.
The asymptotes will be the x- and y-axes.

Step 3 Since the domain is restricted to nonnegative values, only choose x-values on the right side of the vertical asymptote.

Depth of Diamond (mm)	2	10	20	45
Brilliancy Ratio	45	9	4.5	2

Step 4 Plot the points and connect them with smooth curves.



CHECK IT OUT!

4. A librarian has a budget of \$500 to buy copies of a software program. She will receive 10 free copies when she sets up an account with the supplier. The number of copies y of the program that she can buy is given by $y = \frac{500}{x} + 10$, where x is the price per copy.

a. Describe the reasonable domain and range values.

b. Graph the function.

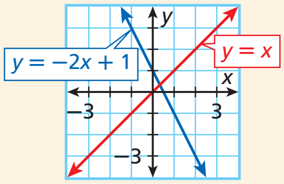
The table shows some of the properties of the four families of functions you have studied and their graphs.

Families of Functions

LINEAR FUNCTION

$$y = mx + b$$

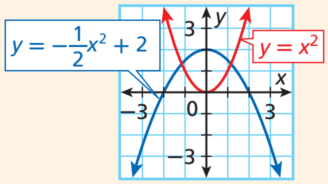
- Parent function: $y = x$
- m is the slope. It rotates the graph about $(0, b)$.
- b is the y-intercept. It translates the graph of $y = x$ **vertically**.



QUADRATIC FUNCTION

$$y = ax^2 + bx + c$$

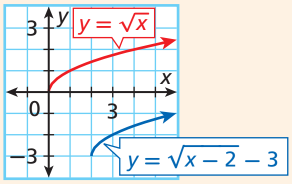
- Parent function: $y = x^2$
- a determines the width of the parabola and the direction it opens.
- c translates the graph of $y = ax^2$ **vertically**.
- The axis of symmetry is the vertical line $x = -\frac{b}{2a}$.



SQUARE-ROOT FUNCTION

$$y = \sqrt{x - a} + b$$

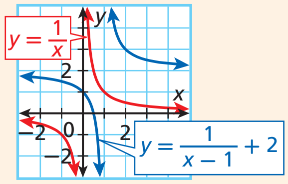
- Parent function: $y = \sqrt{x}$
- a translates the graph of $y = \sqrt{x}$ **horizontally**.
- b translates the graph of $y = \sqrt{x}$ **vertically**.



RATIONAL FUNCTION

$$y = \frac{a}{x - b} + c$$

- Parent function: $y = \frac{1}{x}$
- b translates the graph $y = \frac{1}{x}$ **horizontally**.
- c translates the graph of $y = \frac{1}{x}$ **vertically**.



THINK AND DISCUSS

1. Does $y = \frac{1}{x-5}$ have any excluded values? Explain.

2. Tell how to find the vertical and horizontal asymptotes of $y = \frac{1}{x+9} - 5$.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, find the asymptotes for the given rational function.

Rational Functions

- $y = \frac{1}{x-2}$
- $y = \frac{1}{x+2}$
- $y = \frac{1}{x} + 2$
- $y = \frac{1}{x} - 2$

12-2 Exercises

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Homework Help Online
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Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

1. **Vocabulary** An x-value that makes a function undefined is a(n) ___?___. (*asymptote* or *excluded value*)

SEE EXAMPLE 1 p. 878

Identify the excluded value for each rational function.

2. $y = \frac{4}{x}$
3. $y = \frac{2}{x+3}$
4. $y = -\frac{2}{x}$
5. $y = \frac{16}{x-4}$

SEE EXAMPLE 2 p. 879

Identify the asymptotes.

6. $y = \frac{1}{x-3}$
7. $y = \frac{4}{3x+15}$
8. $y = \frac{2}{3x-5} + 2$
9. $y = \frac{1}{x+9} - 10$

SEE EXAMPLE 3 p. 880

Graph each function.

10. $y = \frac{2}{x+6}$
11. $y = \frac{1}{x-2} - 6$
12. $y = \frac{1}{x} + 2$
13. $y = \frac{1}{x-3} - 2$

SEE EXAMPLE 4 p. 881

14. **Catering** A caterer has \$100 in her budget for fruit. Slicing and delivery of each pound of fruit costs \$5. If x represents the cost per pound of the fruit itself, then $y = \frac{100}{x+5}$ represents the number of pounds y she can buy.
 a. Describe the reasonable domain and range values.
 b. Graph the function.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
15–18	1
19–22	2
23–26	3
27	4

Extra Practice
Skills Practice p. S26
Application Practice p. S39

Identify the excluded value for each rational function.

15. $y = \frac{7}{x}$
16. $y = \frac{1}{x-4}$
17. $y = -\frac{15}{x}$
18. $y = \frac{12}{x-5}$

Identify the asymptotes.

19. $y = \frac{9}{x-4}$
20. $y = \frac{2}{x+4}$
21. $y = \frac{7}{4x-12} + 4$
22. $y = \frac{7}{3x+5} - 9$

Graph each function.

23. $y = \frac{5}{x-5}$
24. $y = \frac{1}{x+5} - 6$
25. $y = \frac{1}{x+4}$
26. $y = \frac{1}{x-4} + 2$

27. **Business** A wholesaler is buying auto parts. He has \$200 to spend. He receives 5 parts free with the order. The number of parts y he can buy, if the average price of the parts is x dollars, is $y = \frac{200}{x} + 5$.
 a. Describe the reasonable domain and range values.
 b. Graph the function.

Find the excluded value for each rational function.

28. $y = \frac{4}{x}$
29. $y = \frac{1}{x-7}$
30. $y = \frac{2}{x+4}$
31. $y = \frac{3}{2x+1}$

Graph each rational function. Show the asymptotes.

32. $y = \frac{1}{x-2}$
33. $y = \frac{2}{x} + 3$
34. $y = \frac{3}{x+1} + 2$
35. $y = \frac{1}{x-4} - 1$

36. **Multi-Step** The function $y = \frac{60}{x^2}$ relates the luminescence in lumens y of a 60-watt lightbulb viewed from a distance of x ft. Graph the function. Use the graph to find the luminescence of a 60-watt lightbulb viewed from a distance of 6 ft.

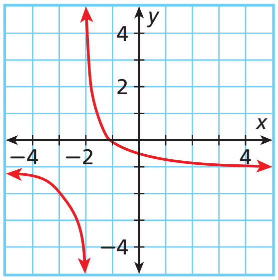
Identify the asymptotes of each rational function.

37. $y = \dfrac{7}{x+1}$ **38.** $y = \dfrac{1}{x} - 5$ **39.** $y = \dfrac{12}{x-2} + 5$ **40.** $y = \dfrac{18}{x+3} - 9$

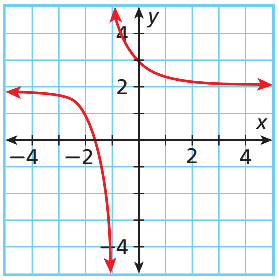
Match each graph with one of the following functions.

A. $y = \dfrac{1}{x+1} + 2$ **B.** $y = \dfrac{1}{x+2} - 1$ **C.** $y = \dfrac{1}{x-2} + 1$

41.



42.



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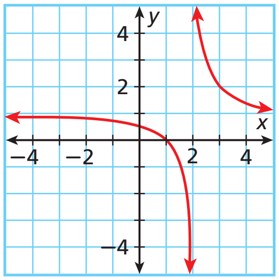


44. **ERROR ANALYSIS** In finding the horizontal asymptote of $y = \frac{1}{x+2} - 3$, student A said the asymptote is at $y = -3$, and student B said it is at $y = -2$. Who is incorrect? Explain the error.

45. Finance The time in months y that it will take to pay off a bill of \$1200, when x dollars are paid each month and the finance charge is \$15 per month, is $y = \frac{1200}{x-15}$. Describe the reasonable domain and range values and graph the function.

46. The table shows how long it takes different size landscaping teams to complete a project.

a. Graph the data.

b. Write a rational function to represent the data.

c. How many hours would it take 12 landscapers to complete the project?

Landscapers	Time (h)
2	30
4	15
5	12
10	6

Graph each function. Compare its graph to the graph of $y = \frac{1}{x}$.

47. $y = \dfrac{1}{x-6}$ **48.** $y = \dfrac{1}{x+7}$ **49.** $y = \dfrac{1}{x} + 4$ **50.** $y = \dfrac{1}{x-2} - 9$

Find the domain that makes the range positive.

51. $y = \dfrac{10}{x-2}$ **52.** $y = \dfrac{10}{x+2}$ **53.** $y = \dfrac{5}{5x+1}$ **54.** $y = \dfrac{4}{3x-7}$

55. Critical Thinking In which quadrants would you find the graph of $y = \frac{a}{x}$ when a is positive? when a is negative?

MULTI-STEP TEST PREP

56. This problem will prepare you for the Multi-Step Test Prep on page 896.

It takes a total of 250 workdays to build a house for charity. For example, if 2 workers build the house, it takes them 125 actual construction days. If 10 workers are present, it takes 25 construction days to build the house.

a. Write a function that represents the number of construction days to build as a function of the number of workers.

b. What is the domain of this function?

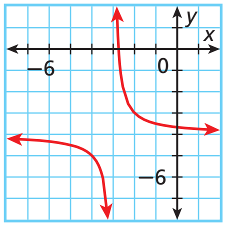
c. Sketch a graph of the function.

57. Write About It Graph each pair of functions on a graphing calculator. Then make a conjecture about the relationship between the graphs of the rational functions $y = \frac{k}{x}$ and $y = \frac{-k}{x}$.

a. $y = \frac{1}{x};\ y = \frac{-1}{x}$ **b.** $y = \frac{3}{x};\ y = \frac{-3}{x}$ **c.** $y = \frac{5}{x};\ y = \frac{-5}{x}$

TEST PREP

58. Which function is graphed?



Ⓐ $y = \frac{2}{x+3} - 4$ Ⓒ $y = \frac{2}{x-3} + 4$

Ⓑ $y = \frac{2}{x+4} - 3$ Ⓓ $y = \frac{2}{x-4} + 3$

59. Which rational function has a graph with the horizontal asymptote $y = -1$?

Ⓕ $y = \frac{-1}{x}$ Ⓗ $y = \frac{1}{x+1}$

Ⓖ $y = \frac{1}{x-1}$ Ⓙ $y = \frac{1}{x} - 1$

60. Short Response Write a rational function whose graph is the same shape as the graph of $f(x) = \frac{1}{x}$, but is translated 2 units left and 3 units down. Graph the function.

CHALLENGE AND EXTEND

61. Graph the equation $y = \frac{1}{x^2 + 1}$.

a. Does this equation represent a rational function? Explain.

b. What is the domain of the function?

c. What is the range of the function?

d. Is the graph discontinuous?

62. Graphing Calculator Are the graphs of $f(x) = \frac{(x-3)(x-1)}{x-3}$ and $g(x) = x - 1$ identical? Explain. (*Hint:* Are there any excluded values?)

63. Critical Thinking Write the equation of the rational function that has a horizontal asymptote at $y = 3$ and a vertical asymptote at $x = -2$ and contains the point $(1, 4)$.

SPIRAL REVIEW

Solve each equation by graphing the related function. *(Lesson 9-5)*

64. $4 - x^2 = 0$ **65.** $3x^2 = x^2 + 2x + 12$ **66.** $-x^2 = -6x + 9$

67. In the first five stages of a fractal design, a line segment has the following lengths, in centimeters: 240, 120, 60, 30, 15. Use this pattern and your knowledge of geometric sequences to determine the length of the segment in the tenth stage. *(Lesson 11-1)*

Determine whether each function represents an inverse variation. Explain. *(Lesson 12-1)*

68. $x + y = 12$

69.

x	30	60	90	120
y	10	20	30	40

70. $xy = -4$

12-3 Simplifying Rational Expressions

Objectives

Simplify rational expressions.

Identify excluded values of rational expressions.

Vocabulary

rational expression

Why learn this?

The shapes and sizes of plants and animals are partly determined by the ratio of surface area to volume.

If an animal's body is small and its surface area is large, the rate of heat loss will be high. Hummingbirds must maintain a high metabolism to compensate for the loss of body heat due to having a high surface-area-to-volume ratio. Formulas for surface-area-to-volume ratios are *rational expressions*.

A **rational expression** is an algebraic expression whose numerator and denominator are polynomials. The value of the polynomial expression in the denominator cannot be zero since division by zero is undefined. This means that rational expressions may have excluded values.

Remember!

To review the Zero Product Property, see Lesson 9-6.

To review factoring trinomials, see Chapter 8.

EXAMPLE 1 **Identifying Excluded Values**

Find any excluded values of each rational expression.

A $\dfrac{5}{8r}$

$8r = 0$ — *Set the denominator equal to 0.*

$r = \dfrac{0}{8} = 0$ — *Solve for r by dividing both sides by 8.*

The excluded value is 0.

B $\dfrac{9d+1}{d^2-2d}$

$d^2 - 2d = 0$ — *Set the denominator equal to 0.*

$d(d-2) = 0$ — *Factor.*

$d = 0$ or $d - 2 = 0$ — *Use the Zero Product Property.*

$d = 2$ — *Solve for d.*

The excluded values are 0 and 2.

C $\dfrac{x+4}{x^2+5x+6}$

$x^2 + 5x + 6 = 0$ — *Set the denominator equal to 0.*

$(x+3)(x+2) = 0$ — *Factor.*

$x + 3 = 0$ or $x + 2 = 0$ — *Use the Zero Product Property.*

$x = -3$ or $x = -2$ — *Solve each equation for x.*

The excluded values are −3 and −2.

CHECK IT OUT! **Find any excluded values of each rational expression.**

1a. $\dfrac{12}{t+5}$ **1b.** $\dfrac{3b}{b^2+5b}$ **1c.** $\dfrac{3k^2}{k^2+7k+12}$

A rational expression is in its simplest form when the numerator and denominator have no common factors except 1. Remember that to simplify fractions you can divide out common factors that appear in both the numerator and the denominator. You can do the same to simplify rational expressions.

EXAMPLE 2 Simplifying Rational Expressions

Simplify each rational expression, if possible. Identify any excluded values.

Caution!

Be sure to use the original denominator when finding excluded values. The excluded values may not be "seen" in the simplified denominator.

A $\dfrac{3t^3}{12t}$

$\dfrac{3t^3}{3 \cdot 4t}$ *Factor 12.*

$\dfrac{{}^1\cancel{3}\cancel{t^3}^{t^2}}{{}_1\cancel{3} \cdot 4\cancel{t}^1}$ *Divide out common factors. Note that if t = 0, the expression is undefined.*

$\dfrac{t^2}{4};\ t \neq 0$ *Simplify. The excluded value is 0.*

B $\dfrac{3x^2 - 9x}{x - 3}$

$\dfrac{3x(x-3)}{x-3}$ *Factor the numerator.*

$\dfrac{3x\cancel{(x-3)}^1}{\cancel{x-3}^1}$ *Divide out common factors. Note that if x = 3, the expression is undefined.*

$3x;\ x \neq 3$ *Simplify. The excluded value is 3.*

C $\dfrac{c}{c+5}$

$\dfrac{c}{c+5};\ c \neq -5$ *The numerator and denominator have no common factors. The excluded value is −5.*

CHECK IT OUT! **Simplify each rational expression, if possible. Identify any excluded values.**

2a. $\dfrac{5m^2}{15m}$ **2b.** $\dfrac{6p^3 + 12p}{p^2 + 2}$ **2c.** $\dfrac{3n}{n-2}$

From now on in this chapter, you may assume that the values of the variables that make the denominator equal to 0 are excluded values. You do not need to include excluded values in your answers unless they are asked for.

EXAMPLE 3 Simplifying Rational Expressions with Trinomials

Simplify each rational expression, if possible.

A $\dfrac{k+1}{k^2 - 4k - 5}$

$\dfrac{k+1}{(k+1)(k-5)}$ *Factor the numerator and the denominator when possible.*

$\dfrac{\cancel{k+1}^1}{\cancel{(k+1)}^1(k-5)}$ *Divide out common factors.*

$\dfrac{1}{k-5}$ *Simplify.*

B $\dfrac{y^2 - 16}{y^2 - 8y + 16}$

$\dfrac{(y+4)(y-4)}{(y-4)(y-4)}$

$\dfrac{(y+4)\cancel{(y-4)}^1}{(y-4)\cancel{(y-4)}^1}$

$\dfrac{y+4}{y-4}$

CHECK IT OUT! Simplify each rational expression, if possible.

3a. $\dfrac{r+2}{r^2+7r+10}$ **3b.** $\dfrac{b^2-25}{b^2+10b+25}$

Recall from Chapter 8 that opposite binomials can help you factor polynomials. Recognizing opposite binomials can also help you simplify rational expressions.

Consider $\frac{x-3}{3-x}$. The numerator and denominator are opposite binomials. Therefore,

$$\frac{x-3}{3-x}=\frac{x-3}{-x+3}=\frac{\cancel{x-3}^{1}}{-1\cancel{(x-3)}^{1}}=\frac{1}{-1}=-1.$$

EXAMPLE 4 Simplifying Rational Expressions Using Opposite Binomials

Simplify each rational expression, if possible.

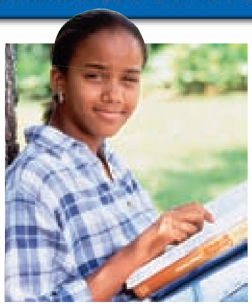
A $\dfrac{2x-10}{25-x^2}$		**B** $\dfrac{2-2m}{2m^2+2m-4}$
$\dfrac{2(x-5)}{(5-x)(5+x)}$	*Factor.*	$\dfrac{2(1-m)}{2(m+2)(m-1)}$
$\dfrac{2(x-5)}{(5-x)(5+x)}$	*Identify opposite binomials.*	$\dfrac{2(1-m)}{2(m+2)(m-1)}$
$\dfrac{2(x-5)}{-1(x-5)(5+x)}$	*Rewrite one opposite binomial.*	$\dfrac{2(1-m)}{2(m+2)(-1)(1-m)}$
$\dfrac{2\cancel{(x-5)}^{1}}{-1\cancel{(x-5)}^{1}(5+x)}$	*Divide out common factors.*	$\dfrac{\cancel{2}^{1}\cancel{(1-m)}^{1}}{\cancel{2}^{1}(m+2)(-1)\cancel{(1-m)}^{1}}$
$-\dfrac{2}{5+x}$	*Simplify.*	$-\dfrac{1}{m+2}$

CHECK IT OUT! Simplify each rational expression, if possible.

4a. $\dfrac{3x-12}{16-x^2}$ **4b.** $\dfrac{6-2x}{2x^2-4x-6}$ **4c.** $\dfrac{3x-33}{x^2-121}$

Student to Student

Simplifying Rational Expressions



Tanika Brown,
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When I can't tell if I'm allowed to divide out part of an expression, I substitute a number into the original expression and simplify it. Then I simplify the original expression by dividing out the term in question. I check by seeing if the results are the same.

For example, I'll use $x = 2$ to see if I can divide out $4x$ in $\frac{4x}{4x-7}$.

Substitute $x = 2$.	*Divide out $4x$.*
$\dfrac{4x}{4x-7}$	$\dfrac{\cancel{4}^{1}\cancel{x}^{1}}{\cancel{4}^{1}\cancel{x}^{1}-7}$
$\dfrac{4(2)}{4(2)-7}$	$\dfrac{1}{1-7}$
$\dfrac{8}{8-7}=\dfrac{8}{1}=8$	$\dfrac{1}{-6}$

$4x$ cannot be divided out because $8 \neq \frac{1}{-6}$.

EXAMPLE 5 Biology Application

Desert plants must conserve water. Water evaporates from the surface of a plant. The volume determines how much water is in a plant. So the greater the surface-area-to-volume ratio, the less likely a plant is to survive in the desert. A barrel cactus is a desert plant that is close to spherical in shape.

a. What is the surface-area-to-volume ratio of a spherical barrel cactus? (*Hint:* For a sphere, $S = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$.)

$\dfrac{4\pi r^2}{\frac{4}{3}\pi r^3}$ *Write the ratio of surface area to volume.*

$\dfrac{4\cancel{\pi}^1 r^2}{\frac{4}{3}\cancel{\pi}^1 r^3}$ *Divide out common factors.*

$\dfrac{4\cancel{r^2}^1}{\frac{4}{3}\cancel{r^3}^r}$ *Use properties of exponents.*

$\dfrac{4}{r} \cdot \dfrac{3}{4}$ *Multiply by the reciprocal of $\frac{4}{3}$.*

$\dfrac{\cancel{4}^1}{r} \cdot \dfrac{3}{\cancel{4}^1}$ *Divide out common factors.*

$\dfrac{3}{r}$ *Simplify.*

b. Which barrel cactus has a greater chance of survival in the desert, one with a radius of 4 inches or one with a radius of 7 inches? Explain.

$\dfrac{3}{r} = \dfrac{3}{4}$ $\quad$ $\dfrac{3}{r} = \dfrac{3}{7}$ *Write the ratio of surface area to volume twice. Substitute 4 and 7 for r.*

$\dfrac{3}{4} > \dfrac{3}{7}$ *Compare the ratios.*

The barrel cactus with a radius of 7 inches has a greater chance of survival. Its surface-area-to-volume ratio is less than for a cactus with a radius of 4 inches.

Remember!

For two fractions with the same numerator, the value of the fraction with a greater denominator is less than the value of the other fraction.

$9 > 3$

$\frac{2}{9} < \frac{2}{3}$

CHECK IT OUT!

5. Which barrel cactus has less of a chance to survive in the desert, one with a radius of 6 inches or one with a radius of 3 inches? Explain.

THINK AND DISCUSS

1. Write a rational expression that has an excluded value that cannot be identified when the expression is in its simplified form.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write and simplify one of the given rational expressions using the most appropriate method. $\frac{x-3}{x^2-6x+9}, \frac{5x^4}{x^2}, \frac{4-x}{x-4}, \frac{4x^2-4x}{8x}$

Ways of Simplifying Rational Expressions

- Using properties of exponents
- Using opposite binomials
- Factoring the numerator
- Factoring the denominator

12-3 Exercises

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KEYWORD: MA7 12-3
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

1. **Vocabulary** What is true about both the numerator and denominator of rational expressions?

SEE EXAMPLE 1 p. 886

Find any excluded values of each rational expression.

2. $\dfrac{5}{m}$ 3. $\dfrac{x+2}{x^2-8x}$ 4. $\dfrac{p^2}{p^2-2p-15}$

SEE EXAMPLE 2 p. 887

Simplify each rational expression, if possible. Identify any excluded values.

5. $\dfrac{4a^2}{8a}$ 6. $\dfrac{2d^2+12d}{d+6}$ 7. $\dfrac{2}{y+3}$

8. $\dfrac{10}{5-y}$ 9. $\dfrac{2h}{2h+4}$ 10. $\dfrac{3(x+4)}{6x}$

Simplify each rational expression, if possible.

SEE EXAMPLE 3 p. 887

11. $\dfrac{b+4}{b^2+5b+4}$ 12. $\dfrac{s^2-4}{s^2+4s+4}$ 13. $\dfrac{c^2+5c+6}{(c+3)(c-4)}$

14. $\dfrac{(x-2)(x+1)}{x^2+4x+3}$ 15. $\dfrac{j^2-25}{j^2+2j-15}$ 16. $\dfrac{p+1}{p^2-4p-5}$

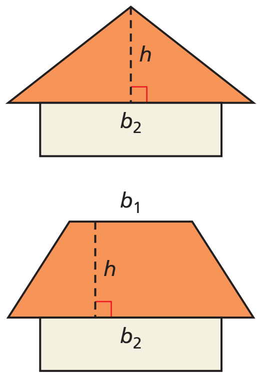
SEE EXAMPLE 4 p. 888

17. $\dfrac{2n-16}{64-n^2}$ 18. $\dfrac{8-4x}{2x^2-12x+16}$ 19. $\dfrac{10-5r}{r^2-4r-12}$

20. $\dfrac{2x-14}{49-x^2}$ 21. $\dfrac{5q-50}{100-q^2}$ 22. $\dfrac{36-12a}{a^2+2a-15}$

SEE EXAMPLE 5 p. 889

23. **Construction** The side of a triangular roof will have the same height h and base b_2 as the side of a trapezoidal roof.



a. What is the ratio of the area of the triangular roof to the area of the trapezoidal roof?
$\left(\textit{Hint:} \text{ For a triangle, } A = \frac{1}{2}b_2h.\right.$
$\left.\text{For a trapezoid, } A = \frac{b_1+b_2}{2}h.\right)$

b. Compare the ratio from part **a** to what the ratio will be if b_1 is doubled for the trapezoidal roof and b_2 is doubled for both roofs.

PRACTICE AND PROBLEM SOLVING

Find any excluded values of each rational expression.

24. $\dfrac{c}{c^2+c}$ 25. $\dfrac{2}{-3x}$ 26. $\dfrac{4}{x^2-3x-10}$ 27. $\dfrac{n^2-1}{2n^2-7n-4}$

Simplify each rational expression, if possible. Identify any excluded values.

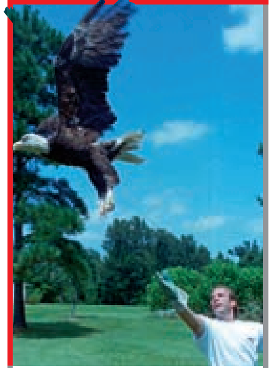
28. $\dfrac{4d^3+4d^2}{d+1}$ 29. $\dfrac{3m^2}{m-4}$ 30. $\dfrac{10y^4}{2y}$ 31. $\dfrac{2t^2}{16t}$

Independent Practice	
For Exercises	See Example
24–27	1
28–31	2
32–37	3
38–40	4
41	5

Extra Practice
Skills Practice p. S26
Application Practice p. S39

LINK

Biology



As a result of a nation-wide policy of protection and reintroduction, the population of bald eagles in the lower 48 states grew from 417 nesting pairs in 1963 to more than 6400 nesting pairs in 2000.

Source: U.S. Fish and Wildlife Service

Simplify each rational expression, if possible.

32. $\dfrac{q-6}{q^2-9q+18}$ **33.** $\dfrac{z^2-2z+1}{z^2-1}$ **34.** $\dfrac{t-3}{t^2-5t+6}$

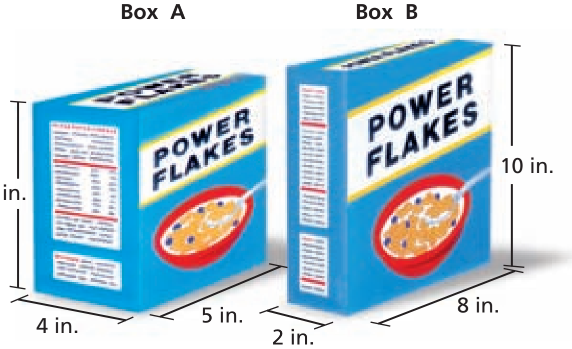
35. $\dfrac{p^2-6p-7}{p^2-4p-5}$ **36.** $\dfrac{x^2-1}{x^2+4x+3}$ **37.** $\dfrac{2x-4}{x^2-6x+8}$

38. $\dfrac{20-4x}{x^2-25}$ **39.** $\dfrac{3-3b}{3b^2+18b-21}$ **40.** $\dfrac{3v-36}{144-v^2}$

41. Geometry When choosing package sizes, a company wants a package that uses the least amount of material to hold the greatest volume of product.

a. What is the surface-area-to-volume ratio for a rectangular prism? (*Hint:* For a rectangular prism, $S = 2\ell w + 2\ell h + 2wh$ and $V = \ell wh$.)

b. Which box should the company choose? Explain.



42. Biology The table gives information on two populations of animals that were released into the wild. Suppose 16 more predators and 20 more prey are released into the area. Write and simplify a rational expression to show the ratio of predator to prey.

	Predator	Prey
Original Population	x	x
Population 5 Years Later	$4x$	$5x$

Simplify each rational expression, if possible.

43. $\dfrac{p^2+12p+36}{12p+72}$ **44.** $\dfrac{3n^3+33n^2+15n}{3n^3+15n}$ **45.** $\dfrac{a}{2a+a}$

46. $\dfrac{j-5}{j^2-25}$ **47.** $\dfrac{6w^2+11w-7}{6w-3}$ **48.** $\dfrac{n^2-n-56}{n^2-16n+64}$

49. $\dfrac{(x+1)^2}{x^2+2x+1}$ **50.** $\dfrac{5}{(x+5)^2}$ **51.** $\dfrac{25-x^2}{x^2-3x-10}$

MULTI-STEP TEST PREP

52. This problem will prepare you for the Multi-Step Test Prep on page 896.

It takes 250 workdays to build a house. The number of construction days is determined by the size of the crew. The crew includes one manager who supervises workers and checks for problems, but does not do any building.

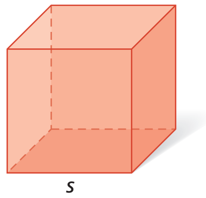
a. The table shows the number of construction days as a function of the number of workers. Copy and complete the table.

b. Use the table to write a function that represents the number of construction days.

c. Identify the excluded values of the function.

Crew Size (x)	$\dfrac{\text{Workdays}}{\text{Workers}}$	Construction Days (y)
2	$\dfrac{250}{2-1}$	250
3	$\dfrac{250}{3-1}$	▒
6	▒	▒
11	▒	25

53. **Geometry** Let s represent the length of an edge of a cube.



a. Write the ratio of a cube's surface area to volume in simplified form. (*Hint:* For a cube, $S = 6s^2$.)

b. What is the ratio of the cube's surface area to volume when $s = 2$?

c. What is the ratio of the cube's surface area to volume when $s = 6$?

54. **Write About It** Explain how to find excluded values for a rational expression.

55. **Critical Thinking** Give an example of a rational expression that has x in both the numerator and denominator, but cannot be simplified.

TEST PREP

56. Which expression is undefined for $x = 4$ and $x = -1$?

(A) $\dfrac{x-1}{x+4}$ (B) $\dfrac{x-4}{x+4}$ (C) $\dfrac{x}{x^2+3x-4}$ (D) $\dfrac{x}{x^2-3x-4}$

57. Which expression is the ratio of the area of a triangle to the area of a rectangle that has the same base and height?

(F) $\dfrac{1}{2}$ (G) $\dfrac{bh}{2}$ (H) $\dfrac{(bh)^2}{2}$ (J) 2

58. **Gridded Response** What is the excluded value for $\dfrac{x-4}{x^2-8x+16}$?

CHALLENGE AND EXTEND

Tell whether each statement is sometimes, always, or never true. Explain.

59. A rational expression has an excluded value.

60. A rational expression has a variable in a square root in the numerator.

61. The graph of a rational function has at least one asymptote.

Simplify each rational expression.

62. $\dfrac{9v-6v^2}{4v^2-4v-3}$ 63. $\dfrac{2a^2-7a+3}{2a^2+9a-5}$ 64. $\dfrac{0.25y-0.10}{0.25y^2-0.04}$

Identify any excluded values of each rational expression.

65. $\dfrac{\frac{1}{4}x^2-7x+49}{\frac{1}{4}x^2-49}$ 66. $\dfrac{-80x+40x^2+40}{-30-30x^2+60x}$ 67. $\dfrac{6x+12}{12x+6x^2}$

SPIRAL REVIEW

68. A rectangle has an area of 24 square feet. Every dimension is multiplied by a scale factor, and the new rectangle has an area of 864 square feet. What is the scale factor? *(Lesson 2-8)*

Use intercepts to graph the line described by each equation. *(Lesson 5-2)*

69. $5x - 3y = -15$ 70. $y = 8x - 8$ 71. $\frac{1}{2}x + y = 2$

For each of the following, let y vary inversely as x. *(Lesson 12-1)*

72. $x_1 = 2$, $y_1 = 4$, and $x_2 = 1$. Find y_2. 73. $x_1 = 2$, $y_1 = -1$, and $y_2 = \frac{1}{3}$. Find x_2.

12-3 Technology LAB

Graph Rational Functions

You can use a graphing calculator to graph rational functions and to compare graphs of rational functions before and after they are simplified.

Use with Lesson 12-3

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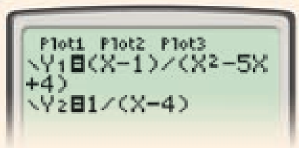
Activity

Simplify $y = \frac{x-1}{x^2 - 5x + 4}$ and give any excluded values. Then graph both the original function and the simplified function and compare the graphs.

1. Simplify the function and find the excluded values.

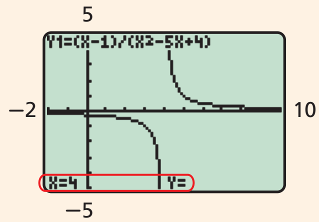
   $\frac{x-1}{x^2 - 5x + 4} = \frac{x-1}{(x-1)(x-4)} = \frac{1}{x-4}$; excluded values: 4, 1

2. Enter $y = \frac{x-1}{x^2-5x+4}$ and $y = \frac{1}{x-4}$ into your calculator as shown and press GRAPH.



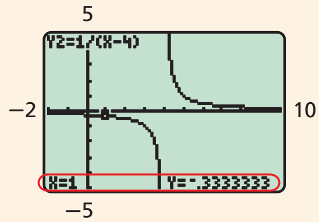
3. To compare the graphs, press TRACE. At the top of the screen, you can see which graph the cursor is on. To change between graphs, press ▲ and ▼.

4. The graphs appear to be the same, but check the excluded values, 4 and 1. While on **Y1,** press 4 ENTER. Notice that there is no y-value at $x = 4$. The function is undefined.



5. Press ▼ to switch to **Y2** and press 4 ENTER. This function is also undefined at $x = 4$. The graphs are the same at this excluded value.

6. Return to **Y1** and press 1 ENTER. This function is undefined at $x = 1$. However, this is not a vertical asymptote. Instead, this graph has a "hole" at $x = 1$.



7. Switch to **Y2** and press 1 ENTER. This function is defined at $x = 1$. So the two graphs are the same except at $x = 1$.

Try This

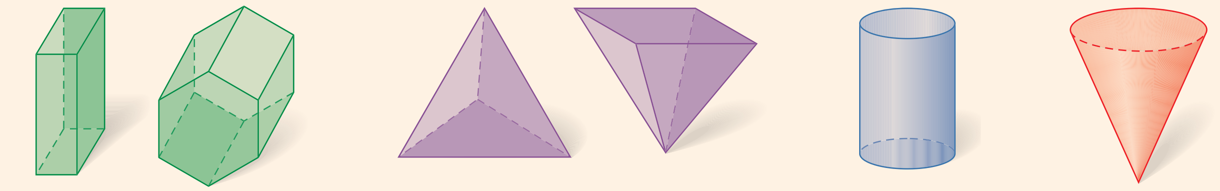
1. Why is $x = 1$ an excluded value for one function but not for the other?
2. Are the functions $y = \frac{x-1}{x^2-5x+4}$ and $y = \frac{1}{x-4}$ truly equivalent for all values of x? Explain.
3. **Make a Conjecture** Complete each statement.
   a. If a value of x is excluded from a function and its simplified form, it appears on the graph as a(n) ___?___.
   b. If a value of x is excluded from a function but not its simplified form, it appears on the graph as a ___?___.

Connecting Algebra to Geometry

Representing Solid Figures

A *net* is a flat pattern that can be folded to make a solid figure. The net shows all of the faces and surfaces of the solid.

To identify the solid shown by a net, remember these properties of solids.

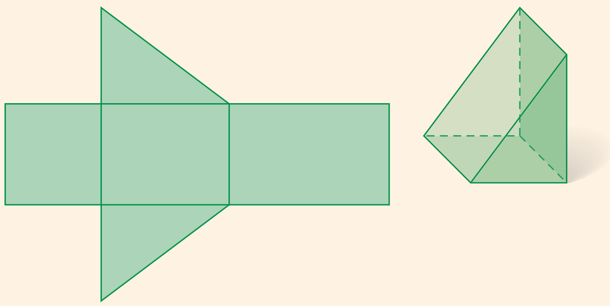


Prisms Pyramids Cylinder Cone

- A ***prism*** has two parallel, congruent bases that are polygons. The other faces are rectangles or parallelograms.
- A base of a ***pyramid*** is a polygon. The other faces are triangles.
- A ***cylinder*** has two congruent circles as its bases.
- A ***cone*** has one circle as its base.

Example 1

Identify the solid shown by this net.

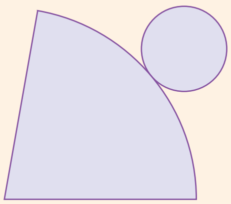


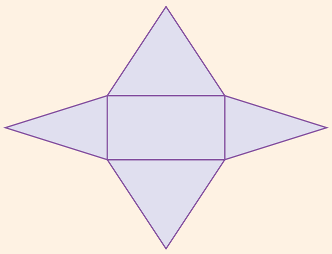
The faces are all polygons, so this is either a prism or a pyramid. Look for a pair of congruent polygons. There are two congruent right triangles. The solid is a *prism*.

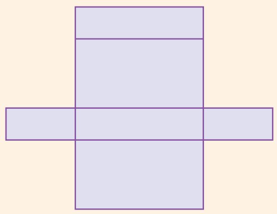
A prism is named by the shape of its bases, so this is a *triangular prism*.

Try This

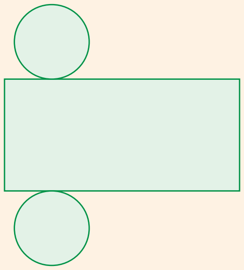
Identify the solid shown by each net.

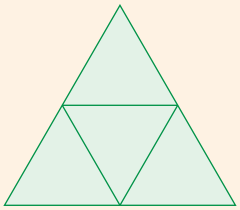
1. 

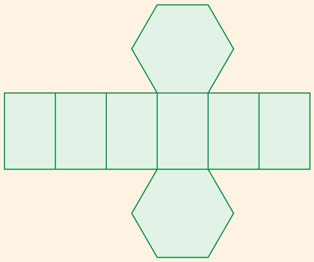
2. 

3. 

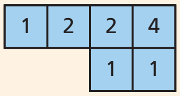
Identify the solid shown by each net.

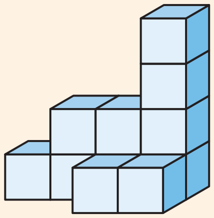
4. 

5. 

6. 

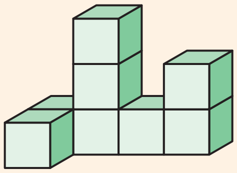
A foundation plan is a drawing that represents a solid made from cubes. The squares in the foundation plan are a top view of the solid. The number in each square shows the height of the solid at that point. The foundation plan shown is like a set of instructions for building the solid next to it.



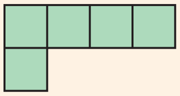


Example 2

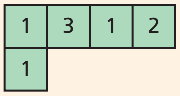
Draw a foundation plan for this solid figure.



Use squares to show a top view of the solid.

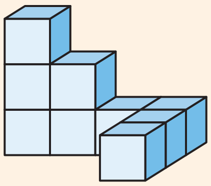


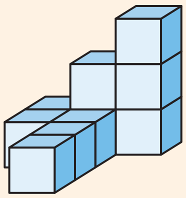
Put a number in each square to show the height at that point.

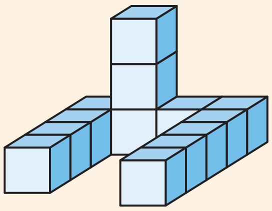


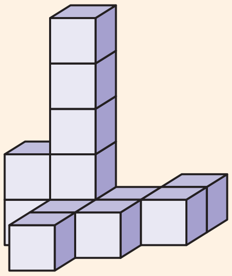
Try This

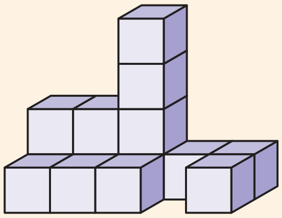
Draw a foundation plan for each solid figure.

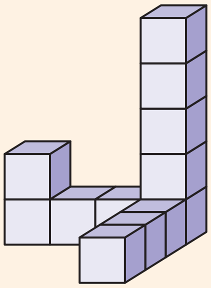
7. 

8. 

9. 

10. 

11. 

12. 

CHAPTER 12

SECTION 12A

MULTI-STEP TEST PREP

Rational Functions and Expressions

Construction Daze

Robert is part of a volunteer crew constructing houses for low-income families. The table shows how many construction days it takes to complete a house for work crews of various sizes.

Crew Size	Construction Days	Workdays
2	100	200
4	50	200
8	25	200
10	20	200
20	10	200

1. Working at the same rate, how many construction days should it take a crew of 40 people to build the house?
2. Express the number of construction days as a function of the crew size. Define the variables. What type of relationship is formed in the situation?
3. Explain how the crew size affects the number of construction days.
4. About how many construction days would it take a crew of 32 to complete a house?
5. If a crew can complete a house in 12.5 days, how many people are in the crew?
6. What are a reasonable domain and range of the function?
7. Suppose there are two managers that do not contribute to the work of building the house, yet are counted as part of the crew. Express the number of construction days as a function of the crew size. What are the asymptotes of this function? Graph the function.

READY TO GO ON?

CHAPTER 12

SECTION 12A

Quiz for Lessons 12-1 Through 12-3

12-1 Inverse Variation

Tell whether each relationship represents an inverse variation. Explain.

1.

x	−5	−4	−3
y	10	−8	6

2.

x	18	9	6
y	2	4	6

3. $y = \frac{3}{x}$ **4.** $y + x = \frac{3}{4}$ **5.** $xy = -2$ **6.** $y = \frac{x}{5}$

7. Write and graph the inverse variation in which $y = 3$ when $x = 2$.

8. Write and graph the inverse variation in which $y = 4$ when $x = -1$.

9. The number of calculators Mrs. Hopkins can buy for the classroom varies inversely as the cost of each calculator. She can buy 24 calculators that cost \$60 each. How many calculators can she buy if they cost \$80 each?

12-2 Rational Functions

Identify the excluded value and the vertical and horizontal asymptotes for each rational function. Then graph each function.

10. $y = \frac{12}{x}$ **11.** $y = \frac{6}{x + 2}$ **12.** $y = \frac{4}{x - 1}$ **13.** $y = \frac{2}{x + 1} - 3$

14. Jeff builds model train layouts. He has \$75 to spend on packages of miniature landscape items. He receives 6 free packages with each order. The number of packages y that Jeff can buy is given by $y = \frac{75}{x} + 6$, where x represents the cost of each package in dollars. Describe the reasonable domain and range values and graph the function.

12-3 Simplifying Rational Expressions

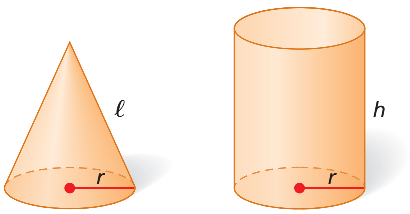
Find any excluded values of each rational expression.

15. $\frac{15}{n}$ **16.** $\frac{p}{p - 8}$ **17.** $\frac{x + 2}{x^2 + 6x + 8}$ **18.** $\frac{t - 1}{t^2 + t}$

Simplify each rational expression, if possible. Identify any excluded values.

19. $\frac{3x^2}{6x^3}$ **20.** $\frac{2n}{n^2 - 3n}$ **21.** $\frac{s + 1}{s^2 - 4s - 5}$ **22.** $\frac{12 - 3x}{x^2 - 8x + 16}$

23. Suppose a cone and a cylinder have the same radius and that the slant height ℓ of the cone is the same as the height h of the cylinder. Find the ratio of the cone's surface area to the cylinder's surface area.



$S = \pi r\ell + \pi r^2$ $S = 2\pi rh + 2\pi r^2$

12-4 Multiplying and Dividing Rational Expressions

Objective
Multiply and divide rational expressions.

Why learn this?
You can multiply rational expressions to determine the probabilities of winning prizes at carnivals. (See Example 5.)

The rules for multiplying rational expressions are the same as the rules for multiplying fractions. You multiply the numerators, and you multiply the denominators.

Know it! Note

Multiplying Rational Expressions

If a, b, c, and d are nonzero polynomials, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.

EXAMPLE 1 **Multiplying Rational Expressions**

Multiply. Simplify your answer.

A $\frac{a+3}{2} \cdot \frac{6}{3a+9}$

$\frac{6(a+3)}{2(3a+9)}$ *Multiply the numerators and denominators.*

$\frac{6(a+3)}{2 \cdot 3(a+3)}$ *Factor.*

$\frac{\cancel{6}^1\cancel{(a+3)}^1}{\cancel{6}^1\cancel{(a+3)}^1}$ *Divide out the common factors.*

1 *Simplify.*

Remember!
See the Quotient of Powers Property in Lesson 7-4.
$\frac{a^m}{a^n} = a^{m-n}$

B $\frac{12b^3c^2}{5ac} \cdot \frac{15a^2b}{3b^2c}$

$\frac{(12)(15)a^2(b^3 \cdot b)c^2}{(5)(3)ab^2(c \cdot c)}$ *Multiply the numerators and the denominators. Arrange the expression so like variables are together.*

$\frac{180a^2b^4c^2}{15ab^2c^2}$ *Simplify.*

$12a^1b^2c^0$ *Divide out common factors. Use properties of exponents.*

$12ab^2$ *Simplify. Remember that $c^0 = 1$.*

C $\frac{5x^2}{2y^3} \cdot \frac{3x}{2y^2}$

$\frac{15x^3}{4y^5}$ *Multiply. There are no common factors, so the product cannot be simplified.*

CHECK IT OUT!

Multiply. Simplify your answer.

1a. $\dfrac{(c-4)}{5} \cdot \dfrac{45}{(-4c+16)}$ **1b.** $\dfrac{5y^5z}{3xy^2z} \cdot \dfrac{2x^4y^2}{4xy}$

EXAMPLE 2 Multiplying a Rational Expression by a Polynomial

Remember!

Just as you can write an integer as a fraction, you can write any expression as a rational expression by writing it with a denominator of 1.

Multiply $(x^2+8x+15)\frac{4}{2x+6}$. Simplify your answer.

$\dfrac{x^2+8x+15}{1} \cdot \dfrac{4}{2x+6}$ *Write the polynomial over 1.*

$\dfrac{(x+3)(x+5)}{1} \cdot \dfrac{4}{2(x+3)}$ *Factor the numerator and denominator.*

$\dfrac{\cancel{(x+3)}^1(x+5)\cancel{4}^2}{\cancel{2}^1\cancel{(x+3)}^1}$ *Divide out common factors.*

$2x+10$ *Multiply remaining factors.*

CHECK IT OUT!

2. Multiply $\frac{m-5}{m^2-4m-12} \cdot (3m+6)$. Simplify your answer.

There are two methods for simplifying rational expressions. You can **simplify first** by dividing out common factors and **then multiply** the remaining factors. You can also **multiply first** and **then simplify**. Using either method will result in the same answer.

EXAMPLE 3 Multiplying Rational Expressions Containing Polynomials

Multiply $\frac{4d^3+4d}{16f} \cdot \frac{2f}{7d^2f+7f}$. Simplify your answer.

Method 1 Simplify first.

$\dfrac{4d^3+4d}{16f} \cdot \dfrac{2f}{7d^2f+7f}$

$\dfrac{4d(d^2+1)}{16f} \cdot \dfrac{2f}{7f(d^2+1)}$ *Factor.*

$\dfrac{\cancel{4}^1d\cancel{(d^2+1)}^1}{\cancel{16}^2\cancel{f}^1} \cdot \dfrac{\cancel{2}^1\cancel{f}^1}{7f\cancel{(d^2+1)}^1}$ *Divide out common factors.*

Then multiply.

$\dfrac{d}{14f}$ *Simplify.*

Method 2 Multiply first.

$\dfrac{4d^3+4d}{16f} \cdot \dfrac{2f}{7d^2f+7f}$

$\dfrac{(4d^3+4d)2f}{16f(7d^2f+7f)}$ *Multiply.*

$\dfrac{8d^3f+8df}{112d^2f^2+112f^2}$ *Distribute.*

Then simplify.

$\dfrac{8df(d^2+1)}{112f^2(d^2+1)}$ *Factor.*

$\dfrac{\cancel{8}^1d\cancel{f}^1\cancel{(d^2+1)}^1}{\cancel{112}^{14}\cancel{f^2}^f\cancel{(d^2+1)}^1}$ *Divide out common factors.*

$\dfrac{d}{14f}$ *Simplify.*

CHECK IT OUT!

Multiply. Simplify your answer.

3a. $\dfrac{n-5}{n^2+4n} \cdot \dfrac{n^2+8n+16}{n^2-3n-10}$ **3b.** $\dfrac{p+4}{p^2+2p} \cdot \dfrac{p^2-3p-10}{p^2+16}$

The rules for dividing rational expressions are the same as the rules for dividing fractions. To divide by a rational expression, multiply by its reciprocal.

Know it! Note

Dividing Rational Expressions

If a, b, c, and d are nonzero polynomials, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$.

EXAMPLE 4 Dividing by Rational Expressions and Polynomials

Divide. Simplify your answer.

A $\frac{1}{x} \div \frac{x-2}{2x}$

$\frac{1}{x} \cdot \frac{2x}{x-2}$ *Write as multiplication by the reciprocal.*

$\frac{1(2x)}{x(x-2)}$ *Multiply the numerators and the denominators.*

$\frac{2\cancel{x}^1}{\cancel{x}^1(x-2)}$ *Divide out common factors.*

$\frac{2}{x-2}$ *Simplify.*

B $\frac{x^2-2x}{x} \div \frac{2-x}{x^2+2x+1}$

$\frac{x^2-2x}{x} \cdot \frac{x^2+2x+1}{2-x}$ *Write as multiplication by the reciprocal.*

$\frac{x(x-2)}{x} \cdot \frac{(x+1)(x+1)}{2-x}$ *Factor.*

$\frac{x(x-2)}{x} \cdot \frac{(x+1)(x+1)}{-1(x-2)}$ *Rewrite one opposite binomial.*

$\frac{\cancel{x}^1\cancel{(x-2)}^1}{\cancel{x}^1} \cdot \frac{(x+1)(x+1)}{-1\cancel{(x-2)}^1}$ *Divide out common factors.*

$-(x+1)^2$ *Multiply.*

C $\frac{3a^2b}{b} \div (3a^2+6a)$

$\frac{3a^2b}{b} \div \frac{3a^2+6a}{1}$ *Write the binomial over 1.*

$\frac{3a^2b}{b} \cdot \frac{1}{3a^2+6a}$ *Write as multiplication by the reciprocal.*

$\frac{3a^2b}{b(3a^2+6a)}$ *Multiply the numerators and the denominators.*

$\frac{\cancel{3}^1\cancel{a^2}^a\cancel{b}^1}{\cancel{b}^1[\cancel{3}^1\cancel{a}^1(a+2)]}$ *Factor. Divide out common factors.*

$\frac{a}{a+2}$ *Simplify.*

CHECK IT OUT!

Divide. Simplify your answer.

4a. $\frac{3}{x^2} \div \frac{x^3}{x-5}$

4b. $\frac{18vw^2}{6v} \div \frac{3v^2x^4}{2w^4x}$

4c. $\frac{x^2-x}{x+2} \div (x^2+2x-3)$

EXAMPLE 5 *Probability Application*

Marty is playing a carnival game. He needs to pick two items out of a bag without looking. The bag has red and blue items. There are three more red items than blue items.

a. Write and simplify an expression that represents the probability that Marty will pick two blue items without replacing the first item.

Let $x =$ the number of blue items.

Blue	+	Red	=	Total
x	+	$x + 3$	=	$2x + 3$

Write expressions for the number of each color item and for the total number of items.

The probability of picking a blue item and then another blue item is the product of the probabilities of the individual events.

1st pick: blue items

2nd pick: blue items

$$P(\text{blue, blue}) = \frac{x}{2x+3} \cdot \frac{x-1}{2x+2}$$

1st pick: total items

2nd pick: total items

$$= \frac{x}{2x+3} \cdot \frac{x-1}{2(x+1)}$$

$$= \frac{x(x-1)}{2(2x+3)(x+1)}$$

b. What is the probability that Marty picks two blue items if there are 10 blue items in the bag before his first pick?

Use the probability of picking two blue items. Since x represents the number of blue items, substitute 10 for x.

$$P(\text{blue, blue}) = \frac{10(10-1)}{2(2 \cdot 10 + 3)(10+1)}$$ *Substitute.*

$$= \frac{10(9)}{2(23)(11)} = \frac{90}{506} \approx 0.18$$ *Use the order of operations to simplify.*

The probability of picking two blue items if there are 10 blue items in the bag before the first pick is approximately 0.18.

CHECK IT OUT!

5. **What if...?** There are 50 blue items in the bag before Marty's first pick. What is the probability that Marty picks two blue items?

Remember!

For a review of probability topics, see Chapter 10.

THINK AND DISCUSS

1. Explain how to divide by a polynomial.

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, describe how to perform the operation with rational expressions.

Know it! Note

Rational Expressions

Multiplying | Dividing

12-4 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MA7 12-4
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

SEE EXAMPLE 1 p. 898

Multiply. Simplify your answer.

1. $\frac{4hj^2}{10j^3} \cdot \frac{3h^3k}{h^3k^3}$ **2.** $\frac{4y}{x^5} \cdot \frac{2yz^2}{9x^2}$ **3.** $\frac{x-2}{x+3} \cdot \frac{4x+12}{6}$

4. $\frac{ab}{c} \cdot \frac{2a^2}{3c}$ **5.** $\frac{7c^4d}{10c} \cdot \frac{5a}{21c^3d}$ **6.** $\frac{12p^2q}{5p} \cdot \frac{15p^4q^3}{12q}$

SEE EXAMPLE 2 p. 899

7. $\frac{12}{4y+8}\left(y^2-4\right)$ **8.** $\frac{x+2}{6x^2}(5x+10)$ **9.** $\frac{3m}{6m+18}\left(m^2-7m-30\right)$

10. $\frac{4p}{8p+16}\left(p^2-5p-14\right)$ **11.** $\frac{a^2}{a}\left(a^2+10a+25\right)$ **12.** $\frac{-c}{4c+4}\left(c^2-c-2\right)$

SEE EXAMPLE 3 p. 899

13. $\frac{a^2+6ab}{b} \cdot \frac{5+3a}{3a^2b+5ab}$ **14.** $\frac{x^2+5x+4}{x-4} \cdot \frac{x^2-2x-8}{x^2+6x+8}$

15. $\frac{j-1}{j^2-4j+3} \cdot \frac{j^2-5j+6}{2j-4}$ **16.** $\frac{p^3+4pq}{p} \cdot \frac{6q^3-8}{2q}$

17. $\frac{r^2+15r+14}{r^2-16} \cdot \frac{2r+8}{r+1}$ **18.** $\frac{y-8}{y^2-1} \cdot \frac{y+2}{y^2-49}$

SEE EXAMPLE 4 p. 900

Divide. Simplify your answer.

19. $\frac{3a^4b}{2a^2c^3} \div \frac{12a^2c}{8c^4}$ **20.** $\frac{2m^3+2m}{m^2-2m} \div \frac{4m^2+4}{m-1}$ **21.** $\frac{x^2+4x-5}{3x-3} \div \left(x^2-25\right)$

SEE EXAMPLE 5 p. 901

22. Probability While playing a game, Rachel pulls two tiles out of a bag without looking and without replacing the first tile. The bag has two colors of tiles—black and white. There are 10 more white tiles than black tiles.

a. Write and simplify an expression that represents the probability that Rachel will pick a black tile, then a white tile.

b. What is the probability that Rachel pulls a black tile and then a white tile if there are 5 black tiles in the bag before her first pick?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
23–25	1
26–28	2
29–31	3
32–34	4
35	5

Extra Practice
Skills Practice p. S27
Application Practice p. S39

Multiply. Simplify your answer.

23. $\frac{p^6q^2}{7r^3} \cdot \frac{-3p^2}{r}$ **24.** $\frac{3r^2t}{6st^3} \cdot \frac{2r^2s^3t^2}{8r^4s^2}$ **25.** $\frac{10}{y+5} \cdot \frac{y+2}{3}$

26. $\frac{3}{2a+6}\left(a^2+4a+3\right)$ **27.** $\frac{4m^2-8m}{m^2+6m-16}\left(m^2+7m-8\right)$

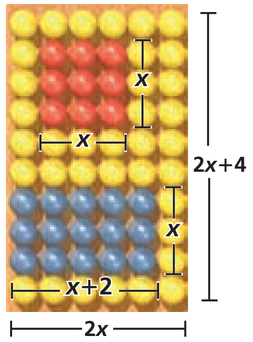
28. $\frac{x}{2x^2-12x+18}\left(2x^2-4x-6\right)$ **29.** $\frac{6n^2+18n}{n^2+9n+8} \cdot \frac{n^2-1}{2n+6}$

30. $\frac{3a^2b}{5a^3+10a^2b} \cdot \frac{2a+4b}{6a^3b+6a^2b^2}$ **31.** $\frac{t^2-100}{5t+50} \cdot \frac{5}{t-10}$

Divide. Simplify your answer.

32. $\frac{6j^2k^5}{5j} \div \frac{4j^3k^3}{3j}$ **33.** $\frac{a-4}{a^2} \div \left(8a-2a^2\right)$ **34.** $\frac{x^2-9}{x^2+6x+9} \div \frac{4x^2-12x}{16x}$

35. Entertainment A carnival game board is covered completely in small balloons. You throw darts at the board and try to pop the balloons.



a. Write and simplify an expression describing the probability that the next two balloons popped are red and then blue. (*Hint:* Write the probabilities as ratios of the areas of rectangles.)

b. What is the probability that the next two balloons popped are red and then blue if $x = 3$?

36. ///ERROR ANALYSIS/// Which is incorrect? Explain the error.

A

$$\frac{4a^2 - b^2}{a^2} \cdot \frac{a}{2a - b}$$

$$\frac{{}^{2}\cancel{4}\cancel{a^2} - b^2}{\cancel{a^2}} \cdot \frac{\cancel{a}}{\cancel{2}\cancel{a} - b} = \frac{2 - b^2}{b}$$

B

$$\frac{4a^2 - b^2}{a^2} \cdot \frac{a}{2a - b}$$

$$\frac{\cancel{(2a - b)}(2a + b)}{{}_{a}\cancel{a^2}} \cdot \frac{\cancel{a}}{\cancel{2a - b}} = \frac{2a + b}{a}$$

37. Critical Thinking Which of the following expressions is NOT equivalent to the other three? Explain why.

a. $\dfrac{4x^2}{x^2 - 3x} \cdot \dfrac{2x - 6}{8y^2}$

b. $\dfrac{6xy^2}{x^2} \div \dfrac{3y^4}{2x^2}$

c. $\dfrac{10x^4y}{5xy^2} \div 2x^2y$

d. $\dfrac{4x}{xy^2 + 2y^2} \cdot \dfrac{x^2 - 4}{4x - 8}$

Multiply or divide. Simplify your answer.

38. $\dfrac{5p^3}{p^2q} \cdot \dfrac{2q^3}{p^2}$

39. $\dfrac{6m^2 - 18m}{12m^3 + 12m^2} \div \dfrac{m^2 - 9}{m^2 + 4m + 3}$

40. $\dfrac{2x^2}{4x - 8} \cdot \dfrac{x^2 - 5x + 6}{x^5}$

41. $\dfrac{x^2 - 9}{4x} \div \left(4x^2 - 36\right)$

42. $\dfrac{33m - 3m^2}{-2m - 4} \div \dfrac{6m - 66}{m^2 - 4m}$

43. $\dfrac{12w^4x^7}{3w^3} \cdot \dfrac{w^{-1}x^{-7}}{4}$

44. Write About It Explain how to divide $\frac{1}{m} \div \frac{3}{4m}$.

MULTI-STEP TEST PREP

45. This problem will prepare you for the Multi-Step Test Prep on page 926.

The size of an image projected on a screen depends on how far the object is from the lens, the magnification of the lens, and the distance between the image and the lens. Magnification of a lens is $M = \frac{I}{O} = \frac{y}{x}$ where I is the height of the image, O is the height of the object, x is the distance of the object from the lens, and y is the distance of the image from the lens.

a. If an object 16 cm high is placed 15 cm from the lens, it forms an image 60 cm from the lens. What is the height of the image?

b. Marie moves the same object to a distance of 20 cm from the lens. If the image is the same size as part **a**, what is the distance between the image and the lens?

c. What is the magnification of the lens?

TEST PREP

46. Which expression is equivalent to $\frac{t+4}{9} \cdot \frac{t+4}{3}$?

Ⓐ $\frac{(t+4)^2}{27}$ Ⓑ $\frac{t^2+16}{27}$ Ⓒ $\frac{1}{3}$ Ⓓ $\frac{1}{27}$

47. Identify the product $-\frac{20b^2}{a^2} \cdot \frac{3ab}{15b}$.

Ⓕ $-\frac{a}{4b^2}$ Ⓖ $-4b^2$ Ⓗ $-\frac{4b^2}{a}$ Ⓙ $-\frac{b^2}{4a}$

48. Which of the following is equivalent to $\frac{2x}{x+5}$?

Ⓐ $\frac{x-2}{8x} \cdot \frac{4}{x^2+3x-10}$

Ⓒ $\frac{x-2}{4} \div \frac{x^2+3x-10}{8x}$

Ⓑ $\frac{x^2-3x-10}{8x} \cdot \frac{4}{x-2}$

Ⓓ $\frac{x^2-3x-10}{4} \div \frac{x-2}{8x}$

49. **Short Response** Simplify $\frac{x^2-10x+24}{3x^2-12x} \div (x^2-3x-18)$. Show your work.

CHALLENGE AND EXTEND

Simplify.

50. $\frac{x-3}{3x-6} \cdot \frac{3x+12}{x+1} \cdot \frac{2x-4}{x^2+x-12}$

51. $\frac{x^2-1}{x+2} \div \frac{3x+3}{x+2} \div (x-1)$

A *complex fraction* is a fraction that contains one or more fractions in the numerator or the denominator. Simplify each complex fraction.

$\left(\textit{Hint:} \text{ Use the rule } \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d}.\right)$

52. $\dfrac{\frac{c+5}{c^2-4}}{\frac{c^2+6c+5}{c+2}}$

53. $\dfrac{\frac{x^2y}{xz^3}}{\frac{x^2y}{x^2z}}$

54. $\dfrac{\frac{x^2}{2} \cdot \frac{x}{3}}{\frac{x}{6}}$

55. $\dfrac{\frac{a+1}{a^2+6a+5}}{\frac{2a+2}{a+5}}$

SPIRAL REVIEW

56. Jillian's mother told her to preheat the oven to at least 325 °F. When Jillian went into the kitchen, the oven was already set to 200 °F. Write and solve an inequality to determine how many more degrees Jillian should increase the temperature. *(Lesson 3-2)*

57. Pierce has \$30 to spend on a night out. He already spent \$12 on dinner and \$9 on a movie ticket. He will spend some money m on snacks. Write and solve an inequality that will show all the values of m that Pierce can spend on snacks. *(Lesson 3-2)*

Simplify each radical. Then add or subtract, if possible. *(Lesson 11-7)*

58. $\sqrt{18} - \sqrt{8}$

59. $\sqrt{3t} + 5\sqrt{3t} + \sqrt{12t}$

60. $\sqrt{20} - \sqrt{80} + \sqrt{3}$

Identify the excluded value and asymptotes for each rational function. *(Lesson 12-2)*

61. $y = \frac{4}{x-3}$

62. $y = \frac{1}{2x+4}$

63. $y = -\frac{1}{x} + 3$

64. $y = -\frac{2}{x+5}$

65. $y = \frac{12}{4x}$

66. $y = -\frac{1}{3x-2}$

12-5 Adding and Subtracting Rational Expressions

Objectives

Add and subtract rational expressions with like denominators.

Add and subtract rational expressions with unlike denominators.

Who uses this?

Kayakers can use rational expressions to figure out travel time for different river trips. (See Example 5.)

The rules for adding rational expressions are the same as the rules for adding fractions. If the denominators are the same, you add the numerators and keep the common denominator.

$$\frac{3}{8}+\frac{2}{8}=\frac{3+2}{8}=\frac{5}{8}$$

Know it! Note

Adding Rational Expressions with Like Denominators

If a, b, and c represent polynomials and $c \neq 0$, then $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$.

EXAMPLE 1 Adding Rational Expressions with Like Denominators

Add. Simplify your answer.

A $\frac{3b}{b^2}+\frac{5b}{b^2}$

$\frac{3b+5b}{b^2}=\frac{8\cancel{b}^1}{\cancel{b^2}^{b}}$ — *Combine like terms in the numerator. Divide out common factors.*

$=\frac{8}{b}$ — *Simplify.*

B $\frac{x^2-8x}{x-4}+\frac{2x+8}{x-4}$

$\frac{x^2-8x+2x+8}{x-4}=\frac{x^2-6x+8}{x-4}$ — *Combine like terms in the numerator.*

$=\frac{(x-2)\cancel{(x-4)}^1}{\cancel{x-4}^1}$ — *Factor. Divide out common factors.*

$=x-2$ — *Simplify.*

C $\frac{2m+4}{m^2-9}+\frac{2}{m^2-9}$

$\frac{2m+4+2}{m^2-9}=\frac{2m+6}{m^2-9}$ — *Combine like terms in the numerator.*

$=\frac{2\cancel{(m+3)}^1}{(m-3)\cancel{(m+3)}^1}$ — *Factor. Divide out common factors.*

$=\frac{2}{m-3}$ — *Simplify.*

CHECK IT OUT! Add. Simplify your answer.

1a. $\frac{n}{2n}+\frac{3n}{2n}$

1b. $\frac{3y^2}{y+1}+\frac{3y}{y+1}$

To subtract rational expressions with like denominators, remember to add the opposite of *each* term in the second numerator.

Caution!

Make sure you add the opposite of all the terms in the numerator of the second expression when subtracting rational expressions.

EXAMPLE 2 Subtracting Rational Expressions with Like Denominators

Subtract. Simplify your answer.

$$\frac{3m-6}{m^2+m-6}-\frac{-m+2}{m^2+m-6}$$

$$\frac{3m-6-(-m+2)}{m^2+m-6}=\frac{3m-6+m-2}{m^2+m-6}$$ *Subtract numerators.*

$$=\frac{4m-8}{m^2+m-6}$$ *Combine like terms.*

$$=\frac{4\cancel{(m-2)}^1}{(m+3)\cancel{(m-2)}^1}$$ *Factor. Divide out common factors.*

$$=\frac{4}{m+3}$$ *Simplify.*

CHECK IT OUT! **Subtract. Simplify your answer.**

2a. $\frac{5a+2}{a^2-4}-\frac{2a-4}{a^2-4}$ **2b.** $\frac{2b+14}{b^2+3b-4}-\frac{-2b+2}{b^2+3b-4}$

As with fractions, rational expressions must have a common denominator before they can be added or subtracted. If they do not have a common denominator, you can use the least common multiple, or LCM, of the denominators to find one.

To find the LCM, write the prime factorization of both expressions. Use each factor the greatest number of times it appears in either expression.

$$6x^2=2\cdot\boxed{3}\cdot\boxed{x\cdot x}$$
$$8x=\boxed{2\cdot 2\cdot 2}\cdot x$$
$$\text{LCM}=2\cdot 2\cdot 2\cdot 3\cdot x\cdot x=24x^2$$

$$5x+15=\boxed{5}(x+3)$$
$$x^2-9=\boxed{(x+3)}\boxed{(x-3)}$$
$$\text{LCM}=5(x+3)(x-3)$$

EXAMPLE 3 Identifying the Least Common Multiple

Find the LCM of the given expressions.

A $24a^3, 4a$

$$24a^3=2\cdot 2\cdot 2\cdot 3\cdot a\cdot a\cdot a$$
$$4a=2\cdot 2\cdot a$$
$$\text{LCM}=2\cdot 2\cdot 2\cdot 3\cdot a\cdot a\cdot a=24a^3$$

Write the prime factorization of each expression. Use every factor of both expressions the greatest number of times it appears in either expression.

B $2d^2+10d+12, d^2+7d+12$

$$2d^2+10d+12=2(d^2+5d+6)$$ *Factor each expression.*

$$=2(d+3)(d+2)$$

$$d^2+7d+12=(d+3)(d+4)$$

Use every factor of both expressions the greatest number of times it appears in either expression.

$$\text{LCM}=2(d+3)(d+2)(d+4)$$

CHECK IT OUT! **Find the LCM of the given expressions.**

3a. $5f^2h, 15fh^2$ **3b.** $x^2-4x-12, (x-6)(x+5)$

The LCM of the denominators of fractions or rational expressions is also called the least common denominator, or LCD. You use the same method to add or subtract rational expressions.

Adding or Subtracting Rational Expressions
Step 1 Identify the LCD.
Step 2 Multiply each expression by an appropriate form of 1 so that each term has the LCD as its denominator.
Step 3 Write each expression using the LCD.
Step 4 Add or subtract the numerators, combining like terms as needed.
Step 5 Factor.
Step 6 Simplify as needed.

EXAMPLE 4

Adding and Subtracting with Unlike Denominators

Add or subtract. Simplify your answer.

A $\dfrac{3x}{6x^2} + \dfrac{2x}{4x}$

$6x^2 = 2 \cdot 3 \cdot x \cdot x$

Step 1 $4x = 2 \cdot 2 \cdot x$ *Identify the LCD.*

$\text{LCD} = 2 \cdot 2 \cdot 3 \cdot x \cdot x = 12x^2$

Step 2 $\dfrac{3x}{6x^2}\left(\dfrac{2}{2}\right) + \dfrac{2x}{4x}\left(\dfrac{3x}{3x}\right)$ *Multiply each expression by an appropriate form of 1.*

Step 3 $\dfrac{6x}{12x^2} + \dfrac{6x^2}{12x^2}$ *Write each expression using the LCD.*

Step 4 $\dfrac{6x + 6x^2}{12x^2}$ *Add the numerators.*

Step 5 $\dfrac{\cancel{6}^1\cancel{x}^1(1+x)}{\cancel{6}^1 \cdot 2\cancel{x^2}^x}$ *Factor and divide out common factors.*

Step 6 $\dfrac{1+x}{2x}$ *Simplify.*

B $\dfrac{1}{m-3} - \dfrac{5}{3-m}$

Step 1 The denominators are opposite binomials.
The LCD can be either $m - 3$ or $3 - m$. *Identify the LCD.*

Step 2 $\dfrac{1}{m-3} - \dfrac{5}{3-m}\left(\dfrac{-1}{-1}\right)$ *Multiply the second expression by $\frac{-1}{-1}$ to get an LCD of $m - 3$.*

Step 3 $\dfrac{1}{m-3} - \dfrac{-5}{m-3}$ *Write each expression using the LCD.*

Step 4 $\dfrac{1-(-5)}{m-3}$ *Subtract the numerators.*

Steps 5, 6 $\dfrac{6}{m-3}$ *No factoring is needed, so just simplify.*

Remember!

Expressions like $m - 3$ and $3 - m$ are opposite binomials.

$3 - m = -1(m - 3)$

and

$m - 3 = -1(3 - m)$

CHECK IT OUT! **Add or subtract. Simplify your answer.**

4a. $\dfrac{4}{3d} - \dfrac{2d}{2d^3}$ **4b.** $\dfrac{a^2+4a}{a^2+2a-8} + \dfrac{8}{a-2}$

EXAMPLE 5 Recreation Application

Katy wants to find out how long it will take to kayak 1 mile up a river and return to her starting point. Katy's average paddling rate is 4 times the speed of the river's current.

a. Write and simplify an expression for the time it will take Katy to kayak the round-trip in terms of the rate of the river's current.

Step 1 Write expressions for the distances and rates in the problem.

The distance in both directions is **1 mile.**

Let x represent the rate of the current, and let $4x$ represent Katy's paddling rate.

Katy's **rate against the current** is $4x - x$, or $3x$.

Katy's **rate with the current** is $4x + x$, or $5x$.

Step 2 Use a table to write expressions for time.

Direction	Distance (mi)	Rate (mi/h)	Time (h) $= \frac{\text{distance}}{\text{rate}}$
Upstream (against current)	1	$3x$	$\frac{1}{3x}$
Downstream (with current)	1	$5x$	$\frac{1}{5x}$

Helpful Hint

To write expressions for time in terms of distance and rate, solve $d = rt$ for t.

$$t = \frac{d}{r}$$

Step 3 Write and simplify an expression for the total time.

total time = time upstream + time downstream

$$\text{total time} = \frac{1}{3x} + \frac{1}{5x}$$ *Substitute known values.*

$$= \frac{1}{3x}\left(\frac{5}{5}\right) + \frac{1}{5x}\left(\frac{3}{3}\right)$$ *Multiply each fraction by an appropriate form of 1.*

$$= \frac{5}{15x} + \frac{3}{15x}$$ *Write each expression using the LCD, 15x.*

$$= \frac{8}{15x}$$ *Add the numerators.*

b. If the rate of the river is 2 miles per hour, how long will it take Katy to kayak round trip?

$$\frac{8}{15(2)} = \frac{4}{15}$$ *Substitute 2 for x. Simplify.*

It will take Katy $\frac{4}{15}$ of an hour, or 16 minutes, to kayak the round-trip.

CHECK IT OUT!

5. What if?... Katy's average paddling rate increases to 5 times the speed of the current. Now how long will it take Katy to kayak the round trip?

THINK AND DISCUSS

1. Explain how to find the least common denominator of rational expressions.

Know it! Note

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, compare and contrast operations with fractions and rational expressions.

Numerical Fractions and Rational Expressions

Adding: Similarities | Differences

Subtracting: Similarities | Differences

12-5 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MA7 12-5
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

SEE EXAMPLE 1 p. 905

Add. Simplify your answer.

1. $\dfrac{y}{3y^2} + \dfrac{5y}{3y^2}$
2. $\dfrac{4m+30}{m+5} + \dfrac{m^2+8m+5}{m+5}$
3. $\dfrac{x}{x^2-16} + \dfrac{4}{x^2-16}$

SEE EXAMPLE 2 p. 906

Subtract. Simplify your answer.

4. $\dfrac{7}{2x^3} - \dfrac{3}{2x^3}$
5. $\dfrac{7a-2}{a^2+3a+2} - \dfrac{5a-6}{a^2+3a+2}$
6. $\dfrac{3x^2+1}{2x+2} - \dfrac{2x^2-2x}{2x+2}$

SEE EXAMPLE 3 p. 906

Find the LCM of the given expressions.

7. $3xy^2,\ 6x^3yz$
8. $x^2+9x+20,\ (x+5)(x-4)$
9. $y^2-16,\ (y+9)(y-4)$

SEE EXAMPLE 4 p. 907

Add or subtract. Simplify your answer.

10. $\dfrac{3}{c} - \dfrac{4}{3c}$
11. $\dfrac{x^2+x}{x^2+3x+2} + \dfrac{3}{x+2}$
12. $\dfrac{2x}{x-5} + \dfrac{x}{5-x}$

SEE EXAMPLE 5 p. 908

13. **Travel** The Escobar family went on a car trip. They drove 100 miles on country roads and 240 miles on the highway. They drove 50% faster on the highway than on the country roads. Let r represent their rate on country roads in miles per hour.
    a. Write and simplify an expression that represents the number of hours it took the Escobar family to complete their trip in terms of r. (*Hint:* 50% faster means 150% of the original rate.)
    b. Find their total travel time if they drove the posted speed limit.

PRACTICE AND PROBLEM SOLVING

Independent Practice	
For Exercises	See Example
14–16	1
17–19	2
20–25	3
26–31	4
32	5

Extra Practice
Skills Practice p. S27
Application Practice p. S39

Add. Simplify your answer.

14. $\dfrac{4y}{y^3} + \dfrac{4y}{y^3}$
15. $\dfrac{a^2-3}{a+3} + \dfrac{2a}{a+3}$
16. $\dfrac{4x-13}{x^2-5x+6} + \dfrac{1}{x^2-5x+6}$

Subtract. Simplify your answer.

17. $\dfrac{m^2}{m-6} - \dfrac{6m}{m-6}$
18. $\dfrac{c+3}{4c^2-25} - \dfrac{-c+8}{4c^2-25}$
19. $\dfrac{-2a^2-9a}{a-2} - \dfrac{-5a^2-4a+2}{a-2}$

Find the LCM of the given expressions.

20. $4jk^4m,\ 25jm$
21. $12a^2+4a,\ 27a+9$
22. $p^2-3p,\ pqr^2$
23. $5xy^2z,\ 10y^3$
24. $5x^2,\ 7x-14$
25. $y^2+7y+10,\ y^2+9y+20$

Add or subtract. Simplify your answer.

26. $\dfrac{2x}{5x} + \dfrac{10x}{3x^2}$
27. $\dfrac{y^2-y}{y^2-4y+3} - \dfrac{2y-2}{3y-9}$
28. $\dfrac{-3t}{t-4} - \dfrac{2t+4}{4-t}$
29. $\dfrac{z}{3z^2} + \dfrac{4}{7z}$
30. $\dfrac{5x}{2x-6} + \dfrac{x+2}{3-x}$
31. $\dfrac{3m}{4m-8} - \dfrac{m^2}{m^2-4m+4}$

32. Fitness Ira walks one mile from his house to the recreation center. After playing basketball, he walks home at only 85% of his normal walking speed. Let w be Ira's normal rate of walking.

a. Write an expression to represent Ira's round-trip walking time.

b. If Ira's normal rate of walking is 3 miles per hour, how long did it take for him to complete his walking?

Travel

The first transcontinental railroad was completed in Utah on May 10, 1869. The occasion was commemorated with a golden spike that connected the eastern and western tracks.

33. Travel A train travels 500 miles across the Midwest—50 miles through cities and 450 miles through open country. As it passes through cities, it slows to one-fifth the speed it travels through open country. Let r represent the rate in open country in miles per hour.

a. Write and simplify an expression in terms of r that represents the number of hours it takes the train to travel 500 miles.

b. Find the total travel time if the train's rate through open country is 50 miles per hour.

c. Critical Thinking If you knew the time it took the train to make the trip, how could you find its average rate?

Add or subtract. Simplify your answer.

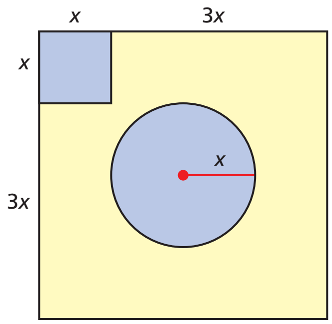
34. $\frac{10}{5+y} + \frac{2y}{5+y}$ **35.** $\frac{7}{49-c^2} - \frac{c}{49-c^2}$ **36.** $\frac{6a}{a-12} + \frac{4}{12-a}$

37. $\frac{b}{2b^3} + \frac{3}{3b^2}$ **38.** $\frac{r^2+2r}{r+3} - \frac{2r+9}{r+3}$ **39.** $\frac{x^2-2x}{3x-15} - \frac{8x-25}{3x-15}$

40. $\frac{2y}{8y^2} + \frac{9}{4y^3}$ **41.** $\frac{2}{x+2} + \frac{6}{x+4}$ **42.** $\frac{2y}{3y-9} - \frac{y+1}{y^2-9}$

43. ERROR ANALYSIS Two students were asked to find the excluded values of the expression $\frac{p}{p^2-p-12} - \frac{4}{p^2-p-12}$. Student A identified the excluded value as $p = -3$. Student B identified the excluded values as $p = -3$ and $p = 4$. Who is incorrect? What is the error?

44. Multi-Step At the spring fair there is a square Velcro target as shown. A player tosses a ball, which will stick to the target in some random spot. If the ball sticks to a spot in either the small square or the circle, the player wins a prize. What is the probability that a player will win a prize, assuming the ball sticks somewhere on the target? Round your answer to the nearest hundredth.



45. Critical Thinking Write two expressions whose sum is $\frac{x}{x+1}$.

MULTI-STEP TEST PREP

46. This problem will prepare you for the Multi-Step Test Prep on page 926.

Jonathan is studying light in his science class. He finds that a magnifying glass can be used to project upside-down images onto a piece of paper. The equation $\frac{1}{f} = \frac{1}{x} + \frac{1}{y}$ relates the focal length of the lens f, the distance of the object from the lens x, and the distance of the image from the lens y. The focal length of Jonathan's lens is 12 cm.

a. Jonathan wants to write y, the distance of the image from the lens, as a function of x, the distance of the object from the lens. To begin, he rewrote the equation above as $\frac{1}{y} = \frac{1}{12} - \frac{1}{x}$. Explain how he did this.

b. Explain how Jonathan simplified the equation in part **a** to $\frac{1}{y} = \frac{x-12}{12x}$.

47. **Critical Thinking** Identify three common denominators that could be used to add $\frac{3}{2x^2}$ to $\frac{3}{4x}$.

48. **Write About It** Explain how to find the least common denominator of two rational expressions when the denominators are opposite binomials.

TEST PREP

49. What is the LCD of $\frac{6}{3p+3}$ and $\frac{4}{p+1}$?

Ⓐ $p+1$ Ⓑ 12 Ⓒ $3p+1$ Ⓓ $3p+3$

50. Simplify $\frac{4}{2x} - \frac{1}{x}$.

Ⓕ $\frac{1}{x}$ Ⓖ $\frac{3}{x}$ Ⓗ $\frac{5}{x}$ Ⓙ $\frac{3}{2x}$

51. Which of the following is equivalent to $\frac{2x}{x-2}$?

Ⓐ $\frac{x}{x+2} + \frac{x}{x-2}$

Ⓑ $\frac{2x}{x^2-4} + \frac{4}{x-2}$

Ⓒ $\frac{x^2+4x}{x^2-4} + \frac{x}{x+2}$

Ⓓ $\frac{x}{x+2} + \frac{x^2+6x}{x^2-4}$

52. **Extended Response** Andrea biked 3 miles to the post office and 5 miles to the library. The rate at which she biked to the library was three times faster than her rate to the post office *r*.

a. Write an expression that represents Andrea's total biking time in hours. Explain what each part of your expression means in the situation.

b. Simplify the expression.

c. How long did it take Andrea to bike the 8 miles if her biking rate to the post office was 3 miles per hour?

CHALLENGE AND EXTEND

Add or subtract and simplify. Find the excluded values.

53. $\frac{3}{x+y} - \frac{2x+y}{x^2-y^2}$ 54. $\frac{3}{2m} + \frac{4}{m^2} + \frac{2}{5m}$ 55. $\frac{a}{xy} + \frac{b}{xz} + \frac{c}{yz}$

56. Simplify the complex fraction $\frac{\frac{1}{x} - \frac{1}{y}}{\frac{x}{xy} - \frac{1}{x}}$. (*Hint:* Simplify the numerator and denominator of the complex fraction first.)

SPIRAL REVIEW

Sketch a graph for each situation. *(Lesson 4-1)*

57. Snow falls lightly at first, then falls heavily at a steady rate.

58. Snow melts quickly during the afternoon, then stops melting at night.

59. Snow falls heavily, is shoveled away, and then a light snow falls.

Solve each quadratic equation by factoring. *(Lesson 9-6)*

60. $d^2 - 4d - 12 = 0$ 61. $2g^2 - 9g = -4$ 62. $9x^2 + 6x + 1 = 0$

Simplify each rational expression, if possible. Identify any excluded values. *(Lesson 12-3)*

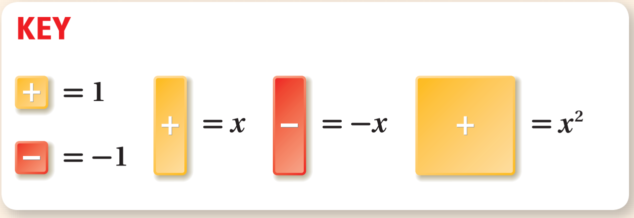
63. $\frac{2t^2-8}{t^2-4}$ 64. $\frac{n^2+5n}{n^2+3n-10}$ 65. $\frac{4-x}{x^2-16}$

12-6 Algebra LAB

Model Polynomial Division

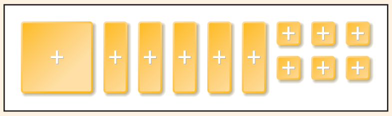
Some polynomial divisions can be modeled by algebra tiles. If a polynomial can be modeled by a rectangle, then its factors are represented by the length and width of the rectangle. If one factor is a divisor, then the other factor is a quotient.

Use with Lesson 12-6

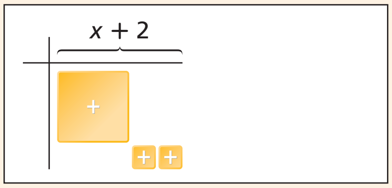


Activity 1

Use algebra tiles to find the quotient $(x^2 + 5x + 6) \div (x + 2)$.

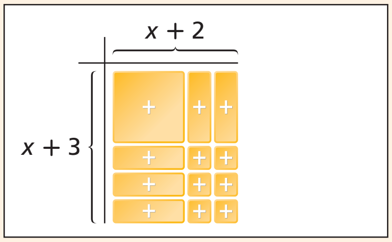


Model $x^2 + 5x + 6$.



Try to form a rectangle with a length of $x + 2$.

Place the x^2-tile in the upper-left corner. Then place two unit tiles in a row at the lower-right corner.



Try to use all the remaining tiles to complete a rectangle.

If you can complete a rectangle, then the width of the rectangle is the quotient.

The rectangle has length $x + 2$ and width $x + 3$. So, $(x^2 + 5x + 6) \div (x + 2) = x + 3$.

You can check your answer by multiplying.	$(x + 3)(x + 2)$	
	$x^2 + 2x + 3x + 6$	*Use the FOIL method.*
	$x^2 + 5x + 6$ ✓	

Try This

Use algebra tiles to find each quotient.

1. $(x^2 + 5x + 4) \div (x + 1)$
2. $(x^2 + 7x + 10) \div (x + 5)$
3. $(x^2 + 4x - 5) \div (x - 1)$
4. $(2x^2 + 5x + 2) \div (x + 2)$
5. $(x^2 - 6x + 8) \div (x - 2)$
6. $(2x^2 - x - 3) \div (x + 1)$
7. Describe what happens when you try to model $(x^2 - 4x + 3) \div (x + 1)$.

12-6 Dividing Polynomials

Objective
Divide a polynomial by a monomial or binomial.

Why learn this?
Division of polynomials can be used to compare the energy produced by solar panels. (See Exercise 51.)

The electrical power (in watts) produced by a solar panel is directly proportional to the surface area of the solar panel. Division of polynomials can be used to compare energy production by solar panels of different sizes.

To divide a polynomial by a monomial, you can first write the division as a rational expression. Then divide each term in the polynomial by the monomial.

EXAMPLE 1 Dividing a Polynomial by a Monomial

Divide $\left(6x^3 + 8x^2 - 4x\right) \div 2x$.

$\dfrac{6x^3 + 8x^2 - 4x}{2x}$ *Write as a rational expression.*

$\dfrac{6x^3}{2x} + \dfrac{8x^2}{2x} - \dfrac{4x}{2x}$ *Divide each term in the polynomial by the monomial 2x.*

$\dfrac{\cancel{6}^3 \cancel{x^3}^{x^2}}{\cancel{2}^1 \cancel{x}^1} + \dfrac{\cancel{8}^4 \cancel{x^2}^{x}}{\cancel{2}^1 \cancel{x}^1} - \dfrac{\cancel{4}^2 \cancel{x}^1}{\cancel{2}^1 \cancel{x}^1}$ *Divide out common factors.*

$3x^2 + 4x - 2$ *Simplify.*

CHECK IT OUT! **Divide.**

1a. $\left(8p^3 - 4p^2 + 12p\right) \div \left(-4p^2\right)$

1b. $\left(6x^3 + 2x - 15\right) \div 6x$

Division of a polynomial by a binomial is similar to division of whole numbers.

Know it! Note **Dividing Polynomials**

	WORDS	NUMBERS	POLYNOMIALS
Step 1	Factor the numerator and/or denominator if possible.	$\dfrac{168}{3} = \dfrac{56 \cdot 3}{3}$	$\dfrac{r^2 + 3r + 2}{r + 2} = \dfrac{(r+2)(r+1)}{(r+2)}$
Step 2	Divide out any common factors.	$\dfrac{56 \cdot \cancel{3}}{\cancel{3}}$	$\dfrac{\cancel{(r+2)}(r+1)}{\cancel{(r+2)}}$
Step 3	Simplify.	56	$r + 1$

EXAMPLE 2 Dividing a Polynomial by a Binomial

Divide.

A $\dfrac{c^2 + 4c - 5}{c - 1}$

$\dfrac{(c + 5)(c - 1)}{c - 1}$ *Factor the numerator.*

$\dfrac{(c + 5)\cancel{(c - 1)}^1}{\cancel{(c - 1)}^1}$ *Divide out common factors.*

$c + 5$ *Simplify.*

B $\dfrac{3x^2 - 10x - 8}{4 - x}$

$\dfrac{(3x + 2)(x - 4)}{4 - x}$ *Factor the numerator.*

$\dfrac{(3x + 2)(x - 4)}{-1(x - 4)}$ *Factor one opposite binomial.*

$\dfrac{(3x + 2)\cancel{(x - 4)}^1}{-1\cancel{(x - 4)}^1}$ *Divide out common factors.*

$-3x - 2$ *Simplify.*

Helpful Hint

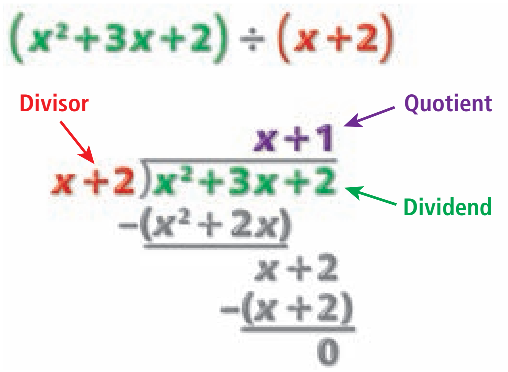
Put each term of the numerator over the denominator only when the denominator is a monomial. If the denominator is a polynomial, try to factor first.

CHECK IT OUT! **Divide.**

2a. $\dfrac{10 + 7k + k^2}{k + 2}$ **2b.** $\dfrac{b^2 - 49}{b + 7}$ **2c.** $\dfrac{s^2 + 12s + 36}{s + 6}$

Recall how you used long division to divide whole numbers as shown at right. You can also use long division to divide polynomials. An example is shown below.

$$\begin{array}{r} 15 \\ 23\overline{)345} \\ \underline{-23} \\ 115 \\ \underline{-115} \\ 0 \end{array}$$



Using Long Division to Divide a Polynomial by a Binomial	
Step 1	Write the binomial and polynomial in long division form.
Step 2	Divide the first term of the dividend by the first term of the divisor. This is the first term of the quotient.
Step 3	Multiply this first term of the quotient by the binomial divisor and place the product under the dividend, aligning like terms.
Step 4	Subtract the product from the dividend.
Step 5	Bring down the next term in the dividend.
Step 6	Repeat Steps 2–5 as necessary until you get 0 or until the degree of the remainder is less than the degree of the binomial.

EXAMPLE 3 Polynomial Long Division

Divide using long division.

A $(x^2 + 2 + 3x) \div (x + 2)$

Step 1 $x + 2\overline{)x^2 + 3x + 2}$

Write in long division form with expressions in standard form.

Step 2
$$\begin{array}{r} x \\ x + 2\overline{)x^2 + 3x + 2} \end{array}$$

Divide the first term of the dividend by the first term of the divisor to get the first term of the quotient.

Step 3
$$\begin{array}{r} x \\ x + 2\overline{)x^2 + 3x + 2} \\ \underline{x^2 + 2x} \end{array}$$

Multiply the first term of the quotient by the binomial divisor. Place the product under the dividend, aligning like terms.

Step 4
$$\begin{array}{r} x \\ x + 2\overline{)x^2 + 3x + 2} \\ \underline{-(x^2 + 2x)} \\ 0 + x \end{array}$$

Subtract the product from the dividend.

Step 5
$$\begin{array}{r} x \\ x + 2\overline{)x^2 + 3x + 2} \\ \underline{-(x^2 + 2x)} \downarrow \\ x + 2 \end{array}$$

Bring down the next term in the dividend.

Step 6
$$\begin{array}{r} x + 1 \\ x + 2\overline{)x^2 + 3x + 2} \\ \underline{-(x^2 + 2x)} \\ x + 2 \\ \underline{-(x + 2)} \\ 0 \end{array}$$

Repeat Steps 2–5 as necessary.
The remainder is 0.

Check Multiply the answer and the divisor.

$$(x + 2)(x + 1)$$
$$x^2 + x + 2x + 2$$
$$x^2 + 3x + 2 \checkmark$$

B $\dfrac{x^2 + 4x + 3}{x + 1}$

$x + 1\overline{)x^2 + 4x + 3}$ *Write in long division form.*

$$\begin{array}{r} x + 3 \\ x + 1\overline{)x^2 + 4x + 3} \\ \underline{-(x^2 + x)} \downarrow \\ 3x + 3 \\ \underline{-(3x + 3)} \\ 0 \end{array}$$

$x^2 \div x = x$
Multiply $x \cdot (x + 1)$. Subtract.
Bring down the 3. $3x \div x = 3$
Multiply $3(x + 1)$. Subtract.
The remainder is 0.

Check Multiply the answer and the divisor.

$$(x + 1)(x + 3)$$
$$x^2 + 3x + 1x + 3$$
$$x^2 + 4x + 3 \checkmark$$

Helpful Hint

You can check your simplified answer by multiplying it by the divisor. When the remainder is 0, the product should be the numerator.

CHECK IT OUT!

Divide using long division.

3a. $(2y^2 - 5y - 3) \div (y - 3)$ **3b.** $(a^2 - 8a + 12) \div (a - 6)$

Sometimes the divisor is not a factor of the dividend, so the remainder is not 0. Then the remainder can be written as a rational expression.

EXAMPLE 4 Long Division with a Remainder

Divide $(2x^2 + 3x - 6) \div (x - 2)$.

$x - 2\overline{)2x^2 + 3x - 6}$	*Write in long division form.*
$\begin{array}{r} 2x + 7 \\ x - 2\overline{)2x^2 + 3x - 6} \\ -(2x^2 - 4x) \\ \hline 7x - 6 \\ -(7x - 14) \\ \hline 8 \end{array}$	*$2x^2 \div x = 2x$* *Multiply $2x(x - 2)$. Subtract.* *Bring down the -6. $7x \div x = 7$* *Multiply $7(x - 2)$. Subtract.* *The remainder is 8.*
$\frac{8}{x - 2}$	*Write the remainder as a rational expression using the divisor as the denominator.*
$2x + 7 + \frac{8}{x - 2}$	*Write the quotient with the remainder.*

CHECK IT OUT! **Divide.**

4a. $(3m^2 + 4m - 2) \div (m + 3)$ **4b.** $(y^2 + 3y + 2) \div (y - 3)$

Sometimes you need to write a placeholder for a term using a zero coefficient. This is best seen if you write the polynomials in standard form.

EXAMPLE 5 Dividing Polynomials That Have a Zero Coefficient

Divide $(3x - 4x^3 - 15) \div (2x + 3)$.

$(-4x^3 + 3x - 15) \div (2x + 3)$	*Write the polynomials in standard form.*
$2x + 3\overline{)-4x^3 + 0x^2 + 3x - 15}$	*Write in long division form. Use $0x^2$ as a placeholder for the x^2 term.*
$\begin{array}{r} -2x^2 + 3x - 3 \\ 2x + 3\overline{)-4x^3 + 0x^2 + 3x - 15} \\ -(-4x^3 - 6x^2) \\ \hline 6x^2 + 3x \\ -(6x^2 + 9x) \\ \hline -6x - 15 \\ -(-6x - 9) \\ \hline -6 \end{array}$	*$-4x^3 \div 2x = -2x^2$* *Multiply $-2x^2(2x + 3)$. Subtract.* *Bring down $3x$. $6x^2 \div 2x = 3x$* *Multiply $3x(2x + 3)$. Subtract.* *Bring down -15. $-6x \div 2x = -3$* *Multiply $-3(2x + 3)$. Subtract.* *The remainder is -6.*

$$(3x - 4x^3 - 15) \div (2x + 3) = -2x^2 + 3x - 3 + \frac{-6}{2x + 3}.$$

Remember!

Recall from Chapter 7 that a polynomial in one variable is written in standard form when the degrees of the terms go from greatest to least.

CHECK IT OUT! **Divide.**

5a. $(1 - 4x^2 + x^3) \div (x - 2)$

5b. $(4p - 1 + 2p^3) \div (p + 1)$

THINK AND DISCUSS

1. When dividing a polynomial by a binomial, what does it mean when the remainder is 0?
2. Suppose that the final answer to a polynomial division problem is $x - 5 + \frac{3}{x+2}$. Find an excluded value. Justify your answer.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, show an example.

Know it! Note

Long Division

Polynomials | Whole numbers

12-6 Exercises

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GUIDED PRACTICE

Divide.

SEE EXAMPLE 1 p. 913

1. $(4x^2 - x) \div 2x$
2. $(16a^4 - 4a^3) \div 4a$
3. $(21b^2 - 14b + 24) \div 3b$
4. $(18r^2 - 12r + 6) \div (-6r)$
5. $(6x^3 + 12x^2 + 9x) \div 3x^2$
6. $(5m^4 + 15m^2 - 10) \div 5m^3$

SEE EXAMPLE 2 p. 914

7. $\dfrac{2x^2 - x - 3}{x + 1}$
8. $\dfrac{a^2 - a - 12}{a - 4}$
9. $\dfrac{6y^2 + 11y - 10}{3y - 2}$
10. $\dfrac{t^2 - 6t + 8}{t - 4}$
11. $\dfrac{x^2 + 16x + 15}{x + 15}$
12. $\dfrac{p^2 - p - 20}{p + 4}$

Divide using long division.

SEE EXAMPLE 3 p. 915

13. $(c^2 + 7c + 12) \div (c + 4)$
14. $(3s^2 - 12s - 15) \div (s - 5)$
15. $\dfrac{x^2 + 5x - 14}{x + 7}$
16. $\dfrac{x^2 + 4x - 12}{x - 2}$

SEE EXAMPLE 4 p. 916

17. $(a^2 + 4a + 3) \div (a + 2)$
18. $(2r^2 + 11r + 5) \div (r - 3)$
19. $(n^2 + 8n + 15) \div (n + 4)$
20. $(2t^2 - t + 4) \div (t - 1)$
21. $(8n^2 - 6n - 7) \div (2n + 1)$
22. $(b^2 - b + 1) \div (b + 2)$

SEE EXAMPLE 5 p. 916

23. $(3x - 2x^3 - 10) \div (3 + x)$
24. $(3p^3 - 2p^2 - 4) \div (p - 2)$
25. $(m^2 + 2) \div (m - 1)$
26. $(3x^2 + 4x^3 - 5) \div (5 + x)$
27. $(4k^3 - 2k - 8) \div (k + 1)$
28. $(j^3 + 6j + 2) \div (j + 4)$

PRACTICE AND PROBLEM SOLVING

Divide.

29. $(9t^3 + 12t^2 - 6t) \div 3t^2$
30. $(5n^3 - 10n + 15) \div (-5n)$
31. $(-16p^4 + 4p^3 + 8) \div 4p^3$
32. $\dfrac{4r^2 - 9r + 2}{r - 2}$
33. $\dfrac{8t^2 + 2t - 3}{2t - 1}$
34. $\dfrac{3g^2 + 7g - 6}{g + 3}$

Independent Practice	
For Exercises	See Example
29–31	1
32–34	2
35–38	3
39–41	4
42–44	5

Extra Practice
Skills Practice p. S27
Application Practice p. S39

Divide using long division.

35. $\left(x^2 - 5x + 6\right) \div (x - 2)$

36. $\left(2m^2 + 8m + 8\right) \div (m + 2)$

37. $\left(6a^2 + 7a - 3\right) \div (2a + 3)$

38. $\left(3x^2 - 10x - 8\right) \div (x - 4)$

39. $\left(3x^2 - 2x + 6\right) \div (x - 2)$

40. $\left(2m^2 + 5m + 8\right) \div (m + 1)$

41. $\left(6x^2 - x - 3\right) \div (2x - 1)$

42. $\left(2m^3 - 4m - 30\right) \div (2m - 10)$

43. $\left(6t^3 + 21t + 9\right) \div (3t + 9)$

44. $\left(p^4 - 7p^2 + p + 1\right) \div (p - 3)$

45. Multi-Step Find the value of n, so that $x - 4$ is a factor of $x^2 + x + n$.

Geometry The area of each of three rectangles is $2x^2 - 3x - 2$ cm². Below are the different widths of the rectangles. Find each corresponding length.

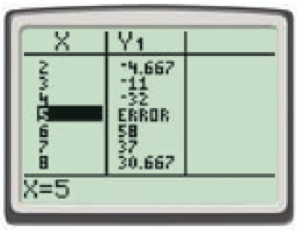
46. $x - 2$ **47.** $x + 1$ **48.** $2x + 1$

49. Graphing Calculator Use the table of values for $f(x) = \dfrac{x^2 + 3x + 4}{x - 5}$ to answer the following.

a. Describe what is happening to the values of y as x increases from 2 to 4.

b. Describe what is happening to the values of y as x increases from 6 to 8.

c. Explain why there is no value in the y column when x is 5.



50. Estimation Estimate the value of $\dfrac{x^2 + 10x + 25}{x^2 - 25} \div \dfrac{x^4 - 4x^3 - 45x^2}{x^2 - 14x + 45}$ for $x = 2.88$.

LINK Solar Energy

NASA is researching unmanned solar-powered aircraft. In the future, these aircraft may be able to stay in the air indefinitely.

51. Solar Energy The greater the area of a solar panel, the greater the number of watts of energy produced. The area of two solar panels A and B, in square meters, can be represented by $A = m^2 + 3m + 2$ and $B = 2m + 2$. Divide the polynomials to find an expression that represents the ratio of the area of A to the area of B.

52. ERROR ANALYSIS Two students attempted to divide $\dfrac{4x^2 - 6x + 12}{-2x}$. Which is incorrect? Explain the error.

A

$$\frac{4x^2 - 6x + 12}{-2x}$$

$$\frac{\overset{2}{\cancel{4}}\cancel{x^2}^{1}}{-\underset{1}{\cancel{2x}}} - \frac{\overset{3}{\cancel{6x}}}{-\underset{1}{\cancel{2x}}} + \frac{\overset{6}{\cancel{12}}}{-\underset{1}{\cancel{2}}x}$$

$$-2x + 3 - \frac{6}{x}$$

B

$$\frac{4x^2 - 6x + 12}{-2x}$$

$$\frac{\overset{2}{\cancel{4}}\cancel{x^2}^{1}}{-\underset{1}{\cancel{2x}}} - \frac{\overset{3}{\cancel{6x}}}{-\underset{1}{\cancel{2x}}} + \frac{\overset{6}{\cancel{12}}}{-\underset{1}{\cancel{2}}x}$$

$$-2x - 3 - \frac{6}{x}$$

MULTI-STEP TEST PREP

53. This problem will prepare you for the Multi-Step Test Prep on page 926.

Jonathan continues to study lenses and uses the equation $\frac{1}{y} = \frac{x - 12}{12x}$.

a. Jonathan wants to write y, the distance of the image from the lens, as a function of x, the distance of the object from the lens. What is the equation solved for y?

b. Use a graphing calculator to create a table of values for the function $y(x)$. For which value of x is the function undefined?

54. **Write About It** When dividing a polynomial by a binomial, what does it mean when there is a remainder?

55. **Critical Thinking** Divide $2x + 3\overline{)2x^2 + 7x + 6}$. Find a value for each expression by substituting 10 for x in the original problem. Repeat the division. Compare the results of each division.

56. **Write About It** Is $3x + 2$ a factor of $3x^2 + 14x + 8$? Explain.

TEST PREP

57. Which expression has an excluded value of $-\frac{1}{2}$?

Ⓐ $\frac{4x^2 - 2x - 2}{4x - 2}$ Ⓑ $\frac{4x^2 - 2x - 2}{2x - 4}$ Ⓒ $\frac{4x^2 - 2x - 2}{4x + 2}$ Ⓓ $\frac{4x^2 - 2x - 2}{2x + 4}$

58. Find $(x^2 - 1) \div (x + 2)$.

Ⓕ $x - 2 + \frac{-5}{x + 2}$ Ⓖ $x - 2 + \frac{3}{x + 2}$ Ⓗ $x + 2 + \frac{-5}{x - 2}$ Ⓙ $x + 2 + \frac{3}{x - 2}$

59. Which expression is a factor of $x^2 - 4x - 5$?

Ⓐ $x - 1$ Ⓑ $x + 1$ Ⓒ $x - 4$ Ⓓ $x + 5$

60. Which of the following expressions is equivalent to $(x^3 + 2x^2 + 3x + 1) \div (x - 1)$?

Ⓕ $x^2 + 3x + 6 + \frac{7}{x - 1}$ Ⓗ $x^2 + 3x + 6 + \frac{-5}{x - 1}$

Ⓖ $x^2 + x + 2 + \frac{-1}{x - 1}$ Ⓙ $x^2 + x + 2 + \frac{3}{x - 1}$

CHALLENGE AND EXTEND

Divide. Simplify your answer.

61. $(6x^3y - x^2 + 4xy^2) \div (2x^2y)$

62. $(x^3 - 1) \div (x - 1)$

63. $(x^3 + 2x^2 - x - 2) \div (x^2 - 1)$

64. $(x^3 + 8) \div (x + 2)$

65. **Geometry** The base of a triangle is $2x + 4$ m and the area is $2x^2 + 5x + 2$ m^2. How much longer is the base than the height?

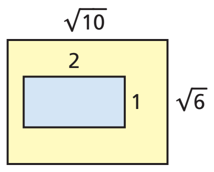
66. **Geometry** The formula for finding the volume of a cylinder is $V = BH$, where B is the area of the base of the cylinder and H is the height.

a. Find the height of the cylinder given that $V = \pi(x^3 + 4x^2 + 5x + 2)$ and $B = \pi(x^2 + 2x + 1)$.

b. Find an expression for the radius of the base.

SPIRAL REVIEW

67. Find the probability that a point within the boundaries of the larger rectangle is in the shaded region. Express your answer as a simplified radical expression. *(Lesson 11-8)*



Multiply. Write each product in simplest form. *(Lesson 11-8)*

68. $3\sqrt{3} \cdot \sqrt{6}$

69. $\sqrt{5}(6 - \sqrt{10})$

70. $(\sqrt{3} + 2)(\sqrt{3} + 5)$

Multiply. Simplify your answer. *(Lesson 12-4)*

71. $(x^2 + 4x + 3) \cdot \frac{8}{2(x + 3)}$

72. $\frac{9xy^2}{2x^3} \cdot \frac{8y}{3x^4}$

73. $\frac{2k^2 + 4k^3}{k + 1} \cdot \frac{k^2 + 3k + 2}{2k^2}$

12-7 Solving Rational Equations

Objectives
Solve rational equations.

Identify extraneous solutions.

Vocabulary
rational equation

Why learn this?
Rational equations can be used to find how much time it takes two or more people working together to complete a job. (See Example 3.)

A **rational equation** is an equation that contains one or more rational expressions. If a rational equation is a proportion, it can be solved using the Cross Product Property.

EXAMPLE 1 Solving Rational Equations by Using Cross Products

Solve $\frac{3}{t-3} = \frac{2}{t}$. Check your answer.

$\frac{3}{t-3} = \frac{2}{t}$ *Use cross products.*

$3t = (t-3)(2)$

$3t = 2t - 6$ *Distribute 2 on the right side.*

$t = -6$ *Subtract 2t from both sides.*

Check

$$\frac{3}{t-3} = \frac{2}{t}$$

$\frac{3}{-6-3}$	$\frac{2}{-6}$
$\frac{3}{-9}$	$\frac{2}{-6}$
$-\frac{1}{3}$	$-\frac{1}{3}$ ✓

CHECK IT OUT! **Solve. Check your answer.**

1a. $\frac{1}{n} = \frac{3}{n+4}$ **1b.** $\frac{4}{h+1} = \frac{2}{h}$ **1c.** $\frac{21}{x-7} = \frac{3}{x}$

Some rational equations contain sums or differences of rational expressions. To solve these, you must find the LCD of all the rational expressions in the equation.

EXAMPLE 2 Solving Rational Equations by Using the LCD

Solve $\frac{1}{c} + \frac{3}{2c} = \frac{2}{c+1}$.

Step 1 Find the LCD.

$2c(c+1)$ *Include every factor of the denominators.*

Step 2 Multiply both sides by the LCD.

$$2c(c+1)\left(\frac{1}{c} + \frac{3}{2c}\right) = 2c(c+1)\left(\frac{2}{c+1}\right)$$

$$2c(c+1)\left(\frac{1}{c}\right) + 2c(c+1)\left(\frac{3}{2c}\right) = 2c(c+1)\left(\frac{2}{c+1}\right)$$ *Distribute on the left side.*

Step 3 Simplify and solve.

$$2\cancel{c}^1(c+1)\left(\frac{1}{\cancel{c}^1}\right) + \cancel{2c}^1(c+1)\left(\frac{3}{\cancel{2c}^1}\right) = 2c\cancel{(c+1)}^1\left(\frac{2}{\cancel{c+1}^1}\right)$$ *Divide out common factors.*

$$2(c+1) + (c+1)3 = (2c)2$$ *Simplify.*

$$2c + 2 + 3c + 3 = 4c$$ *Distribute and multiply.*

$$5c + 5 = 4c$$ *Combine like terms.*

$$c + 5 = 0$$ *Subtract 4c from both sides.*

$$c = -5$$ *Subtract 5 from both sides.*

Check Verify that your solution is not extraneous.

$$\begin{array}{c|c} \frac{1}{c} + \frac{3}{2c} & = \frac{2}{c+1} \\ \hline \frac{1}{-5} + \frac{3}{2(-5)} & \frac{2}{-5+1} \\ \frac{2}{-10} + \frac{3}{-10} & \frac{2}{-4} \\ -\frac{5}{10} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2}\ \checkmark \end{array}$$

CHECK IT OUT! **Solve each equation. Check your answer.**

2a. $\frac{2}{a+1} + \frac{1}{a+1} = \frac{4}{a}$

2b. $\frac{6}{j+2} - \frac{10}{j} = \frac{4}{2j}$

2c. $\frac{8}{t+3} = \frac{1}{t} + 1$

EXAMPLE 3

PROBLEM SOLVING

Problem-Solving Application

Greg can clean a house in 5 hours. It takes Armin 7 hours to clean the same house. How long will it take them to clean the house if they work together?

1 Understand the Problem

The **answer** will be the number of hours h Greg and Armin need to clean the house.

List the important information:

- Greg cleans the house in 5 hours, so he cleans $\frac{1}{5}$ of the house per hour.
- Armin cleans the house in 7 hours, so he cleans $\frac{1}{7}$ of the house per hour.

2 Make a Plan

The part of the house Greg cleans plus the part of the house Armin cleans equals the complete job. Greg's rate times the number of hours worked plus Armin's rate times the number of hours worked will give the complete time to clean the house. Let h represent the number of hours worked.

$$(\text{Greg's rate})\ h + (\text{Armin's rate})\ h = \text{complete job}$$

$$\frac{1}{5}h + \frac{1}{7}h = 1$$

3 Solve

$$35\left(\frac{1}{5}h + \frac{1}{7}h\right) = 35(1)$$ *Multiply both sides by the LCD, 35.*

$$7h + 5h = 35$$ *Distribute 35 on the left side.*

$$12h = 35$$ *Combine like terms.*

$$h = \frac{35}{12} = 2\frac{11}{12}$$ *Divide by 12 on both sides.*

Greg and Armin working together can clean the house in $2\frac{11}{12}$ hours, or 2 hours 55 minutes.

4 Look Back

Greg cleans $\frac{1}{5}$ of the house per hour and Armin cleans $\frac{1}{7}$ of the house per hour. So, in $2\frac{11}{12}$ hours, Greg cleans $\frac{35}{12} \cdot \frac{1}{5} = \frac{7}{12}$ of the house and Armin cleans $\frac{35}{12} \cdot \frac{1}{7} = \frac{5}{12}$ of the house. Together, they clean $\frac{7}{12} + \frac{5}{12} = 1$ house.

CHECK IT OUT!

3. Cindy mows a lawn in 50 minutes. It takes Sara 40 minutes to mow the same lawn. How long will it take them to mow the lawn if they work together?

When you multiply each side of an equation by the LCD, you may get an extraneous solution. Recall from Chapter 11 that an extraneous solution is a solution to a resulting equation that is not a solution to the original equation.

EXAMPLE 4 Extraneous Solutions

Solve $\frac{x-9}{x^2-9} = \frac{-3}{x-3}$. Identify any extraneous solutions.

Step 1 Solve.

$$(x-9)(x-3) = -3(x^2-9)$$ *Use cross products.*

$$x^2 - 12x + 27 = -3x^2 + 27$$ *Multiply the left side. Distribute −3 on the right side.*

$$4x^2 - 12x + 27 = 27$$ *Add $3x^2$ to both sides.*

$$4x^2 - 12x = 0$$ *Subtract 27 from both sides.*

$$4x(x-3) = 0$$ *Factor the quadratic expression.*

$$4x = 0 \text{ or } x - 3 = 0$$ *Use the Zero Product Property.*

$$x = 0 \text{ or } x = 3$$ *Solve for x.*

Step 2 Find extraneous solutions.

$$\begin{array}{c|c} \frac{x-9}{x^2-9} & \frac{-3}{x-3} \\ \hline \frac{0-9}{0^2-9} & \frac{-3}{0-3} \\ \frac{-9}{-9} & \frac{-3}{-3} \\ 1 & 1\ \checkmark \end{array} \qquad \begin{array}{c|c} \frac{x-9}{x^2-9} & \frac{-3}{x-3} \\ \hline \frac{3-9}{3^2-9} & \frac{-3}{3-3} \\ \frac{-6}{0} & \frac{-3}{0} \end{array}$$

Because both $\frac{-6}{0}$ and $\frac{-3}{0}$ are undefined, 3 is not a solution.

The only solution is 0, so 3 is an extraneous solution.

Helpful Hint

Extraneous solutions may be introduced by squaring both sides of an equation or by multiplying both sides of an equation by a variable expression.

CHECK IT OUT!

Solve. Identify any extraneous solutions.

4a. $\frac{3}{x-7} = \frac{x-2}{x-7}$ **4b.** $\frac{x+1}{x-2} = \frac{4}{x-3}$ **4c.** $\frac{9}{x^2+2x} = \frac{6}{x^2}$

THINK AND DISCUSS

1. Why is it important to check your answers to rational equations?

2. For what values of x are the rational expressions in the equation $\frac{x}{x-3} = \frac{2}{x+3}$ undefined?

3. Explain why some rational equations, such as $\frac{x}{x-4} = \frac{4}{x-4}$, have no solutions.

4. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write the solution and check.

Solving Rational Equations

Solve by using cross products.	Solve by using the LCD.
$\frac{3}{x} = \frac{2}{x+1}$	$\frac{7}{x-1} - \frac{4}{x-1} = \frac{6}{x}$

12-7 Exercises

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GUIDED PRACTICE

1. Vocabulary A(n) ___?___ contains one or more rational expressions. (*extraneous solution* or *rational equation*)

Solve. Check your answer.

SEE EXAMPLE 1
p. 920

2. $\frac{3}{x+4} = \frac{2}{x}$ **3.** $\frac{5}{s-6} = \frac{4}{s}$ **4.** $\frac{20}{p+100} = \frac{-10}{2p}$

5. $\frac{4}{j} = \frac{1}{j+2}$ **6.** $\frac{3}{x-4} = \frac{9}{x-2}$ **7.** $\frac{6}{2x-1} = 3$

SEE EXAMPLE 2
p. 920

8. $\frac{6}{x} - \frac{5}{x} = \frac{1}{3}$ **9.** $\frac{a}{9} + \frac{1}{3} = \frac{2}{5}$ **10.** $\frac{3}{x+1} = \frac{2}{x} + \frac{3}{x}$

11. $\frac{8}{d} = \frac{1}{d+2} - \frac{3}{d}$ **12.** $\frac{3}{s-6} = \frac{4}{s} + \frac{1}{2s}$ **13.** $\frac{7}{r} + \frac{2}{r-1} = \frac{-1}{2r}$

14. $\frac{3}{a-4} = \frac{a}{a-2}$ **15.** $\frac{r}{2} - \frac{2}{r} = \frac{5}{6}$ **16.** $\frac{6}{n} = \frac{7}{n^2} - 1$

17. $\frac{4}{x+1} = x - 2$ **18.** $\frac{5}{a^2} = \frac{-4}{a} + 1$ **19.** $\frac{1}{p} = \frac{-3}{p^2} + 2$

SEE EXAMPLE 3
p. 921

20. Painting Summer can paint a room in 3 hours. Louise can paint the room in 5 hours. How many hours will it take to paint the room if they work together?

21. Technology Lawrence's old robotic vacuum can clean his apartment in $1\frac{1}{2}$ hours. His new robotic vacuum can clean his apartment in 45 minutes. How long will it take both vacuums, working together, to clean his apartment?

SEE EXAMPLE 4
p. 922

Solve. Identify any extraneous solutions.

22. $\frac{3}{c-4} = \frac{c-1}{c-4}$ **23.** $\frac{w+3}{w^2-1} - \frac{2w}{w-1} = 1$ **24.** $\frac{3x-7}{x-5} + \frac{x}{2} = \frac{8}{x-5}$

PRACTICE AND PROBLEM SOLVING

Independent Practice	
For Exercises	See Example
25–28	1
29–36	2
37	3
38–41	4

Extra Practice

Skills Practice p. S27
Application Practice p. S39

Solve. Check your answer.

25. $\frac{8}{x-2} = \frac{2}{x+1}$ **26.** $\frac{12}{3n-1} = \frac{3}{n}$ **27.** $\frac{x}{x+4} = \frac{x}{x-1}$ **28.** $\frac{9}{x+5} = \frac{4}{x}$

29. $\frac{6}{s} - \frac{2}{s} = 5$ **30.** $\frac{1}{2x} + \frac{1}{4x} = \frac{7}{8x}$ **31.** $\frac{7}{c} - \frac{2}{c} = \frac{4}{c-1}$ **32.** $\frac{9}{m} - \frac{3}{2m} = \frac{15}{m}$

33. $\frac{3}{x^2} = \frac{2}{x}$ **34.** $\frac{r}{3} - 3 = -\frac{6}{r}$ **35.** $\frac{6}{x^2} = \frac{1}{2} + \frac{1}{2x}$ **36.** $\frac{8}{3x^2} = \frac{2}{x} - \frac{1}{3}$

37. Mel can carpet a floor in 10 hours. Sandy can carpet the same floor in 15 hours. How many hours will it take them to carpet the floor if they work together?

Solve. Identify any extraneous solutions.

38. $\frac{5x}{x-3} = 8 + \frac{15}{x-3}$ **39.** $\frac{3t}{t-3} = \frac{t+4}{t-3}$

40. $\frac{2}{x} = \frac{x+1}{x^2-1}$ **41.** $\frac{1}{x} = \frac{x-4}{x^2-16}$

42. Multi-Step Clancy has been keeping his free throw statistics. Use his data to write the ratio of the number of free throws Clancy has made to the number of attempts.

Clancy's Free Throws

Attempts	Made
45	39

a. What percentage has he made?

b. Write and solve an equation to find how many free throws f Clancy would have to make in a row to improve his free-throw percentage to 90%. (*Hint:* Clancy needs to make f more free throws in f more attempts.)

43. Travel A passenger train travels 20 mi/h faster than a freight train. It took the passenger train 2 hours less time than the freight train to travel 240 miles. The freight train took t hours. Copy and complete the chart. Then find the rate of the freight train.

	Distance	Rate	Time
Passenger train	240	$\frac{240}{t-2}$	
Freight train	240		t

Suspecting that just such a word problem might be on the algebra midterm, Gary came prepared.

44. Pipe A fills a storage tank with a certain chemical in 12 hours. Pipe B fills the tank in 18 hours. How long would it take both pipes to fill the tank?

MULTI-STEP TEST PREP

45. This problem will prepare you for the Multi-Step Test Prep on page 926.

Blanca sets up a lens with a focal length f of 15 cm and places a candle 24 cm from the lens. She knows that $\frac{1}{f} = \frac{1}{x} + \frac{1}{y}$ where x is the distance of the object from the lens and y is the distance of the image from the lens.

a. Write the equation using the given values.

b. For the values of f and x given above, how far will the image appear from the lens?

c. How will distance between the image and the lens be affected if Blanca uses a lens with a focal length of 18 cm?

46. **Critical Thinking** Can you cross multiply to solve all rational equations? If so, explain. If not, how do you identify which ones can be solved using cross products?

47. **Write About It** Solve $\frac{1}{x} + \frac{3}{x} = \frac{3}{x-1}$. Explain each step and why you chose the method you used.

TEST PREP

48. Which value is an extraneous solution to $\frac{x}{x+4} - \frac{4}{x-4} = \frac{x^2+16}{x^2-16}$?

Ⓐ -16 Ⓑ -4 Ⓒ 4 Ⓓ 16

49. Which is a solution to $\frac{x+2}{x-3} - \frac{1}{x} = \frac{3}{x^2-3x}$?

Ⓕ -1 Ⓖ 0 Ⓗ 1 Ⓙ 3

50. What are the solutions of $\frac{5}{x^2} = \frac{1}{3} + \frac{2}{3x}$?

Ⓐ -3 and 5 Ⓑ -3 and 2 Ⓒ 2 and 3 Ⓓ 3 and -5

CHALLENGE AND EXTEND

51. Below is a solution to a rational equation. Use an algebraic property to justify each step.

Solve $\frac{3}{x} = \frac{6}{x+4}$.

Statements	Reasons
a. $3(x+4) = 6x$	
b. $3x + 12 = 6x$	
c. $12 = 3x$	
d. $4 = x$	

52. For what value of a will the equation $\frac{x+4}{x-a} = \frac{7}{x-a}$ have no solution?

53. Luke, Eddie, and Ryan can do a job in 1 hour and 20 minutes if they work together. Working alone, it takes Ryan 1 hour more to do the job than it takes Luke, and Luke does the job twice as fast as Eddie. How much time would it take each to do the job working alone?

SPIRAL REVIEW

Identify which lines are parallel and which lines are perpendicular. *(Lesson 5-9)*

54. $y = \frac{1}{3}x;\ y = 3x + 1;\ y = 3x - 1$

55. $y = -2x;\ y = 2x - 2;\ y = \frac{1}{2}x + 4$

56. $y = -x - 3;\ y = x - 2;\ y = x + 3$

57. $y = -\frac{2}{3}x + 2;\ y = \frac{3}{2}x + 3;\ y = -\frac{3}{2}x - 1$

Solve each equation. Check your answer. *(Lesson 11-9)*

58. $2\sqrt{x} = 24$

59. $\sqrt{x+15} = \sqrt{4x}$

60. $\sqrt{2-x} = x$

Graph each rational function. *(Lesson 12-2)*

61. $y = \frac{4}{x}$

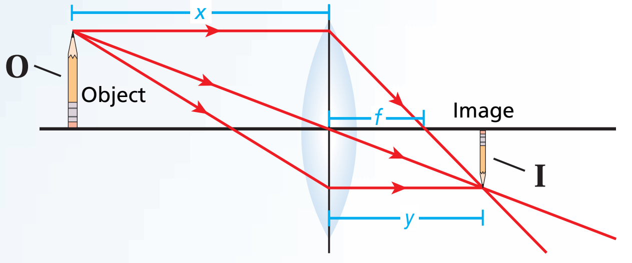
62. $y = \frac{2}{x+1}$

63. $y = -\frac{1}{x} + 3$

MULTI-STEP TEST PREP

Operations with Rational Expressions and Equations

An Upside-Down World Jamal is studying lenses and their images for a science project. He finds in a science book that a magnifying glass can be used to project upside-down images on a screen. The equation $\frac{1}{f} = \frac{1}{x} + \frac{1}{y}$ relates the focal length of the lens f, the distance of the object from the lens x, and the distance of the image from the lens y. The focal length of Jamal's lens is 10 cm.



1. Solve the given equation for y using the given value of f.
2. Jamal experiments with a candle, the lens, and a screen. Given that the focal length remains constant, use a table for the x-values 0, 2, 4, 6, 8, 10, 12, 14, and 16 cm. For which x-values are the y-values positive?
3. Graph the function $y(x)$. Label the axes.

Magnification for images is the ratio of the height of the image to the height of the object. This is also equal to the ratio of the distance between the image and the lens and the distance between the object and the lens: $M = \frac{I}{O} = \frac{y}{x}$. I is the height of the image, O is the height of the object, y is the distance of the image from the lens, and x is the distance of the object from the lens.

4. If the height of a candle is 15 cm and the projected image of that candle is 37.5 cm, what is the magnification of the lens?
5. As Jamal moves the candle further from the lens (increases x), and the distance between the lens and the screen decreases (y decreases), does the magnification M stay the same, increase, or decrease?

READY TO GO ON?

CHAPTER 12

SECTION 12B

Quiz for Lessons 12-4 Through 12-7

12-4 Multiplying and Dividing Rational Expressions

Multiply. Simplify your answer.

1. $\frac{n+3}{n-5} \cdot (n^2 - 5n)$

2. $\frac{8}{2x+6} \cdot (x^2 + 6x + 9)$

3. $\frac{5a^2b^3}{ab^5} \cdot \frac{2a^4bc^5}{20c}$

4. $\frac{6xy^2}{2x^2y^6} \cdot \frac{6x^4y^4}{9x^3}$

5. $\frac{3h^3 - 6h}{10g^2} \cdot \frac{4g}{gh^2 - 2g}$

6. $\frac{m^2 + m - 2}{m^2 - 2m - 8} \cdot \frac{m^2 - 8m + 16}{3m - 3}$

Divide. Simplify your answer.

7. $\frac{2}{n^3} \div \frac{n-6}{n^5}$

8. $\frac{2x^2 + 8x + 6}{x} \div \frac{2x^2 + 2x}{x^3 - x^2}$

9. $\frac{8b^3c}{b^2c} \div (4b^2 + 4b)$

12-5 Adding and Subtracting Rational Expressions

Add or subtract. Simplify your answer.

10. $\frac{15}{2p} - \frac{13}{2p}$

11. $\frac{3m^2}{4m^5} + \frac{5m^2}{4m^5}$

12. $\frac{x^2 + 8x}{x-2} - \frac{3x + 14}{x-2}$

13. $\frac{2t}{4t^2} + \frac{2}{t}$

14. $\frac{m^2 - m - 2}{m^2 + 6m + 5} - \frac{2}{m+5}$

15. $\frac{4x}{x-2} + \frac{3x}{2-x}$

16. Julianne competes in a biathlon that consists of a 25-mile running leg and a 45-mile biking leg. Julianne averages 3 times the rate of speed on her bike that she does on her feet. Let r represent Julianne's running rate of speed. Write and simplify an expression, in terms of r, that represents the time it takes for Julianne to complete both legs of the race. Then determine how long it will take Julianne to complete the race if she runs an average of 10 mi/h.

12-6 Dividing Polynomials

Divide.

17. $(6d^2 + 4d) \div 2d$

18. $(15x^4 + 3x^3 - x) \div (-3x^2)$

19. $(2x^2 - 7x - 4) \div (2x + 1)$

Divide using long division.

20. $(a^2 + 3a - 10) \div (a - 2)$

21. $(4y^2 - 9) \div (2y - 3)$

22. $(2x^2 + 5x - 8) \div (x + 2)$

12-7 Solving Rational Equations

Solve. Identify any extraneous solutions.

23. $\frac{3}{x} = \frac{4}{x-1}$

24. $\frac{1}{x} = \frac{2}{x^2}$

25. $\frac{2}{t} + \frac{4}{3t} = \frac{4}{t+2}$

26. $\frac{4}{n^2} = \frac{7}{n} + 2$

27. $\frac{d+2}{d+8} = \frac{-6}{d+8}$

28. $\frac{x-6}{x^2-6} = \frac{-4}{x-4}$

29. It takes Dustin 2 hours to shovel the snow from his driveway and sidewalk. It takes his sister 3 hours to shovel the same area. How long will it take them to shovel the walk if they work together?

EXTENSION Trigonometric Ratios

Objectives
Find the three basic trigonometric ratios in a right triangle.

Use trigonometric ratios to find missing lengths.

Vocabulary
trigonometric ratios
sine
cosine
tangent

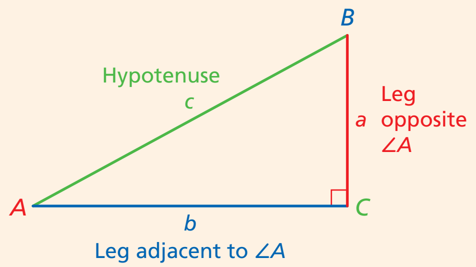
A **trigonometric ratio** is a ratio of the lengths of two sides in a right triangle. Three basic trigonometric ratios are *sine*, *cosine*, and *tangent*, abbreviated *sin*, *cos*, and *tan*, respectively.

Trigonometric Ratios

$$\sin A = \frac{\text{leg opposite } \angle A}{\text{hypotenuse}} = \frac{a}{c}$$

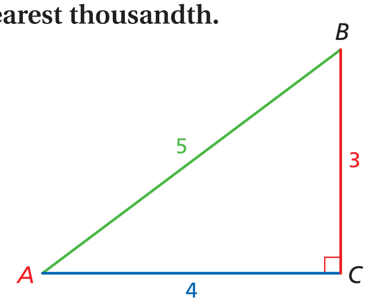
$$\cos A = \frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan A = \frac{\text{leg opposite } \angle A}{\text{side adjacent to } \angle A} = \frac{a}{b}$$



EXAMPLE 1 Finding the Value of a Trigonometric Ratio

Find each trigonometric ratio to the nearest thousandth.



A $\sin A$

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5} = 0.600$$

B $\cos A$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5} = 0.800$$

C $\tan A$

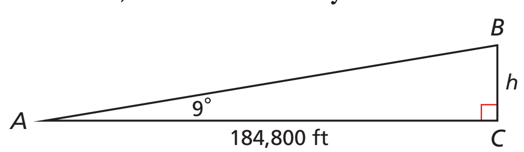
$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4} = 0.750$$

CHECK IT OUT! **1.** Use the figure above to find $\sin B$, $\cos B$ and $\tan B$.

EXAMPLE 2 *Aviation Application*

A plane takes off with a 9° angle of ascent. What is the plane's altitude when it has covered a horizontal distance of 184,800 feet? Round your answer to the nearest foot.

Draw a diagram to model the problem.



$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 9° = \frac{h}{184{,}800}$$

$184{,}800(\tan 9°) = h$ *Multiply both sides by 184,800.*

$29{,}269.4 \approx h$ *Simplify the left side with a calculator.*

The plane's altitude is about 29,269 feet.

Helpful Hint

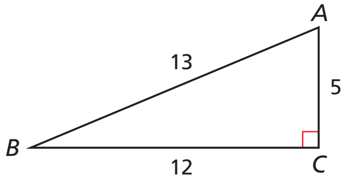
To find trigonometric ratios on a graphing calculator, press SIN, COS, or TAN and then the value of the degree. Be sure your calculator is in Degree mode.

CHECK IT OUT!

2. **Construction** A 14-foot ladder is leaning against a building. The ladder makes a 70° angle with the ground. How far is the base of the ladder from the building? Round your answer to the nearest tenth of a foot.

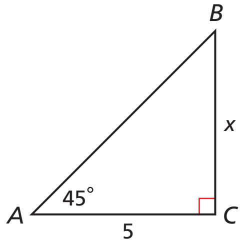
EXTENSION Exercises

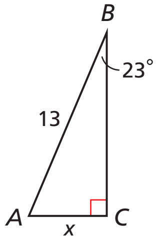
Use the diagram for Exercises 1 and 2.

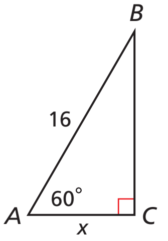


1. Find sin A, cos A, and tan A to the nearest thousandth.
2. Find sin B, cos B, and tan B to the nearest thousandth.

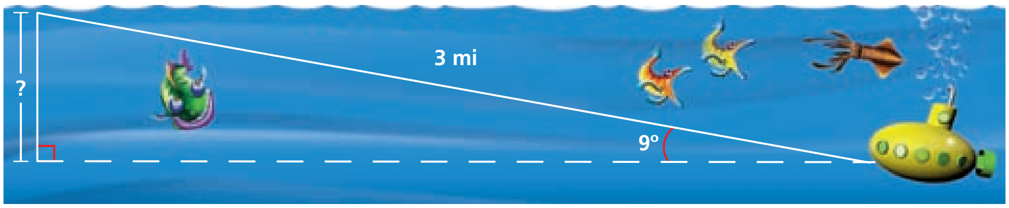
Find the value of x. Round answers to the nearest tenth.

3. 

4. 

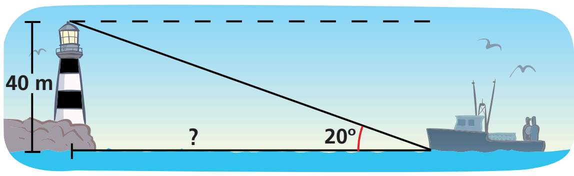
5. 

6. **Diving** If a submarine travels 3 miles while rising to the surface at a 9° angle, how deep was the submarine when it started? Round your answer to the nearest tenth of a mile.



7. **Construction** A wheelchair ramp is to have an angle of 4.5° with the ground. The deck at the top of the ramp is 20 inches above ground level.
   a. Draw a diagram to illustrate the situation.
   b. How long should the ramp be? Round your answer to the nearest tenth of an inch.
   c. How far from the deck should the ramp begin? Round your answer to the nearest tenth of an inch.

8. **Navigation** The top of a lighthouse is 40 meters above sea level. The angle of elevation from a fishing boat to the top of the lighthouse is 20°. How far is the fishing boat from the base of the lighthouse? Round your answer to the nearest tenth of a meter.



9. **Write About It** What would have to be true about the legs of a right triangle for an angle to have a tangent of 1? What would be the measure of that angle?

CHAPTER 12 Study Guide: Review

Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. A(n) ___?___ is an algebraic expression whose numerator and denominator are polynomials.

2. A function whose rule is a quotient of polynomials in which the denominator has a degree of at least 1 is a(n) ___?___ .

3. A(n) ___?___ is an equation that contains one or more rational expressions.

4. A(n) ___?___ is a relationship that can be written in the form $y = \frac{k}{x}$, where k is a nonzero constant.

5. A function is a ___?___ if its graph contains one or more jumps, breaks, or holes.

12-1 Inverse Variation *(pp. 871–877)*

EXAMPLE

■ **Write and graph the inverse variation in which $y = 2$ when $x = 3$.**

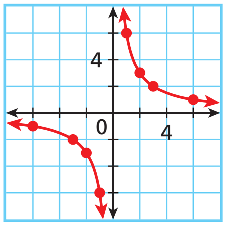
$y = \frac{k}{x}$ *Use the form $y = \frac{k}{x}$.*

$2 = \frac{k}{3}$ *Substitute known values.*

$6 = k$ *Multiply by 3 to find the value of k.*

$y = \frac{6}{x}$ *Substitute 6 for k in $y = \frac{k}{x}$.*

x	−6	−3	−2	−1	0	1	2	3	6
y	−1	−2	−3	−6	und.	6	3	2	1



Make a table of values and plot the points.

EXERCISES

Tell whether each relationship represents an inverse variation. Explain.

6.

x	y
4	−3
−12	1
6	−2

7.

x	y
2	4
6	8
10	12

8. Write and graph the inverse variation in which $y = 4$ when $x = -1$.

9. Write and graph the inverse variation in which $y = \frac{1}{2}$ when $x = 2$.

10. Let $x_1 = 5$, $y_1 = -6$, and $x_2 = 2$. Let y vary inversely as x. Find y_2.

11. The number of fleet vehicles a town can afford to buy varies inversely as the price of each car. If the town can afford 3 cars priced at \$22,000 each, what must the price of a car be in order for the town to purchase 5 of them?

12-2 Rational Functions *(pp. 878–885)*

EXAMPLE

■ **Graph the function $y = \frac{1}{x+1} + 3$.**

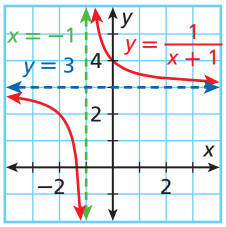
Since the numerator is 1, use the asymptotes and translate $y = \frac{1}{x}$.

Find the asymptotes.

$x = -1$ *$b = -1$*

$y = 3$ *$c = 3$*

Graph the asymptotes. Draw smooth curves to show the translation.



EXERCISES

Identify the excluded values and the vertical and horizontal asymptotes for each rational function.

12. $y = \frac{1}{x+4}$

13. $y = \frac{1}{x+1} + 3$

14. $y = \frac{-5}{2x+6} - 4$

15. $y = \frac{2}{4x-7} + 5$

Graph each function.

16. $y = \frac{3}{x}$

17. $y = \frac{4}{x+5}$

18. $y = \frac{1}{x+4} - 2$

19. $y = \frac{1}{x-6} + 2$

20. A rectangle has an area of 24 cm^2. If x represents the width, then $y = \frac{24}{x}$ represents the length y. Describe the reasonable domain and range values and graph the function.

12-3 Simplifying Rational Expressions *(pp. 886–892)*

EXAMPLE

Simplify the rational expression, if possible. Identify any excluded values.

■ $\frac{x-1}{x^2+2x-3}$

$\frac{x-1}{(x+3)(x-1)}$ *Factor the denominator.*

$\frac{\cancel{x-1}^{1}}{(x+3)\cancel{(x-1)}^{1}}$ *Divide out common factors.*

$\frac{1}{x+3}$ *Simplify.*

Identify the excluded values.

$x^2 + 2x - 3 = 0$ *To find excluded values, set the denominator equal to 0.*

$(x+3)(x-1) = 0$ *Factor.*

$x + 3 = 0$ or $x - 1 = 0$ *Use the Zero Product Property.*

$x = -3$ or $x = 1$ *Solve each equation for x.*

The excluded values are -3 and 1.

EXERCISES

Find any excluded values of each rational expression.

21. $\frac{3}{5p}$

22. $\frac{-2}{r-7}$

23. $\frac{t}{t^2-t}$

24. $\frac{-4}{x^2-4x-5}$

25. $\frac{x-1}{x^2-25}$

26. $\frac{x+4}{x^2-11x+28}$

Simplify each rational expression, if possible. Identify any excluded values.

27. $\frac{7r^2}{21r^3}$

28. $\frac{3k^2}{6k^3-9k^2}$

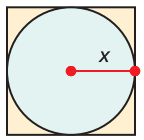
29. $\frac{x+6}{x^2+4x-12}$

30. $\frac{2x-6}{9-x^2}$

31. $\frac{3x+15}{x^2+4x-5}$

32. $\frac{x^2+9x+18}{x^2+x-30}$

33. What is the ratio of the area of the square to the area of the circle?



12-4 Multiplying and Dividing Rational Expressions *(pp. 898–904)*

EXAMPLES

■ **Multiply. Simplify your answer.**

$$\frac{5x^3-10x}{4x}\cdot\frac{6x^2}{7x^4-14x^2}$$

$$\frac{5x(x^2-2)}{4x}\cdot\frac{6x^2}{7x^2(x^2-2)}$$ *Factor.*

$$\frac{5\cancel{x}^1\cancel{(x^2-2)}^1}{\cancel{4}^2\cancel{x}^1}\cdot\frac{\cancel{6}^3\cancel{x^2}^1}{7\cancel{x^2}^1\cancel{(x^2-2)}^1}$$ *Divide out common factors.*

$$\frac{15}{14}$$ *Simplify.*

■ **Divide. Simplify your answer.**

$$\frac{32x^2y^2}{7z}\div\frac{2xy^2}{28xz^3}$$

$$\frac{32x^2y^2}{7z}\cdot\frac{28xz^3}{2xy^2}$$ *Multiply by the reciprocal.*

$$\frac{\cancel{32}^{16}x^2\cancel{y^2}^1}{\cancel{7}^1\cancel{z}^1}\cdot\frac{\cancel{28}^4\cancel{x}^1\cancel{z^3}^{z^2}}{\cancel{2}^1\cancel{x}^1\cancel{y^2}^1}$$ *Divide out common factors.*

$$64x^2z^2$$ *Simplify.*

EXERCISES

Multiply. Simplify your answer.

34. $\frac{2b}{3b-6}\cdot(b^2-b-2)$ **35.** $\frac{4x}{3x+9}\cdot(x^2-9)$

36. $\frac{5ab^2}{2ab}\cdot\frac{3a^2b^2}{a^2b}$ **37.** $\frac{3c}{2d}\cdot\frac{-4c^2d}{8d^2}$

38. $\frac{b+2}{2b^2+12b}\cdot\frac{b^2+2b-24}{b^2-16}$

39. $\frac{n^2-n-12}{n^2+2n-24}\cdot\frac{n^2+3n+2}{n^2-4n-21}$

Divide. Simplify your answer.

40. $\frac{3b+b^2}{b+3b^2}\div\frac{b^2-9}{3b+1}$ **41.** $\frac{7}{y}\div\frac{21}{y^3}$

42. $16n^3\div\frac{4m^2n}{3mn}$ **43.** $\frac{x^2+2x-3}{4x}\div\frac{x^2-4}{x}$

44. A bag contains red and blue marbles. It has 8 more red marbles than blue marbles. Veronica reaches into the bag and selects one marble at random. She sets the marble aside and then selects another. What is the probability that she selects two red marbles?

12-5 Adding and Subtracting Rational Expressions *(pp. 905–911)*

EXAMPLES

Add or subtract. Simplify your answer.

■ $$\frac{7x}{3xy}-\frac{x^2-3x}{3xy}$$

$$\frac{7x-(x^2-3x)}{3xy}$$ *Subtract numerators.*

$$\frac{7x-x^2+3x}{3xy}$$ *Distribute.*

$$\frac{10x-x^2}{3xy}$$ *Simplify.*

■ $$\frac{3w}{w-5}+\frac{4}{w^2-2w-15}$$

$$\frac{3w}{w-5}+\frac{4}{(w-5)(w+3)}$$ *Factor to find the LCD.*

$$\frac{3w(w+3)}{(w-5)(w+3)}+\frac{4}{(w-5)(w+3)}$$ *Write each expression using the LCD.*

$$\frac{3w^2+9w}{(w-5)(w+3)}+\frac{4}{(w-5)(w+3)}$$

$$\frac{3w^2+9w+4}{(w-5)(w+3)}$$ *Add and simplify.*

EXERCISES

Find the LCM of the given expressions.

45. $5a^2b,\ 10ab^2$ **46.** $2x^2-6x,\ 5x-15$

Add or subtract. Simplify your answer.

47. $\frac{b^2}{2b}+\frac{8}{2b}$ **48.** $\frac{3x^2-4}{x^2-2}+\frac{2x}{x^2-2}$

49. $\frac{8p}{p^2-4p+2}-\frac{2}{p^2-4p+2}$

50. $\frac{3b+4}{7-b}-\frac{5-2b}{7-b}$ **51.** $\frac{n-5}{n^2-1}-\frac{n+5}{n^2-1}$

52. $\frac{3}{5m}+\frac{m+2}{10m^2}$ **53.** $\frac{h^2+2h}{h-5}-\frac{3h-1}{5-h}$

54. A scout troop hikes 10 miles to the top of a mountain. Because the return trip is downhill, the troop is able to hike 3 times faster on their way down. Let *r* represent the troop's rate to the mountaintop. Write and simplify an expression for the round-trip hiking time in terms of *r*.

12-6 Dividing Polynomials *(pp. 913–919)*

EXAMPLES

Divide.

- $(6x^4 - 9x^3 + 3x^2) \div 3x$

$\frac{6x^4 - 9x^3 + 3x^2}{3x}$ *Write as a rational expression.*

$\frac{6x^4}{3x} - \frac{9x^3}{3x} + \frac{3x^2}{3x}$ *Divide each term separately.*

$\frac{\cancel{6}^2\cancel{x^4}^{x^3}}{\cancel{3}^1\cancel{x}^1} - \frac{\cancel{9}^3\cancel{x^3}^{x^2}}{\cancel{3}^1\cancel{x}^1} + \frac{\cancel{3}^1\cancel{x^2}^{x}}{\cancel{3}^1\cancel{x}^1}$ *Divide out common factors.*

$2x^3 - 3x^2 + x$ *Simplify.*

- $(4x^3 - 2x^2 + 5x - 1) \div (x - 2)$

$$\begin{array}{r} 4x^2 + 6x + 17 \\ x - 2\,\overline{)\,4x^3 - 2x^2 + 5x - 1} \\ -\,(4x^3 - 8x^2) \\ \hline 6x^2 + 5x \\ -\,(6x^2 - 12x) \\ \hline 17x - 1 \\ -\,(17x - 34) \\ \hline 33 \end{array}$$

$4x^2 + 6x + 17 + \frac{33}{x - 2}$

EXERCISES

Divide.

55. $(4n^3 - 6n^2 - 10n) \div 2n$
56. $(-5x^3 + 10x - 25) \div (-5x^2)$
57. $\frac{x^2 - 8x - 20}{x - 10}$
58. $\frac{6n^2 - 13n - 5}{2n - 5}$
59. $\frac{h^2 - 144}{h - 12}$
60. $\frac{9x^2 + 12x + 4}{3x + 2}$
61. $\frac{m^2 - 2m - 24}{m + 4}$
62. $\frac{3m^2 + m - 4}{m - 1}$

Divide using long division.

63. $(x^2 + 5x + 6) \div (x + 3)$
64. $(x^2 + x - 30) \div (x - 5)$
65. $(p^2 + 2p - 8) \div (p + 4)$
66. $(2x^2 + 3x - 5) \div (x + 2)$
67. $(2n^2 - 3n + 1) \div (n - 5)$
68. $(3b^3 - 4b + 2) \div (b - 2)$
69. $(2x^2 - 4x^3 + 3x) \div (x + 2)$

12-7 Solving Rational Equations *(pp. 920–925)*

EXAMPLE

Solve. Identify any extraneous solutions.

- $\frac{3}{x + 3} - 4 = \frac{2}{x}$

$$x(x+3)\left(\frac{3}{x+3} - 4\right) = x(x+3)\frac{2}{x}$$

$$x(x+3)\left(\frac{3}{x+3}\right) - x(x+3)4 = x(x+3)\frac{2}{x}$$

$$x\cancel{(x+3)}^1\left(\frac{3}{\cancel{x+3}^1}\right) - x(x+3)4 = \cancel{x}^1(x+3)\frac{2}{\cancel{x}^1}$$

$$3x - 4x(x+3) = 2(x+3)$$

$$3x - 4x^2 - 12x = 2x + 6$$

$$0 = 4x^2 + 11x + 6$$

$$0 = (4x+3)(x+2)$$

$4x + 3 = 0$ or $x + 2 = 0$

$x = -\frac{3}{4}$ or $x = -2$ *$x \neq -3$ or 0, so there are no extraneous solutions.*

EXERCISES

Solve. Identify any extraneous solutions.

70. $-4 = \frac{3}{r}$
71. $\frac{6}{7} = \frac{x}{2}$
72. $\frac{6}{b} = \frac{-5}{3 + b}$
73. $\frac{7}{3y^2} = \frac{-2}{y}$
74. $\frac{2}{x - 1} = \frac{3x}{1 - x}$
75. $\frac{2x}{x^2} + \frac{1}{x^2} = 3$
76. $\frac{2}{3} + \frac{4}{x} = \frac{6}{3x}$
77. $-\frac{1}{3x} + \frac{x}{4} = -\frac{1}{12x}$
78. $\frac{2}{3b} + 4 = \frac{1}{3b}$
79. $\frac{4}{x - 4} = \frac{8}{x^2 - 16}$
80. $\frac{5x - 10}{x + 1} = \frac{x}{2}$
81. $\frac{2x}{x + 3} + \frac{x}{4} = \frac{3}{x + 3}$
82. $\frac{9m}{m - 5} = 7 - \frac{3}{m - 5}$
83. $\frac{x - 4}{x^2 - 4} = \frac{-2}{x - 2}$

CHAPTER 12

CHAPTER TEST

1. Write and graph the inverse variation in which $y = -4$ when $x = 2$.
2. The number of posters the Spanish Club can buy varies inversely as the cost of each poster. The club can buy 15 posters that cost \$2.60 each. How many posters can the club buy if they cost \$3.25 each?

Identify the excluded values and the vertical and horizontal asymptotes for each rational function.

3. $y = \dfrac{3}{x+1}$
4. $y = \dfrac{1}{2x-1} + 5$
5. $y = \dfrac{1}{x+3} - 3$

Simplify each rational expression, if possible. Identify any excluded values.

6. $\dfrac{2b}{4b^2}$
7. $\dfrac{x^2-16}{x^2+3x-4}$
8. $\dfrac{b^2-2b-15}{5-b}$
9. $\dfrac{x^2+4x-5}{x^2-25}$

Multiply. Simplify your answer.

10. $\dfrac{-4}{x^2-9} \cdot (x-3)$
11. $\dfrac{2a^2b^2}{5b^3} \cdot \dfrac{15a^2b}{8a^4}$
12. $\dfrac{x^2-x-12}{x^2-16} \cdot \dfrac{x^2+x-12}{x^2+3x+2}$

Divide. Simplify your answer.

13. $\dfrac{4x^2y^4}{3xy^2} \div \dfrac{12xy}{15x^3y^2}$
14. $\dfrac{3b^2-6b}{2b^3+3b^2} \div \dfrac{2b-4}{8b+12}$
15. $\dfrac{x^2+2x-15}{x^2-9} \div \dfrac{x^2-25}{x^2+3x+2}$

Add or subtract. Simplify your answer.

16. $\dfrac{b^2+3}{5b} + \dfrac{4}{5b}$
17. $\dfrac{5x-2}{x^2+2} - \dfrac{2x}{x^2+2}$
18. $\dfrac{2}{3x^2} - \dfrac{5-2x}{3x^2}$
19. $\dfrac{3m}{2m^2} + \dfrac{1}{2m}$
20. $\dfrac{3x}{2x+4} - \dfrac{1}{x+2}$
21. $\dfrac{y^2+4}{y-3} + \dfrac{y^2}{3-y}$

Divide.

22. $(8t^2-2t) \div 2t$
23. $\dfrac{3x^2+2x-8}{x+2}$
24. $\dfrac{k^2-2k-35}{k+5}$

Divide using long division.

25. $(2w^2+5w-12) \div (w+4)$
26. $(x^2-4x+9) \div (x+2)$
27. The area of a rectangle can be modeled by $A(x) = x^3 - 1$. The length is $x - 1$.
    a. Find a polynomial to represent the width of the rectangle.
    b. Find the width when x is 6 cm.

Solve. Identify any extraneous solutions.

28. $\dfrac{2}{x-1} = \dfrac{9}{2x-3}$
29. $\dfrac{3}{n-1} = \dfrac{n}{n+4}$
30. $\dfrac{2}{n+2} = \dfrac{n-4}{n^2-4}$
31. Julio can wash and wax the family car in 2 hours. It takes Leo 4 hours to wash and wax the same car. How long will it take them to wash and wax the car if they work together?

CHAPTER 12

College Entrance Exam Practice

FOCUS ON SAT MATHEMATICS SUBJECT TESTS

The topics covered on each SAT Mathematics Subject Test vary only slightly each time the test is given. Find out the general distribution of test items across topics, and then identify the areas you need to concentrate on while studying.

HOT TIP! To prepare for the SAT Math Subject Tests, start reviewing material several months before your test date. Take sample tests to find the areas you need to focus on. You are not expected to have studied all topics on the test.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

---

1. Which set of ordered pairs satisfies an inverse variation?

(A) $(6, 3)$ and $(8, 4)$

(B) $(2, -3)$ and $(4, 5)$

(C) $(4, -2)$ and $(-5, 10)$

(D) $(2, 6)$ and $(-3, -4)$

(E) $\left(4, \frac{1}{4}\right)$ and $\left(-4, \frac{1}{4}\right)$

---

2. If $\frac{3}{x+3} = \frac{7x}{x^2 - 9}$, what is x?

(A) -12

(B) -3

(C) $-\frac{9}{4}$

(D) $\frac{9}{4}$

(E) 3

---

3. What is h if $(x^3 + 2x^2 - 4x + h) \div (x + 1)$ has a remainder of 15?

(A) -10

(B) -5

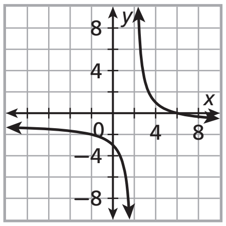
(C) 5

(D) 10

(E) 20

---

4. The graph of which function is shown?



(A) $f(x) = \frac{2}{x+4} + 1$

(B) $f(x) = \frac{4}{x+2} - 1$

(C) $f(x) = \frac{4}{x-2} + 1$

(D) $f(x) = \frac{4}{x-2} - 1$

(E) $f(x) = \frac{2}{x-4} + 1$

---

5. Which function has the same graph as $f(x) = \frac{x^2 - 4x - 5}{x^2 - 3x - 10}$ except at $x = 5$?

(A) $g(x) = \frac{x-1}{x-2}$

(B) $g(x) = \frac{x+1}{x+2}$

(C) $g(x) = \frac{x+1}{(x-5)(x+2)}$

(D) $g(x) = \frac{(x+5)(x-1)}{x+2}$

(E) $g(x) = \frac{(x-5)(x+1)}{x-2}$

CHAPTER 12

TEST TACKLER

Standardized Test Strategies

Multiple Choice: Choose Combinations of Answers

Some multiple-choice test items require selecting a combination of correct answers. The correct response is the most complete option available. To solve this type of test item, determine if each statement is true or false. Then choose the option that includes each correct statement.

EXAMPLE

Which of the following has an excluded value of -5?

I. $\dfrac{5}{x-5}$

II. $\dfrac{8x^2+36x-20}{2(x+5)}$

III. $\dfrac{x^2-10}{5x+25} \cdot \dfrac{5}{x+10}$

IV. $\dfrac{2(x+2)}{2x^2+12x+10}$

(A) I only

(B) II and III

(C) II, III, and IV

(D) III and IV

Look at each statement separately and determine whether it is true. You can keep track of which statements are true in a table.

Statement I

The denominator, $x-5$, equals 0 when $x=5$.

Statement I does not answer the question, so it is false.

Statement II

The denominator, $2(x+5)$, equals 0 when $x=-5$.

Statement II does answer the question, so it is true.

Statement	True/False
I	False
II	True
III	True
IV	True

Statement III

The denominator, $(5x+25)(x+10)$, equals 0 when $x=-5$ or $x=-10$.

Statement III does answer the question, so it is true.

Statement IV

The denominator, $2x^2+12x+10$, can be factored as $2(x+5)(x+1)$. This expression equals 0 when $x=-5$ or $x=-1$.

Statement IV does answer the question, so it is true.

Statements II, III, and IV are all true. Option C is the correct response because it includes all the true statements.

Options B and D contain some of the true statements, but option C is the **most complete** answer.

HOT TIP!

Evaluate all of the statements before deciding on an answer choice. Make a table to keep track of whether each statement is true or false.

Read each test item and answer the questions that follow.

Item A

Which dimensions represent a rectangle that has an area equivalent to the expression $2x^2 + 18x + 16$?

I. $\ell = x + 8$

$w = 2(x + 1)$

II. $\ell = 2x + 2$

$w = \dfrac{x^2 + 3x - 40}{x - 5}$

III. $\ell = x + 2$

$w = \dfrac{(2x + 2)(x + 4)}{1} \cdot \dfrac{3x - 1}{3x^2 - 11x - 4}$

(A) I only

(B) III only

(C) I and II

(D) I, II, and III

1. How do you determine the area of a rectangle?
2. Daisy realized that the area of rectangle I was equivalent to the given area and selected option A as her response. Do you agree? Explain your reasoning.
3. Write a simplified expression for the width of rectangle II.
4. Explain each step for determining the area of rectangle III.
5. If rectangle II has an area equivalent to the given expression, then which options can you eliminate?

Item B

Which expression is undefined for $x = 3$ or $x = -2$?

I. $2x + 12 \cdot \dfrac{4}{2(x - 3)(x + 6)}$

II. $\dfrac{9x - 1}{x^2 + 3}$

III. $\dfrac{x - 2}{(x + 2)(x - 1)}$

IV. $\dfrac{14}{x^2 - x - 6}$

(F) I, III, and IV

(G) I and II

(H) III and IV

(J) I and IV

6. When is an expression undefined?
7. Henry determined that statement I is undefined when $x = 3$. He decides it is an incorrect answer because the expression is defined when $x = -2$. Should he select option H by process of elimination? Explain your reasoning.
8. Make a table to determine the correct response.

Item C

Which rational function has a graph with a horizontal asymptote of $y = 4$?

I. $y = \dfrac{-4}{x}$

II. $y = \dfrac{1}{x} + 4$

III. $y = \dfrac{1}{x - 4}$

IV. $y = -\dfrac{1}{x} + 4$

(A) I and III

(B) II only

(C) I and II

(D) II and IV

9. Where does the horizontal asymptote of the function in statement I occur?
10. Using your answer from Problem 9, which option(s) can you eliminate? Explain your reasoning.
11. Look at the options remaining. Which statement would be best to check next? Explain your reasoning.

CHAPTER 12

STANDARDIZED TEST PREP

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CUMULATIVE ASSESSMENT, CHAPTERS 1–12

Multiple Choice

1. Simplify the expression $4(2d - 1) - 6d$.

Ⓐ $2d$ Ⓒ $-4d + 3$

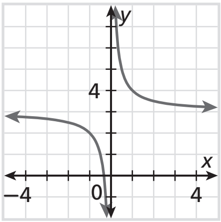
Ⓑ $2d - 4$ Ⓓ $2d - 1$

2. Which equation is the result of solving $3x + 2y = 8$ for y?

Ⓕ $y = \frac{3}{2}x - 4$ Ⓗ $y = -3x + 8$

Ⓖ $y = 3x + 4$ Ⓙ $y = -\frac{3}{2}x + 4$

3. Which function is shown in the graph?



Ⓐ $y = 3x$

Ⓑ $y = \frac{x}{3}$

Ⓒ $y = \sqrt{x} + 3$

Ⓓ $y = \frac{1}{x} + 3$

4. The drama club needs to raise at least \$1400 for a field trip. The club was given \$150 by the school administration. Club members are selling key chains for \$5 each. Which inequality represents the number of key chains k that the drama club needs to sell to go on its field trip?

Ⓕ $150 + 5k \geq 1400$

Ⓖ $5k - 150 \geq 1400$

Ⓗ $5k + 150 \leq 1400$

Ⓙ $150 \leq 5k + 1400$

5. Which expression is NOT equivalent to $\frac{3}{x - 1}$?

Ⓐ $\frac{3x + 6}{x^2 + x - 2}$ Ⓒ $\frac{3x + 3}{x^2 - 1}$

Ⓑ $\frac{3x - 3}{x^2 - 2x + 1}$ Ⓓ $\frac{3x - 3}{x - 1}$

6. Which situation best describes a negative correlation?

Ⓕ The intensity level of an exercise and the number of Calories burned per minute

Ⓖ The amount of time that an electronic game is on and the amount of power remaining in the game's batteries

Ⓗ The height of a tree and the amount of ink in a ballpoint pen

Ⓙ The daytime temperature and the number of people at an ice cream stand

7. Which expression is equivalent to $\frac{3m^2n}{5m} \cdot \frac{20mn}{n^6}$?

Ⓐ $\frac{12m^2}{n^4}$ Ⓒ $12m^2n^3$

Ⓑ $\frac{12m^3}{n^3}$ Ⓓ $\frac{12m}{n}$

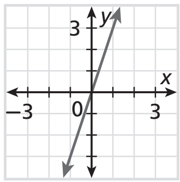
8. Simplify $\frac{3}{x} + \frac{3}{5x}$.

Ⓕ $\frac{1}{x}$ Ⓗ $\frac{18}{5x}$

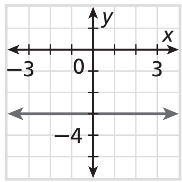
Ⓖ $\frac{9}{5x}$ Ⓙ $\frac{1}{2x}$

9. Which of the following has a slope of -3?

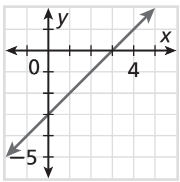
Ⓐ



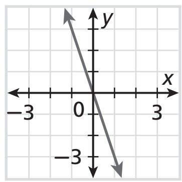
Ⓒ



Ⓑ



Ⓓ



10. What is $(-12x^6 + x) \div (-4x^2)$?

Ⓕ $3x^3 - \frac{1}{4x}$ Ⓗ $3x^5$

Ⓖ $3x^4 - \frac{1}{4x}$ Ⓙ $3x^4 + x$

11. Abe flips two coins. What is the probability of both coins landing heads up?

Ⓐ $\frac{1}{2}$ Ⓒ $\frac{1}{8}$

Ⓑ $\frac{1}{4}$ Ⓓ $\frac{1}{16}$

12. Which is a solution to $\frac{n}{n+2} = \frac{-8}{n}$?

Ⓕ -4 Ⓗ 2

Ⓖ -2 Ⓙ 4

13. Which of the following is equivalent to $\left(\frac{2x^5y^2}{8x}\right)^{-2}$?

Ⓐ $\frac{16}{x^8y^4}$ Ⓒ $\frac{4}{x^4y^2}$

Ⓑ $\frac{x^8}{16y^4}$ Ⓓ $\frac{x^5}{16}$

14. What situation can be modeled by the function $y = \frac{140}{x}$?

Ⓕ The cost of attending a ski trip is $140 for each person who attends.

Ⓖ The area of a rectangle with a width of 140 meters varies directly as its length.

Ⓗ The cost per person of a boat rental is $140 divided by the number of people.

Ⓙ Attendance at this year's concert was 140 people more than at last year's concert.

HOT TIP! **Know the rules for writing gridded-response answers. For example, can you write the fraction $\frac{2}{3}$ as 0.6666 or as 0.67? Different tests may have different rules, so pay close attention to the directions.**

Gridded Response

15. What is the excluded value for the rational expression $\frac{x^2-4}{3x-6}$?

16. What is the next term in the geometric sequence 2000, 1600, 1280, 1024,...?

17. What is the constant of variation if y varies inversely as x and $y = 3$ when $x = 6$?

18. What is the value of $4^0 - (2^{-3})$?

19. Identify the excluded value for $y = \frac{x-4}{x-2}$.

Short Response

20. Mr. Lui wrote $\frac{15-5x}{x^2-9x+18}$ on the board.

a. Explain what kind of expression it is.

b. Simplify the expression. Show your work.

c. Identify any excluded values.

21. Describe the similarities and differences between the graph of $f(x) = x^2 + 4$ and the graph of $g(x) = \frac{1}{2}x^2 + 3$.

22. What are 2 values of b that will make $2x^2 - bx - 20$ factorable? Explain your answer.

23. Brandee makes an hourly wage. In the last pay period, she earned $800 for regular hours and $240 for overtime hours. Her overtime rate of pay is 50% more per hour than her regular rate of pay r. Write and simplify an expression, in terms of r, that represents the number of hours h Brandee worked in the pay period. Show your work.

Extended Response

24. Principal Farley has $200 to pay for some teachers to attend a technology conference. The company hosting the conference is allowing 2 teachers to attend for free. The number of teachers y that can be sent to the conference is given by the function $y = \frac{200}{x} + 2$, where x is the cost per teacher.

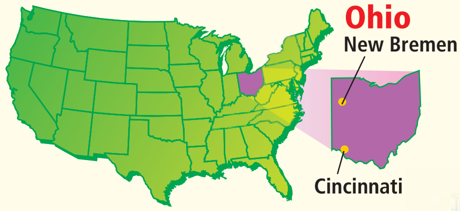
a. Describe the reasonable domain and range values for this function.

b. Identify the vertical and horizontal asymptotes.

c. Graph the function.

d. Give two whole-number solutions to the equation and describe what they mean in the context of this situation.

Real-World CONNECTIONS



Cincinnati Reds

Organized in 1866, the Cincinnati Reds, known then as the Cincinnati Red Stockings, were the first major league baseball team. They now play in a stadium built in 2003 called the Great American Ballpark.

Choose one or more strategies to solve each problem.

1. The table shows the total payroll for the Reds from 2000 to 2005. What percent did the total payroll increase from 2000 to 2005?
2. Assume the payroll percent increase for the next *x* years is the same as from 2004 to 2005. Write an exponential growth function to model this situation.
3. Using the function found in Problem 2, what is the expected total payroll for the Reds in the year 2010?
4. Suppose that at a Cincinnati Reds game, tickets in the blue zone average \$28 and tickets in the green zone average \$20. Suppose twice as many tickets were sold to fans in the green zone as in the blue zone and the total ticket sales for these two zones was \$204,000. Fans bought 14,197 tickets in other zones and sections. How many people had tickets to the game?

Payroll for The Cincinnati Reds

Year	Total Payroll (\$)
2000	44,217,500
2001	48,784,000
2002	45,050,390
2003	59,355,667
2004	46,615,250
2005	61,892,583

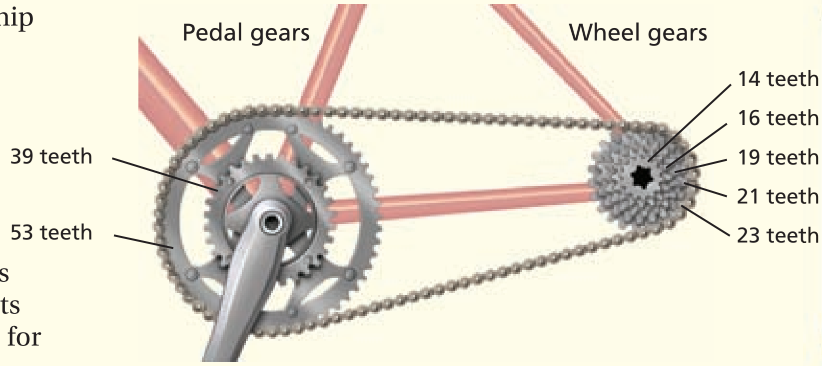
Bicycle Museum of America

The Bicycle Museum of America in New Bremen, Ohio, is home to one of the largest private collections of bicycles and bicycle memorabilia in the world. The collection represents every era, including antique bicycles from the 1800s, balloon-tire classics of the 1940s and 1950s, and banana-seat bikes with high-rise handlebars from the 1960s.

Choose one or more strategies to solve each problem.

1. Alfred Letourner was one of the top racers of his era. On May 17, 1941, at an event sponsored by Schwinn, Letourner rode a bike similar to the one in the photo with a gear ratio of $9\frac{1}{2}$ to 1. The bike only weighed 20 pounds. At this event, Letourner shattered speed records when he rode a mile in 33.05 seconds. At this pace, about how many miles per hour was Letourner riding?

The diagram shows the relationship between the number of teeth on the pedal gear and the number of teeth on the wheel gear of a bicycle. This relationship affects the speed and effort of pedaling. For example, if the pedal gear has 39 teeth and the wheel gear has 17 teeth, then the gear ratio is $\frac{39}{17} = 2.294$. This number represents the number of turns of the wheel for every full turn of the pedal.



2. Which gear combination shown in the diagram would yield the highest number of rotations of the tires to the turn of the pedal? About how many turns of the wheel for every full turn of the pedal can be expected with this combination?

3. If a mountain bike has tires with a diameter of 26 inches, how far, in feet, will a rider travel for each full turn of the pedal if the pedal gear has 39 teeth and the wheel gear has 14 teeth?